Performance-based Pricing Models in Online Advertising: Cost per Click versus Cost per Action

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Abstract

The multibillion-dollar online advertising industry continues to debate whether to use the CPC (cost per click) or CPA (cost per action) pricing model as an industry standard. This article applies the economic framework of incentive contracts to study how these pricing models can lead to risk sharing between the publisher and the advertiser and incentivize them to make efforts that improve the performance of online ads. We find that, compared to the CPC model, the CPA model can better incentivize the publisher to make efforts that can improve the purchase rate. However, the CPA model can cause an adverse selection problem: the winning advertiser tends to have a lower profit margin under the CPA model than under the CPC model. We identify the conditions under which the CPA model leads to higher publisher (or advertiser) payoffs than the CPC model. Whether publishers (or advertisers) prefer the CPA model over the CPC model depends on the advertisers' risk aversion, uncertainty in the product market, and the presence of advertisers with low immediate sales ratios. Our findings indicate a conflict of interest between publishers and advertisers in their preferences for these two pricing models. We further consider which pricing model offers greater social welfare.

Key words: Online advertising, cost-per-click through, cost-per-action, pricing model, incentive, adverse selection, moral hazard.

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1 Introduction

"CPA, or cost per action, is the Holy Grail for targeted advertising."

- Marissa Mayer, President and CEO, *Yahoo!*

The Internet has emerged as an incredibly important advertising medium. According to a recent report, U.S. advertisers spent \$31.7 billion on Internet advertising in 2011, a 22% increase from 2010 (Interactive Advertising Bureau 2012). In the early days of this technology, online advertisers and publishers had simply used a CPM (cost per thousand impressions) model, standard to traditional media advertising, and advertisers paid according to the number of times their advertisement got delivered to consumers. However, the online advertising industry has recently shifted toward performance-based pricing models that tie advertising payments to certain performance metrics. Performance based pricing becomes the most prevalent pricing model since 2006 and approximately 65% of 2011 online advertising revenues were priced on this basis (Interactive Advertising Bureau 2012). The first performance-based pricing model to appear used a *cost per click* (CPC) approach, in which advertisers pay only when viewers click on the advertisement, as invented by Overture (now part of Yahoo!). By 2002, the CPC model had been adopted by both Google and Yahoo! and become the most widely used pricing model in paid search advertising (*The Economist* 2006).

However, the CPC model's current dominance is being challenged by a new performance-based pricing model that relies on CPA (*cost per action*) and calculates advertising payments according to advertiser-specified "actions," such as email sign-ups, downloads, sales leads, or purchases.[1] In 2006, Google attracted media attention when it started to test a CPA model (e.g., Gonsalves 2006; Helft 2007). As the quote that opened this paper reveals, Google regards CPA as the "Holy Grail" of targeted advertising (Gardiner 2007), and many online advertising companies have adopted it, including not only Google (through its Product Listing Ads) and eBay, but also long-time proponents of this model, such as ValueClick and Snap.com. Amazon has also been using the CPA model in its affiliate program (Libai et al. 2003)

The emergence of the CPA model has sparked controversy and debate within the online advertising industry (Cumbrowski 2007; Ezzy 2006; Guanaccia 2006). On one side of the debate, advertisers tend to prefer the CPA model, because the CPC model gives publishers little or no incentive to improve the quality of the clicks delivered (they only seek to drive a high volume of clicks to advertisers). Thus, advertisers worry that consumers who click are not actually interested in the products being sold — a problem exacerbated by the potential for fraudulent clicks by third parties that aim to drive up

[1] Details on how the advertiser defines an "action" and how Google tracks and reports the number of "actions" can be found in Laycock (2007).

advertisers' costs (Wilbur and Zhu 2009). Because the CPA model ties advertising payments to sales of the advertisers' products, publishers must exert some effort to improve the quality of clicks. The CPA model also helps reduce the risk for advertisers, because if an Internet advertisement fails to produce sales, advertisers do not suffer any further financial loss. Therefore, the CPA model is considered to be a preferred model by advertisers, because it shifts the risk almost entirely to publishers and it allows advertisers to easily manage their campaigns' return on investment.

On the other side of the debate stand web publishers, who often prefer the CPC model. They worry that the CPA model gives advertisers minimal incentives to convert clicks into sales, causing a typical moral hazard problem. If an advertising campaign fails and generates no response, the web publisher receives no payment for displaying the advertisement on their web page. Publishers argue that they should be responsible for influencing the consumer, but not closing a deal. Furthermore, some advertisers may take advantage of the CPA arrangement to run a multitude of advertisements that only raise brand awareness, rather than generate immediate sales.[2]

This paper sheds light on this debate over the CPC versus CPA pricing mechanisms. It helps academic researchers and practitioners understand the consequences of adopting one pricing model over the other, as well as the conditions in which each pricing model might perform best. In what circumstances do firms (advertisers or publishers) prefer a particular pricing scheme? Does a CPA model lead to higher purchase rates compared with a CPC model? Does one model produce greater social welfare (defined as the sum of payoffs to all parties) in the online advertising industry?

To the best of our knowledge, this study is the first to investigate the trade-offs between CPC and CPA models. We apply an economic model of incentive contracts to this problem and offer recommendations regarding when advertisers and publishers should adopt either the CPA or the CPC model. Specifically, we solve a game in which different types of advertisers compete in a second-price auction for the right to display their advertisements on one publisher's Web site. First, we assume that the contract goes to the advertiser with the higher bid in the auction, but later we look at more realistic scenario where the publisher chooses the advertiser based on its expected revenue (not necessarily the highest bidder). Then, the winning advertiser and the publisher decide on the levels of their non-contractible efforts to improve the effectiveness of the advertisement. Finally, both parties realize payoffs. We solve the game with both CPC and CPA pricing models, then compare the results and derive several pertinent propositions.

[2]One often-mentioned potential problem with the CPA model is the reliability of the technology that tracks actions generated from a CPA campaign. The CPA model may not be successful if publishers can only rely on advertisers' "truthful" reporting of the actions generated from CPA campaigns. However, recent developments in the tracking technology have enabled publishers and advertisers to overcome this issue. For instance, the publisher often requires the advertiser to install certain program which tracks the actual actions at the advertiser's website and reports them to the publisher (Cumbrowski 2007).

Our model therefore considers the incentive problem for both publishers and advertisers — an issue largely ignored by existing literature on online advertising. We posit that online advertisers and publishers can exert effort to improve the effectiveness of advertising campaigns. For example, advertisers can invest in user interface, easy of navigation, search, and customized landing pages for different keywords; publishers can also invest in user interface with advertising, recommendation and develop better targeting technologies. However, these costly efforts are not contractible and advertisers and publishers would not invest enough unless they have proper incentives to do so.

Moreover, we incorporate two important and realistic features of online advertising that have rarely modeled in prior research. First, we model the effect of delayed response, which is of central concern for both advertisers and publishers but most extant literature has largely overlooked.[3] Delayed response occurs when a consumer who sees the advertiser's offer makes no purchase at that moment but later comes back to the advertiser directly and purchases a product. Such delayed responses can be significant for products that have high value or products that are difficult to be evaluated, such as cars and electronics (Hu 2004). Briggs (2003) reports that an advertiser gets 80 percent of its conversions from these returning consumers. Second, we also allow the possibility of existence of different types of advertisers whose primary goals of advertising campaigns differ (Fulgoni 2009). Some focus on generating a direct and immediate action such as consumer purchase (direct selling advertiser) while others primarily focus on raising awareness about its brand (branding advertiser). By modeling delayed responses and the existence of difference types of advertisers, our analysis reveals that the CPA pricing model increases the possibility that certain types of advertisers win the auction, leading to a potential adverse selection problem.

At first glance, publishers should always prefer the CPC model, and advertisers should always prefer the CPA model. But our results show that in certain conditions, the CPA model leads to higher publisher payoffs than the CPC model and thus, enhance current industry understanding of these two performance-based pricing models. We posit that the CPA model shifts risk away from advertisers, which may cause them to bid more for advertising space. This effect grows even stronger when advertisers are more risk averse and when uncertainty in the product market is higher. In parallel, we identify conditions in which the CPC model produces higher advertiser payoffs than the CPA model. The CPA model increases the probability that a branding advertiser will win the auction, which again creates an adverse selection problem that reduces advertiser payoffs. Finally, we study which pricing model leads to greater social welfare and thus the conditions in which the CPA model is preferable.

[3] Recent paper by Berman (2013) also explores a related issue of performance attribution process where publishers compete to be the last to show an ad prior to conversion.

The remainder of this article proceeds as follows. In Section 2, we review related literature, and then in Section 3, we introduce our basic model. We characterize the equilibrium outcomes for the CPC and CPA pricing schemes and identify the conditions for choosing one pricing scheme over the other in Section 4. In Section 5, we extend our basic model to capture the more realistic situation where the publisher chooses the advertiser based on its expected revenue. Finally, Section 6 concludes this paper.

2 Literature Review

This research contributes to a growing literature on online advertising. Motivated by the real-world models employed by Google and Yahoo!, several analytical studies in economics and marketing have focused on the design of auction mechanisms and advertisers' bidding strategies. Edelman et al. (2007) study the generalized second-price auction mechanisms used in sponsored search advertising and derive many of its properties; in a separate study, Varian (2007) obtains similar results. They all find that the general auction mechanism employed by Google and Yahoo does not have a dominant bidding strategy, but can be reduced to a simple second-price auction under certain conditions. More recently, Athey and Ellison (2011) examine advertisers' bidding strategies, consumers' search strategies, and the division of surplus among consumers, search engines, and advertisers. They find that paid search advertising can provide information about sellers' products to consumers and, thus, provide a welfare benefit by making consumer search more efficient. Agarwal et al. (2010) focus on the new CPA pricing mechanism and study how the CPA mechanism can bias the advertisers' bidding strategies.

There are also several papers which focus mainly on the features of sponsored search advertising. Katona and Sarvary (2010) model the bidding behavior of advertisers and paid ad placements and find an interaction between non-sponsored (or "organic") search results and sponsored search advertising; differences in click-through rates across advertisers can also influence advertisers' bidding behaviors. Wilbur and Zhu (2009) investigate how click fraud influences search engines' revenues in a second-price auction. Feng et al. (2011) compare different mechanisms of ranking advertisers and their bids, whereas Weber and Zheng (2007) build a model of search intermediaries in a vertically differentiated product market and derive advertisers' bids and consumer surplus. While extant research on online advertising mostly take the pricing mechanism as given, we investigate the choice of pricing scheme and its implications on equilibrium behaviors of advertisers and publishers.

Empirical research on online advertising focuses primarily on banner advertising. For example, Sherman and Deighton (2001) use Web site–level data to suggest optimal placements of advertisements. Chatterjee et al. (2003) examine how click-through rates may be influenced by exposure to banner advertisements, and Manchanda et al. (2006) consider the effect of banner advertising on actual purchasing patterns. These findings suggest that the number of exposures, Web sites, and pages all have positive impacts on consumers' purchasing probabilities. More recent empirical studies investigate keyword searches in the context of paid search advertising (Ghose and Yang 2009; Goldfarb and Tucker 2011b; Rutz and Bucklin 2011; Yao and Mela 2011).

Our model follows traditional principal–agent models that recognize moral hazard (Holmstrom 1979; Holmstrom and Milgrom 1987), and this study is one of the first to apply it, together with the economic framework of incentive contracts, to online advertising. We view the CPC or CPA contract between the publisher and the advertiser as a contract that allocates market risks between the parties and that may or may not provide each party with appropriate incentives to make adequate, non-contractible efforts. This new view of the contract between the publisher and the advertiser enables us to find new insights that have important implications for the online advertising industry.

Finally, we note that our paper closely relates to several recent studies of pricing models in online advertising. Hu (2004) is the first paper, which studies online advertising pricing schemes as an optimal contract design problem, but he only compares traditional CPM and CPC models in a monopolistic advertiser–publisher relationship. Asdemir et al. (2012) also compare CPM and CPC models and find several factors that affect the preference of CPM to the CPC using the principal–agent framework. We extend those arguments to the issue of performance-advertising mechanisms (CPC and CPA) under competition. Zhu and Wilbur (2011) study advertisers' bidding strategies in a hybrid auction, in which advertisers can choose a CPM or CPC bid, and derive the unique properties of the mechanism. They find that publishes should offer multiple bid types to advertisers. Liu and Viswanathan (2010) identify conditions under which publishers prefer the CPM model over performance-based models (CPC or CPA). Unlike these studies, we do not study solely the CPM model. Instead, we focus on the incentive problems in performance-based advertising schemes, and therefore, we analyze the trade-offs between CPA and CPC, with a particular focus on the incentive issues (adverse selection and moral hazard) arising under different pricing schemes.

3 Model

We model the advertising contract between multiple online advertisers and an online publisher. Each advertiser sells a product to consumers through the online channel. To boost its sales or brand awareness, an advertiser can launch an online advertising campaign in third party's website or blog (which we call a publisher). The advertiser designs an advertisement and contracts with a publisher, tasking the publisher with delivering the advertiser's advertisement to consumers who visit the publisher's website or blog. Every time the advertisement is delivered to a consumer's browser, the consumer may choose to ignore or click on the advertisement. If he or she clicks, the consumer goes to the advertiser's online store, after which this consumer may make a purchase or leave without purchasing. We define the purchase rate (θ) as the ratio of purchases to clicks.

In the advertising industry, a popular dichotomy differentiates direct response advertising from brand advertising: The former focuses on strategies to drive a particular action, such as purchase, whereas the latter aims to raise awareness and build brand equity (Fulgoni 2009). We assume an advertiser can either be a *direct selling* or a *branding* advertiser. A direct selling advertiser (which we call type D) has a primary goal of generating a direct and immediate action by consumers, such as sale, sign-up, or download, through its advertising campaign. A branding advertiser (type B) instead aims primarily to raise awareness about its brand or build brand equity, which leads to higher future indirect and delayed responses. Of course, the discrete classification of all advertisers into direct selling versus branding advertisers is difficult; most advertising campaigns serve both objectives in practice. Therefore, the classification is based on *relative* terms and the key difference between type D and type B advertisers is whether their advertising goal is relatively to generate a large proportion of direct and immediate sales or a large proportion of delayed responses in the long run.[4]

We consider a stylized model of two advertisers competing for one advertisement slot on the publisher's Web site using a second-price sealed bid auction. This preserves the main incentives of real world CPA and CPC auction while simplifying the analysis significantly (Agarwal et al. 2010; Athey and Levin 2001). The advertisers are heterogeneous in the profits they obtain from each sale (m_i) and the ratio of immediate to total sales (ρ_i). We assume that each advertiser's profit margin m_i is randomly drawn from a uniform distribution on $[0, 1]$. Also, one advertiser is a direct selling, whereas the other is a branding advertiser: $i \in \{B, D\}$. We assume that the direct selling, type D advertiser attains a relatively higher immediate sales ratio of α (i.e., $\rho_D = \alpha$), but that the branding, type B advertiser experiences a relatively lower immediate sales ratio of β (i.e., $\rho_B = \beta$), where $0 < \beta \leq \alpha < 1$. This specification can easily capture the special case in which both advertisers are the same type by setting $\rho_D = \rho_B = \alpha = \beta$.

Incremental efforts for improving purchase rate

Advertisers can greatly influence the purchase rate once those online prospects land through an online advertising campaign. First, the advertiser can affect the purchase rate by improving its online trans-

[4] In reality, the advertisers can exert effort to change the proportion of immediate sales. Our model captures this in a parsimonious way through an adverse selection model with two types of advertisers (who have different immediate sales ratios). In other words, we capture the fact that a certain advertiser lowers its sales ratio through its efforts by the advertiser's type (i.e., it is a branding advertiser).

action process or managing its Web server capacity and bandwidth better. A complicated transaction process or a slow Web server increases consumer inconvenience for check-out and thus reduces immediate purchase rate (Mookerjee 2012, Tillett 2001).[5] Second, the advertiser can improve its purchase rate by having a professional and trustworthy website layout, design or using proper wording (Puscher 2009). Third, setting up customized landing pages and closely linking products to keywords can greatly increase purchase rates (Mitchell 2007). Finally, the advertiser can also use recommendation engines, advanced search and navigation tools to improve stickiness and purchase rates (*Business Wire* 2007). These efforts are costly though, often requiring professional staff or advertising agencies to manage the website. While some of those features can be specified in contract ex ante, most of factors are unobservable to the publisher and are hard to be specified in contract (or at least non-verifiable).[6] We focus on these non-contractible efforts that advertiser i can make and call them e_i.

Similarly, the publisher can affect the purchase rate through an advertising campaign. The key to improving purchase rates is to understand consumer interest and match consumer interest to products.[7] This can be done by various activities such as linking surrounding contents to the product being advertised closely. More importantly, the publisher can automatically match the advertisement to consumers who are most likely to be interested in it by using a targeting technology based on superior knowledge of its consumers' demographics, geographical location, expressed interests, and other information (Beales 2010; Maislin 2001; Needham 1998; Rutz and Bucklin 2011). These efforts, which are rarely specified in the contract between the advertiser and the publisher, represent our main focus, which we refer to as e_p.

Formally, we assume that the purchase rate θ is a linear function of the advertisers' efforts e_i, the publisher's effort e_p, and random noise ε, which is normally distributed with a mean of 0 and a variance of σ^2. The variance σ^2 can be interpreted as sales randomness or risk in the product market. Also, we impose the condition that the purchase rate cannot be lower than 0. Thus,

$$\theta = Max\left[0,\, e_i + e_p + \varepsilon\right], \text{ where } \varepsilon \sim N\left(0, \sigma^2\right). \tag{1}$$

Non-contractible efforts are costly to advertisers and the publisher and become more costly as the

[5] According to the market analysis by TRAC Research, on average \$4,100 of revenues are lost due to website slowdowns as more consumers are becoming increasingly intolerant to slowdowns of web server (http://www.nbcnews.com/technology/technology/).

[6] For example, it is infeasible to contract about the utilization of advertiser's web server ex ante: there can always be unforeseen contingency which prevents the full utilization of the server or causes complete breakdown. It is practically impossible for the publisher to verify in a court that the slowdown of online transaction (which lowers the purchase rate) is due to strategic sabotage of the advertiser.

[7] One can consider the implication of this targeting behavior on consumer behaviors. However, this require a more micro-model of targeting technology, which is beyond the scope of the current research.

total effort level increases. We model the advertisers' cost for incremental efforts with a quadratic cost function, as used widely in research in incentive contracts (e.g., Holmstrom and Milgrom 1987, Hauser et al. 1994, Lafontaine and Slade 1996). Formally, the cost of advertiser i's efforts e_i is $C(e_i) = \frac{e_i^2}{2}$. Similarly, the cost of the publisher's efforts is $C(e_p) = \frac{e_p^2}{2}$.

It is important to note that in practice consumers do not click on every advertisement and thus, it is essential for the publisher to exert sufficient efforts to increase the quantity of clicks. This issue is especially critical for cost per impression (CPM) or display (banner) advertising cases where the payments are based on the number of ads they show during a campaign. However, under the performance-based advertising (CPC and CPA) which we are focused on, the publisher has a strong incentive to increase the quantity of clicks under both CPC and CPA models since they do not receive a payment for just simply showing impressions. The payment is conditional on the clicks (a precondition for both CPC and CPA payment is having a click, because an action can only be realized after a click). Therefore, we assume away the publisher's effort to increase the quantity of clicks and take this effort as given. Our model is only concerned with everything that happens after a click has been generated, and focuses on the efforts that can improve the quality (i.e., purchase rate) of a given click.

Payoffs

We use t to denote the monetary transfer from the winning advertiser to the publisher. The publisher's payoff from "each" click is simply the monetary transfer minus the cost of its efforts, $y_p = t - \frac{e_p^2}{2}$.

Advertiser i obtains a net profit of 0 if it does not win the auction. If it wins, it earns a net profit from each click equal to its profit minus the monetary transfer minus the cost of its efforts,

$$\pi_i = m_i \left(\rho_i + \gamma(1-\rho_i)\right)\theta - t - \frac{e_i^2}{2}, \tag{2}$$

where γ is the time discount rate, which we assume $\gamma = 1$ for simplicity. Because of the randomness of sales in the product market (ε), it is an interesting issue who should bear this market risk and thus, we incorporate risk aversion in the model.[8] We assume that the advertisers have exponential utility functions with a CARA (constant absolute risk aversion) parameter of r, that is, $u(\pi_i) = 1 - exp(-r\pi_i)$. Thus, the advertisers' payoff can be written as the certainty equivalence of their net profit ($CE(\pi)$), which is,

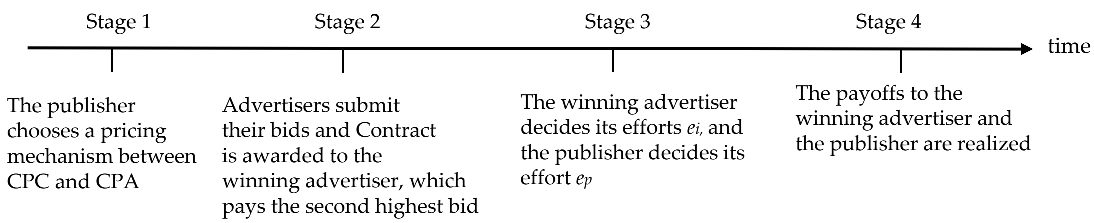
$$y_i = CE(\pi_i) = E(\pi_i) - r\frac{Var(\pi_i)}{2}. \tag{3}$$

[8]There is a large body of literature on firm being risk-averse; for example, Lafontaine and Slade (1999) in franchising setting and Gan et al. (2005) in a supply chain setting. Delegation of control to a risk-averse manager, whose payment is linked to firm performance, may cause the firm to behave in a risk-averse manner (Asplund 2002).

We consider a game in which two advertisers bid on one slot through a second-price sealed bid auction. The timing of the model is following (see Figure 1 below): First, the publisher chooses a pricing mechanism (between CPC and CPA). Second, advertisers submit their bids and the publisher awards the slot (and the contract) to the advertiser with the highest bid, at the price of the second highest bid (i.e., the other advertiser's bid). Later, we look at a more realistic scenario where the publisher chooses the advertiser based on its expected revenue (not necessarily the highest bidder). By assuming a second-price auction, we can focus our analysis on the incentive problems of both the publisher and advertisers. In a second-price auction, a weakly dominant strategy is for advertisers to bid their true value (Vickrey 1961), so we refer to this outcome as the standard result in our analysis.[9]

Third, both the advertiser who wins the slot and the publisher decide the levels of their incremental efforts e_i and e_p. Finally, advertisers and the publisher observe the actual purchase rate and realize their separate payoffs.

Figure 1: Timeline of the game



4 Analysis

We characterize the equilibrium outcomes under two performance-based pricing models: the cost per click (CPC) and the cost per action (CPA). We then compare and investigate the trade-offs between these two models and identify the conditions in which firms (publisher or advertisers) prefer one pricing model over the other, as well as its social welfare implications.

4.1 Cost per click pricing model

In the CPC pricing model, the monetary transfer between the publisher and the winning advertiser is a flat fee of t_c for *each* click. Each advertiser bids on the amount of a payment t_c per click, and this bid, $b(m_i, \rho_i)$, is a function of advertiser i's profit margin m_i and the immediate sales ratio ρ_i. We assumed that the reserve price for the publisher is exogenously given as zero, which implies that the

[9]However, under multiple auction case (i.e., advertising slots), a second-price auction can diverge from the Vickery-Clarke-Groves mechanism, and true-valuation bidding is generally not an equilibrium outcome (Edelman et al. 2007).

publisher always accepts a non-negative winning bid $b(m_i, \rho_i) > 0$, which results in a contract between the two parties. Hereafter, we assume that $r\sigma^2 < 1$, which guarantees non-negative bids from both advertisers in any pricing mechanism.[10]

From Equation (3), we can obtain advertiser i's payoff if it wins the auction, which is $y_i = m_i(e_i + e_p) - t_c - \frac{e_i^2}{2} - \frac{r\sigma^2 m_i^2}{2}$. Also, the publisher's payoff is simply $y_p = t_c - \frac{e_p^2}{2}$.

Lemma 1. *In the CPC pricing model,*

1. *Advertiser i with m_i and ρ_i, bids $b^{CPC}(m_i, \rho_i) = \frac{1}{2}\left(1 - r\sigma^2\right) m_i^2$. Moreover, the probability that type B advertiser wins the auction is $E\left[Pr(w = B)\right]^{CPC} = \frac{1}{2}$.*

2. *The equilibrium effort levels for the winning advertiser and the publisher to improve the purchase rate are $e_i^{CPC} = m_i$ and $e_p^{CPC} = 0$.*

We provide all the proofs in the Appendix. Lemma 1 suggests that the advertiser's profit margin (m_i) has a positive effect on its submitted bid while its bid is negatively influenced by its risk aversion parameter (r) and the level of market risk (σ^2). The advertiser assumes all the market risk under the CPC model since the winning advertiser must pay the publisher for each and every click, even when those clicks fail to lead to any purchases of the winning advertiser's product. Thus, an advertiser with a higher risk aversion parameter and more market risk submits a lower bid.

However, advertisers' bids are independent of their types (i.e., immediate sales ratio); both advertisers have the same bidding strategy. As a result, the winning advertiser is simply the one with greater marginal profit. Therefore, the CPC model provides a level playing field for both types of advertisers, and both of them have an equal probability of winning the auction.

Lemma 1 further shows that the optimal level of the advertiser's effort is independent of its bid or payment (t_c), because when a consumer clicks and enters the advertiser's Web site, the cost of that click becomes a sunk cost. Thus, the advertiser undertake more incremental efforts to increase its purchase rate when it has a greater profit margin per purchase (m_i). In contrast, the publisher has no incentive to make incremental efforts to improve the advertiser's purchase rate under the CPC model, because the publisher's payoff is not tied to purchases. The lack of publisher incentives to improve the purchase rate represents a typical moral hazard problem in contract theory. We discuss how this

[10] It is possible that the publisher can set a non-zero reserve price for the auction and turn down a non-negative winning bid that is below the reserve price. In our Technical Appendix, we endogenize the optimal decision of reserve price of the revenue maximizing publisher which can potentially exclude some bidders from auction and thus increase her expected revenue in both CPC and CPA pricing models. We find that if the opportunity cost for the unsold advertising slot is sufficiently large, then the optimal reserve price for the publisher is zero under both the CPC and CPA cases. This leads to the situation where the publisher always accepts a non-negative bid, which we are assuming here for simplicity. Moreover, we note that this assumption is also consistent with many real world situations (for example, in real Google key word auctions) where the publisher always accepts a non-negative winning bid.

moral hazard problem affects the expected payoff of both advertisers and the publisher subsequently. Again, this zero effort result is only pertaining to the particular type of effort that can improve the quality of each click (which is measured by the purchase rate), and certainly the publisher has all the incentives to exert efforts to increase the quantity of clicks under both CPC and CPA.

Finally, we can explicitly calculate the payoffs to the advertisers and the publisher, respectively in the CPC pricing model (please see the Appendix for the detailed derivation).

$$E(y_i) = \frac{1}{6}\left(1 - r\sigma^2\right), \qquad (4)$$
$$E(y_p) = \frac{1}{12}\left(1 - r\sigma^2\right).$$

In summary, with the CPC model, the advertiser assumes all the risk in the product market. A higher level of market risk or larger risk aversion parameter directly lowers the advertiser's payoff; it also indirectly lowers the publisher's payoff because the auction bids made by advertisers are lower (Lemma 1-(1)). Although an advertiser bears all the risk, this risk affects only its bidding behavior, not its effort level since the cost of that click becomes a sunk cost (Lemma 1-(2)) in our particular setting where the purchase rate is a linear function of the effort.[11]

4.2 Cost per action pricing model

In the CPA pricing model, the monetary transfer between the publisher and the advertiser i that wins the auction is $\rho_i \theta t_a$, where ρ_i is the winning advertiser's immediate purchase ratio, θ is the purchase rate, and t_a is the per-action (i.e., per purchase) payment. From Equation (3), we can obtain advertiser i's payoff if it wins the auction; $y_i^{CPA} = (m_i - \rho_i t_a)(e_i + e_p) - \frac{e_i^2}{2} - \frac{r\sigma^2}{2}(m_i - \rho_i t_a)^2$. The publisher's payoff is simply $y_p^{CPA} = \rho_i t_a (e_i + e_p) - \frac{e_p^2}{2}$.

Lemma 2. *In the CPA pricing model,*

1. *Advertiser i with m_i and ρ_i bids $b^{CPA}(m_i, \rho_i) = \frac{m_i}{\rho_i}$. Moreover, the probability that type B advertiser wins the auction is $E\left[Pr(w = B)\right]^{CPA} = 1 - \frac{\beta}{2\alpha}$.*

2. *The equilibrium effort levels for the winning advertiser and the publisher are $e_i^{CPA} = m_i - \rho_i t$ and $e_p^{CPA} = E(\rho_i) t_a$.*

The bid by advertiser i in the CPA model is equal to m_i/ρ_i. In contrast with the CPC case, the advertiser with lower immediate ratio (i.e., advertiser B) tends to submit a higher bid, thus having a

[11] We acknowledge that this result that the market risk affects the advertiser's bidding behavior, but not the effort depends on our functional form assumption that the purchase rate is a linear function of the effort. However, if we allow more general functional relationship, this may not hold and the market risk may affect the effort levels.

higher probability of winning the auction, than advertiser D in the CPA model. Also, the advertiser's optimal bidding behavior with a CPA contract does not depend on the risk aversion parameter (r) or the level of market risk (σ^2), which is a stark contrast with the CPC model. Because the advertiser pays only if a consumer purchases a product, payment occurs after the market uncertainty is realized. This means all risk arising from purchase uncertainty becomes fully insured. In this sense, the advertiser secures against the unnecessary advertising costs associated with unexpectedly low product sales by transferring the risk to the publisher, which gets paid only when the product sells.

Furthermore, this lemma shows that the winning advertiser's incentive to exert efforts to improve the purchase rate depends on its profit margin and the payment to the publisher. A higher profit margin (m_i), lower immediate purchase ratio (ρ_i), and lower per purchase payment (t_a) all induce the advertiser to undertake greater incremental efforts, because the winning advertiser obtains a profit from each purchase, whether that purchase is immediate or delayed, but it pays the publisher only for each immediate purchase. Therefore, in the CPA model, the branding advertiser (B) with a lower immediate purchase ratio experiences a greater incentive to exert incremental efforts than does advertiser D, who has a higher immediate purchase ratio.

Also unlike the CPC case, the publisher's incentives to exert efforts to improve the purchase rate in the CPA model depend on the per purchase payment (t_a) and the publisher's belief about the winning advertiser's immediate purchase ratio ($E(\rho_i)$). The publisher does not directly observe the wining advertiser's immediate purchase ratio and therefore must form a belief on the basis of the bids submitted by advertisers.[12] The publisher then chooses its efforts according to this belief.

Finally, we can calculate the payoffs to advertisers and the publisher, respectively (see the Appendix for the derivation).

$$\begin{aligned} E(y_i) &= \frac{1}{2}\left(1 - r\sigma^2\right)\left(\frac{\beta}{3\alpha} - \frac{\beta^2}{3\alpha^2} + \frac{\beta^3}{6\alpha^3}\right) + \frac{(\alpha+\beta)^2\beta}{48\alpha^3}, \\ E(y_p) &= \frac{15\beta}{48\alpha} - \frac{\beta^2}{8\alpha^2} - \frac{\beta^3}{48\alpha^3}. \end{aligned} \tag{5}$$

Because the CPA models shifts the market risk from the advertiser to the publisher, the advertiser's bidding behavior is not affected by market risk or risk aversion (Lemma 2-(1)). In turn, the publisher's expected payoff, which is determined by the advertiser's bid, is independent of these factors as well. The publisher's payoff depends only on the advertiser's immediate sales ratio because the total payment is tied solely to immediate sales.

[12] The exact expression for $E(\rho_i)$ is $E(\rho_i) = \frac{\alpha+\beta}{2}$ when the winning bid is less than $\frac{1}{\alpha}$; $E(\rho_i) = \beta$ when the winning bid is greater than $\frac{1}{\alpha}$, (see the Appendix for the derivation for the CPA pricing model).

4.3 Comparing the CPC and CPA pricing models

Adverse selection problem in the CPA pricing model

First, we investigate the issue which types of advertisers would benefit from different pricing schemes. The direct comparison between the CPC and the CPA (from Lemma 1 and 2) reveals that $E(\Pr(w = B)]^{CPC} = \frac{1}{2} < E(\Pr(w = B)]^{CPA} = 1 - \frac{\beta}{2\alpha}$ because $0 < \beta \leq \alpha < 1$. Hence, the probability that type B advertiser wins the auction is greater in the CPA than the CPC model. In the CPC model, both types of advertisers have an equal probability of winning the auction (the winner is simply the advertiser with a higher profit margin m_i) while the CPA pricing model gives the advertiser B a competitive advantage, because it has a smaller immediate purchase ratio ($\beta \leq \alpha$), as is reflected in its bidding function $b(m_i, \rho_i) = \frac{m_i}{\rho_i}$.

Furthermore, because the CPA model gives the branding advertiser a competitive advantage, the advertiser B potentially wins the auction even with a significantly smaller profit margin than the advertiser D (i.e., $m_B < m_D$). This is the CPA pricing model's adverse selection problem. Some publishers have adopted the CPA model, in the hope that adopting this model can help them attract more direct selling advertisers that measure campaign effectiveness by purchases rather than clicks. However, adopting the CPA model can lead to some unintended results for these publishers, in that it attracts branding rather than direct selling advertisers. More importantly, this adverse selection problem increases the possibility that an advertiser with a smaller profit margin wins the auction.

Proposition 1. *The expected profit margin of the winning advertiser is lower in the CPA than the CPC pricing model:* $E(m_i)^{CPA} \leq E(m_i)^{CPC}$.

As we show subsequently, the winning advertiser's profit margin has a positive effect on social welfare in the online advertising industry. Therefore, the adverse selection problem of the CPA model limits its potential to improve social welfare in this industry.

Efforts and purchase rate

With a CPA contract, the winning advertiser must share its sales gains with the publisher. Clearly, this reduces its incentives to make costly incremental efforts, compared with those related to the CPC pricing model: $e_i^{CPC} > e_i^{CPA}$. This classic underinvestment problem arises because the advertiser cannot extract all the surplus it creates from its costly effort. Moreover, as Lemma 2 shows, the optimal effort level depends on the profit margin in a CPA contract ($e_i^{CPA} = m_i - \rho_i t_a$), and the expected profit margin of the winning advertiser is lower for the CPA model (Proposition 1). This further reduces the expected level of incremental efforts made by this winning advertiser under the CPA model.

Also, the publisher's incentives in the two pricing schemes is straightforward. In the CPC model, the publisher's payoff is not tied to purchases, so the publisher has no incentive to exert incremental efforts to improve the purchase rate. In contrast, the CPA pricing model ties the publisher's payoff to purchases, so the publisher has strong incentives to undertake incremental efforts.

We now investigate how those different incentives provided to advertisers and publisher affect the final expected purchase rate under the two different pricing schemes.

Proposition 2. *The expected purchase rate is higher in the CPC than the CPA pricing model:* $E\left[\theta^{CPC}\right] \geq E\left[\theta^{CPA}\right].$

This result is both interesting and counterintuitive. One might expect that the CPA pricing model leads to a higher expected purchase rate than the CPC model, given the fact that the main concern of the CPC pricing model is exactly lower purchase rate due to the lack of proper incentive for the publisher to improve the quality of the clicks delivered. However, there are several forces that we need to take into consideration to fully understand the effects of different pricing mechanisms on the expected purchase rate.

On the one hand, the CPA model enables the winning advertiser and publisher to share the potential payoffs and losses, leading to increased incentives for the publisher to exert efforts. However, this reduces the winning advertiser's incentives at the same time. Furthermore, the CPA model creates additional effect, that is, the adverse selection problem from Proposition 1. This adverse selection problem suggests that the winning advertiser is more likely to be an advertiser with a smaller profit margin (m_i), which implies that the winning advertiser's incentives to undertake efforts further decreases because the optimal effort level of winning advertiser depends on the profit margin in a CPA contract. This in turn lowers the expected purchase rate.

On the other hand, in the CPC model, the winning advertiser has very strong incentives to undertake incremental efforts to improve the expected purchase rate, because it obtains all the potential payoffs from its final product sales. Incorporating all these effects together, we find that the CPA model leads to an expected purchase rate which is lower than that for the CPC model.[13]

[13] An important caveat is that this result may depend on our specific formulation which does not allow the publisher to exert an effort to increase the quantity of clicks. However, if we accept the fact that the publisher has the same incentive to increase the quantity of clicks under both CPC and CPA pricing models (because the publisher does not receive a payment for just simply showing impressions and the payment is conditional on the clicks under both CPC and CPA – without clicking, no action can be realized), we believe that this result is robust.

Uncertainty and risk aversion

Next, we study how various factors influence preferences for one pricing scheme over another. Define $\Delta E(y_p) = E\left(y_p^{CPA}\right) - E\left(y_p^{CPC}\right)$ and $\Delta E(y_i) = E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right)$. From Lemma 1 and 2, we can easily observe that the advertisers' risk aversion parameter (r) and market risk (σ^2) can have negative impacts on the payoffs to both the publisher and advertisers. A unique feature of the CPA pricing model is its ability to enable the winning advertiser to share a portion of the market risk with the publisher. This risk-sharing arrangement can mitigate the negative impact of both the advertisers' risk aversion parameter (r) and market risk (σ^2).

Proposition 3. *As uncertainty in the product market increases or advertisers become more risk averse, the difference in the publisher's and advertisers' expected profits in the CPA versus CPC pricing model monotonically increases:* $\frac{\partial(\Delta E(y_p))}{\partial r} \geq 0$, $\frac{\partial(\Delta E(y_p))}{\partial \sigma^2} \geq 0$, $\frac{\partial(\Delta E(y_i))}{\partial r} \geq 0$, $\frac{\partial(\Delta E(y_i))}{\partial \sigma^2} \geq 0$.

If an advertiser is exposed to greater product market uncertainty, as represented by σ^2, it is less willing to pay and therefore, it bids a lower price per click in the CPC model. Similarly, as advertisers become more risk averse, they offer a lower payment per click to compensate for their own risk, which arises from any given product uncertainty. However, with the CPA model, the burden of bearing the uncertainty risk shifts from the advertiser to the publisher. The advertising payment is tied to purchases and it is independent of σ^2 and the risk aversion factor. Hence, the publisher can benefit from adopting the CPA model.

Proposition 3 sheds some light on which types of advertisers and products represent good candidates for contracts that tie advertising payments to purchases. The CPA model is particularly suitable for advertisers that are risk averse and products that have high levels of market uncertainty. Advertisers that are more risk averse and sell products with high levels of market uncertainty likely make low bids in the CPC pricing model, because they have to shoulder all the market risk. However, with the CPA model, which ties advertising payments to purchases, the risk burden due to product uncertainty shifts from the advertisers to the publisher, so advertisers are more willing to participate and more likely to offer high bids. Such a risk-sharing arrangement directly increases the adertisers' payoff and indirectly increases the publisher's payoff through the advertisers' bids.

Conventional wisdom suggests smaller firms are more risk averse because of their inability to suffer through large market risks. Hence, the CPA model is particularly beneficial to small advertisers that otherwise would have not participated in online advertising, for fear of the market risks involved in CPC deals. In addition, advertisers that sell products with strong seasonality and unpredictable demand are good candidates for CPA deals. These findings are consistent with trends in the online advertising industry. For example, *Affiliate Fuel,* a CPA advertising network, has indicated its great interest in hosting products that are time sensitive and seasonal (Afflliate Fuel 2010).

Social welfare

Finally, we consider how the choice of pricing models might affect expected social welfare, that is, the total sum of the advertisers' and publisher's expected payoffs.

Proposition 4. *Denote the ratio of β to α as k ($k = \frac{\beta}{\alpha}$). There exists a threshold value of k_1 such that, when $k \geq k_1$, expected social welfare is greater for the CPA pricing model than the CPC pricing model:* $E\left[y_i^{CPC} + y_p^{CPC}\right] \leq E\left[y_i^{CPA} + y_p^{CPA}\right]$. *Otherwise,* $E\left[y_i^{CPC} + y_p^{CPC}\right] > E\left[y_i^{CPA} + y_p^{CPA}\right]$.

Proposition 4 relies on the existence of two opposing effects of adopting the CPA model. On the one hand, the CPA model's adverse selection problem leads to a lower expected purchase rate and lower expected profit margins for the winning advertiser. These declines contribute to decreases in the expected social welfare. On the other hand, the CPA model allows the advertiser to share a portion of the market risk with the publisher. Social welfare generally is higher when risk gets shared among different parties, rather than shouldered by one party. This is a standard optimal risk-sharing result in principal agent models (Holmstrom 1979; Holmstrom and Milgrom 1987). In our particular setting where the publisher is risk-neutral and advertiser is risk-averse, to shift risk from a risk-averse agent to a risk-neutral party can generally increase social welfare.

Naturally, the total effect is the sum of a negative effect caused by the CPA model's adverse selection problem and a positive effect caused by risk sharing under the CPA model. Thus, whether the CPA or CPC model leads to greater social welfare depends on the relative size of these two competing effects. When parameter k (ratio of β to α) is relatively high (i.e., the difference between the two advertisers is small), the CPA model's adverse selection problem is not severe, and the social welfare is greater in the CPA model. In contrast, when the parameter k is below a threshold k_1, the CPA model's adverse selection problem becomes severe, and the social welfare is smaller in the CPA.

This suggests an important managerial implications for the online advertising industry. Specifically, if participation in the CPA pricing model is limited to advertisers with sufficiently high immediate purchase rates, the difference between β and α will be small. Therefore, the industry would likely benefit from moving to CPA contracts; it would achieve greater overall social welfare compared with that resulting from the CPC model. The online advertising industry (particularly, networks that strive to maximize total payoffs to all parties, because they serve both advertises and publishers) shares this view and is attempting to develop screening mechanisms for advertisers that wish to use the CPA pricing model. For example, the previously mentioned *Affiliate Fuel* network requires all new advertisers to run a test campaign that demonstrates their likelihood of creating direct purchases before they can enter into a larger-scale contract. *Affiliate Fuel*'s prescreening process also examines advertisers' ads and landing pages to ensure they are designed to convert browsers into buyers (*Affiliate Fuel* 2010).

The choice of pricing models clearly affects advertisers' and the publisher's expected payoff.

Corollary. *There exists a threshold value of k_2 and k_3 such that:*

1. *When $k \geq k_2$, the publisher's expected payoff is higher in the CPA than in the CPC pricing model:, $E\left[y_p^{CPC}\right] \leq E\left[y_p^{CPA}\right]$. Otherwise, $E\left[y_p^{CPC}\right] > E\left[y_p^{CPA}\right]$.*

2. *When $k \geq k_3$, the advertisers' expected payoff is higher in the CPA than in the CPC pricing model: $E\left[y_i^{CPC}\right] \leq E\left[y_i^{CPA}\right]$. Otherwise, $E\left[y_i^{CPC}\right] > E\left[y_i^{CPA}\right]$.*

3. *Moreover, it is always the case that $k_2 \leq k_3$.*

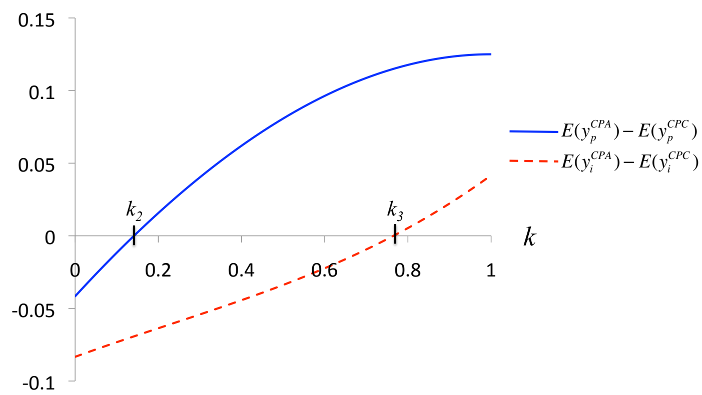
The publisher often appears to be the party resisting CPA adoption, as advertisers seemingly clamor for its adoption. This corollary suggests this is not always the case. For example, when the difference between β and α is very small (i.e., k is large such that $k > k_3$), the advertiser prefers CPA and the publisher's payoff also increases because it is always the case that $k_2 < k_3$. Similarly, when the publisher prefers CPC (i.e., $k < k_2$), both the publisher and the advertiser are better off.[14] However, the corollary also indicates a region of parameter k in which the incentives of the publisher and the advertiser are misaligned (i.e., $k_2 < k < k_3$), such that the publisher prefers CPA but the expected payoff for the advertiser is greater for the CPC model.

To highlight the conflict of interest between the publisher and advertisers, we illustrate the differences in the publisher's and advertisers' expected payoffs for the CPC and CPA settings in Figure 2, for which we set $r = 0.5$ and $\sigma^2 = 1$. The publisher prefers the CPC model if $k < k_2 \simeq 0.142$, and the advertiser prefers the CPC model when $k < k_3 \simeq 0.765$. Thus, when $0.142 < k < 0.765$, the advertisers' expected payoffs is lower in the CPA model than in the CPC model, but the publisher prefers the CPA model. In reality, it is the publisher who chooses the pricing mechanisms[15] and thus,

[14] If we compare all the thresholds levels, k_1, k_2, k_3, from Propositions 4 and Corollary, we can easily confirm that $k_2 < k_1 < k_3$ (it is obvious given that k_1 is the cutoff for social welfare which is the sum of the publisher's and advertisers' expected payoff). That is, when the publisher chooses CPC (i.e., $k < k_2$), the publisher is better off and advertiser's payoff ($k < k_3$) as well as social welfare also increases ($k < k_1$). The opposite reasoning applies to the advertiser's choice of CPA (i.e., $k > k_3$): the advertiser is better off and the publisher's payoff ($k > k_2$) as well as social welfare also increases ($k > k_1$).

[15] In practice, there are several different networks that serve both advertisers and publishers and each network has their own pricing mechanisms (for example, Affiliate Fuel network and Commission Junction network adopt CPA advertising while Google provides both CPA and CPC). The publisher first chooses its network and announces its availability for advertising slot. Then, advertisers choose a publisher where they want to place their ads through the network. Hence, it is effectively consistent with our assumption that the publisher chooses the pricing mechanism first.

Figure 2: The publisher's and advertisers' payoffs



the CPA contract ensues in this case. Hence, even if we observe one particular type of pricing contract in the market, it does not necessarily imply that the preferences of both parties are the same. Also, it is important to note that advertisers make strictly positive expected profits in both CPC and CPA cases since they do not bid more than their expected values for a click. The profit comparison between two pricing mechanisms (CPC vs. CPA) is only relative terms. Hence, even if advertisers prefer the CPC to the CPA model, they still participate in the CPA pricing auction which has been chosen by the publisher.[16]

5 Extension

In this section, we extend our basic model to capture the more realistic situation where the publisher chooses the winning advertiser not based on bid amount, but based on the expected revenue from each advertiser under CPA – thus, the highest bidder is not necessarily chosen by the publisher. Such a process is similar to how Google uses historical data to adjust each advertiser's bid and determine the winner of the auction.

In practice, the publisher adjusts the advertiser's bids using multiple signals about advertisers' types, such as (1) the repeated past interaction with advertisers, or (2) the estimate from a test campaign (*Commission Junction* 2010). However, the publisher's knowledge of advertisers' types is not perfect. For example, even if the publisher knows an advertiser's past performance (such as

[16] Also, in practice advertisers often put their ads on multiple sites, which may have all different pricing mechanisms. Our model suggests that both types of advertiser have always a positive probability to win the auction under both CPC and CPA pricing mechanisms since their bids are a function of their profit margins. Even though one type of advertiser prefers one particular pricing mechanism to the other, it still can benefit from the advertising under less preferred pricing mechanism (again, advertisers make strictly positive expected profits in both CPC and CPA cases in our model). Hence, it is possible that one advertiser entering into a CPA agreement with one publisher and a CPC agreement with another advertiser. We thank an anonymous reviewer for suggesting these broader implications of our model.

immediate conversion rate) through repeated interaction, it is possible that this advertiser adopts different strategies in different product campaigns: it might have used a direct selling strategy in one product campaign and a branding strategy in another. To capture this possibility that the immediate sales ratio can vary not only across advertisers, but also can vary across different campaigns by the same advertiser, we model that the publisher can have an imperfect signal of the advertiser's type since the advertiser can change its immediate sales ratio.

More specifically, we assume that the publisher obtains an *ex ante* signal regarding each advertiser's type. We model the uncertainty of such a noisy signal as ϕ such that the signal is correct with probability ϕ, and incorrect with probability $1-\phi$.[17] In other words, with probability ϕ, type $D\,(B)$ advertiser is correctly identified as direct seller (branding advertiser), and with probability $1-\phi$, type $D\,(B)$ advertiser is incorrectly identified as a branding advertiser (direct seller). Since the publisher's objective is to maximize its expected payoff, the publisher adjusts each advertiser's bid based on advertiser's expected immediate sales ratio. That is, if the publisher receives a signal which indicates it is a type D advertiser, it will adjust the advertiser's bid by multiplying α, which is type D's immediate sales ratio; if the publisher receives a signal which indicates it is a type B advertiser, it will adjust the advertiser's bid by multiplying β, which is type B's immediate sales ratio.[18]

Because there are two advertisers – one with higher immediate sales ration (type D advertiser) and the other with lower immediate sales ratio (type B advertiser), along with a signal, there are two cases to consider: i) with probability ϕ, the publisher correctly identify both type D and type B advertisers; ii) with probability $1-\phi$, the publisher has incorrectly identify type D and type B advertisers. We then investigate how allowing the publisher to use a prior knowledge of advertisers' types changes the adverse selection problem discussed in Proposition 1.

Proposition 5. *Suppose that the publisher receives signals about the advertisers' type and use them to adjust advertisers' biddings. The expected probability that type B advertiser wins the auction in the CPA model is* $E\left[Pr(w=B)\right]^{CPA} = 1-\frac{1}{2}\left[\phi+(1-\phi)\left(\frac{\beta}{\alpha}\right)^2\right]$. *This expected probability is still greater in the CPA than the CPC pricing model:* $E\left[Pr(w=B)\right]^{CPC} \leq E\left[Pr(w=B)\right]^{CPA}$.

The intuition behind this proposition is as follows. In case (i) mentioned above, the publisher has correct signals and can correctly adjust each advertiser's bid. Thus, the expected probability that the type B (branding) advertiser wins the auction in the CPA model becomes the same as that in the

[17]Even if the type is endogenous (i.e., the sales ratio is an endogenous decision of the advertisers), we can capture this aspect in a parsimonious way that there is a probability that they change their types, which is represented by ϕ.

[18]Although we do not know for certain what publishers (for example, Googles or ValueClick) do when choosing the winning bidder, it is well known that publishers maximizes its expected revenue based on advertisers' type. Our specification captures this spirit in a simplest possible way.

CPC model, which is $\frac{1}{2}$. In the CPA model without such signals, the expected probability that the branding advertiser wins the auction is $1 - \frac{\beta}{2\alpha}$. In case (ii), the publisher's incorrect signals regarding both advertisers prevent the publisher from correctly adjusting each advertiser's bid. The expected probability that the branding advertiser wins the auction is $1 - \frac{\beta^2}{2\alpha^2}$, even higher than that in the CPA model without such signals.

Overall, we conclude that our main results are robust even if the publisher can learn about advertisers' types as long as the publisher's knowledge is not perfect, which is always the case in reality. Allowing the publisher to choose the advertiser based on its expected revenue and have knowledge of each advertiser's type would only weaken the adverse selection problem of the CPA model. However, this problem would still persist and sometimes would be more severe when the quality of the publisher's knowledge is low (i.e., ϕ is small).

6 Conclusions

Since 2002, the online advertising industry has increasingly adopted the CPC pricing model, which ties advertising payments to clicks. More recently, several large companies have started to pursue CPA pricing, which calculates advertising payments on the basis of purchases. Which model leads to better outcomes for advertisers, publishers, and the industry as a whole? Is CPA really the future of online advertising? This study offers a first step in understanding this crucial debate. We apply a formal economic framework to analysis of the trade-offs between CPA and CPC, with a particular focus on the non-contractible efforts that publishers and advertisers will exert to improve the product purchase rate for a given click. Unlike existing literature, we view pricing models as contracts that give publishers and advertisers incentives to exert non-contractible efforts, as well as allocate the market risk between advertisers and publishes. This unique angle on these two popular online advertising pricing models leads to several interesting and new insights.

Our results also have important implications for all parties involved in online advertising: advertisers, publishers, and advertising networks. We outline the conditions in which one pricing model is more desirable than the other in terms of increasing the payoffs to each party. We also note which parameters influence the trade-offs between the CPC and CPA models and how the use of different pricing models affects social welfare in this industry. Such insights can help advertising networks design efficient marketplaces for their clients (i.e., advertisers and publishers), as well as help resolve the strident debate about the future of pricing models in online advertising, with billions of advertising dollars in the balance.

There are a number of limitations to the current work and our results could be extended in further research. First, we assume that the publisher has a single slot in spirit of trying to capture the reality that most firms have limited capacity of advertising space in their website. This also allowed us to keep our analysis tractable by guaranteeing the advertisers' bid represent their true valuations in a second-price auction. Nevertheless, most publishers often have more than one slot in which they can place ads in their website. A second-price auction will then diverge from the Vickery-Clarke-Groves mechanism, and true-valuation bidding is generally not an equilibrium strategy (Edelman et al. 2007). Broader analysis encompassing multiple-slots, even if technically challenging, would be insightful for generalization of our findings.

Second, our model does not capture the publisher's efforts to increase the quantity of clicks and take this effort as given. Our model is only concerned with everything that happens after a click has been generated. The issue this paper tackles is "for a given click" how this click should be priced – whether the advertiser pays for every click (CPC) or pays for only high-quality click that leads to the final action (CPA). However, we conjecture that including the publisher's incentive to exert efforts to increase the quantity of clicks would not qualitatively impact our analysis since the publisher always has a strong incentive to increase the quantity of clicks under both CPC and CPA pricing models. Some extensions could be analyzed in the context of how these two different types of efforts (one for increasing the quantity of clicks and the other for increasing the quality of clicks) can be interacted with each other under two different pricing mechanisms.

Third, we do not allow that the immediate sales ratios can be affected by the effort levels of advertisers. Instead, we capture this possibility in a parsimonious way through an adverse selection model with two types of advertisers (who have different immediate sales ratios). Nevertheless, we believe that endogenizing the immediate sales ratio (i.e., the immediate sales ratio can be determined by the advertiser's effort) can be an interesting venue for model extension.

Fourth, we offer several propositions regarding the influence of various factors on the use of CPC and CPA models. These factors, including the advertisers' risk aversion, uncertainty in the product market, and the proportion of delayed responses, differ for various advertisers and publishers. It therefore would be interesting to test these propositions using empirical data. Finally, we predict how the adoption of a CPA model (rather than a CPC model) influences purchase (conversion) rates, firm profits, and social welfare. Additional research should test these predictions empirically as well.

The focus on the current work is not to come up with the optimal auction mechanism of online advertising, but try to highlight the costs and benefits of two different performance-based pricing, which are widely used in practice. By doing so, we are hoping that we shed some insight for the managerial decision about which way the industry should move on.

Appendix

Proof of Lemma 1.

We solve the second part first. In the CPC pricing model, the publisher's payoff is $y_p^{CPC} = t_c - \frac{e_p^2}{2}$. The optimal effort level by the publisher is $e_p^{CPC} = \text{argmax } y_p^{CPC} = 0$. Advertiser i's payoff if it wins the auction is $y_i^{CPC} = m_i(e_i + e_p) - t_c - \frac{e_i^2}{2} - \frac{r\sigma^2 m_i^2}{2}$. The optimal effort level by advertiser i is $e_i^{CPC} = \text{argmax}_{e_i}\left(y_i^{CPC}\right) = m_i$.

Using the result from above that $e_p^{CPC} = 0$ and $e_i^{CPC} = m_i$, advertiser i's payoff if it wins the auction is $y_i^{CPC} = \frac{1}{2}\left(1 - r\sigma^2\right) m_i^2 - t_c$. Because the advertisers bid their true values in a second-price Vickery auction, $b(m_i, \rho_i) = \frac{1}{2}\left(1 - r\sigma^2\right) m_i^2$. The advertiser with a higher m_i wins the auction. Because $m_i(i = D, B)$ is randomly drawn from a standard uniform distribution on $[0, 1]$, the probability that the branding advertiser wins the auction is $E\left[\Pr(w = B)\right]^{CPC} = E\left[\Pr(m_B \geq m_D)\right]^{CPC} = \int_0^1 \int_{m_D}^1 dm_B\, dm_D = \frac{1}{2}$. **Q.E.D.**

Derivation of the payoffs to the advertisers and the publisher in the CPC model.

In the CPC, advertiser i bids $b(m_i, \rho_i) = \frac{1}{2}\left(1 - r\sigma^2\right) m_i^2$. The advertiser with a higher m_i wins the auction and pays the lower bid. Let m_D, m_B each be random draws from a standard uniform distribution on $[0, 1]$. Thus, $t_c^* = \frac{1}{2}\left(1 - r\sigma^2\right) \min\left(m_D^2,\, m_B^2\right)$.

The winning advertiser's profit is $y_i^{CPC} = \frac{1}{2}\left(1 - r\sigma^2\right)\left\{\max\left(m_D^2,\, m_B^2\right) - \min\left(m_D^2,\, m_B^2\right)\right\}$ and the publisher's expected profit is $y_p^{CPC} = t_c^* = \frac{1}{2}\left(1 - r\sigma^2\right)\min\left(m_D^2,\, m_B^2\right)$.

Integrating the advertiser's profit over the two uniform distributions, we get:

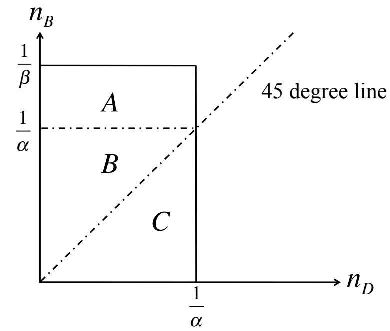
$$
\begin{aligned}
E(y_i^{CPC}) = \frac{1}{2}\left(1 - r\sigma^2\right) \Bigg\{ &\left(\int_0^1 \int_0^{m_D} m_D^2\, dm_B\, dm_D + \int_0^1 \int_{m_D}^1 m_B^2\, dm_B\, dm_D \right) \\
&- \left(\int_0^1 \int_0^{m_D} m_B^2\, dm_B\, dm_D + \int_0^1 \int_{m_D}^1 m_D^2\, dm_B\, dm_D \right) \Bigg\} \\
= \frac{1}{6}\left(1 - r\sigma^2\right).&
\end{aligned} \tag{6}
$$

Similarly, we get: $E(y_p^{CPC}) = \frac{1}{12}\left(1 - r\sigma^2\right)$. **Q.E.D.**

Proof of Lemma 2.

We solve the second part of the Lemma first. In the CPA, given the winning bid t_a, the winning advertiser forms an expectation regarding the publisher's effort e_p since it does not know the publisher's e_p for sure. Thus, the advertiser's payoff, if it wins the auction, becomes $y_i^{CPA} = (m_i - \rho_i t_a)(e_i +$

Figure 3: Advertiser's bidding behavior and equilibrium outcome



$E(e_p)) - \frac{e_i^2}{2} - \frac{r\sigma^2}{2}(m_i - \rho_i t_a)^2$. The optimal effort level by the advertiser i in turn is the solution to $e_i^{CPA} = argmax_{e_i} y_i^{CPA} = m_i - \rho_i t_a$.

Also, the publisher does not know the winning advertiser's ρ_i and e_i, and therefor must form expectations about these values. Thus, the publisher's payoff becomes $y_p^{CPA} = E(\rho_i)t_a(E(e_i) + e_p) - \frac{e_p^2}{2}$, and the optimal effort level by the publisher is the solution to $e_p^{CPA} = argmax_{e_p} E(y_p^{CPA}) = E(\rho_i)t_a$.

Using the result from the above that $e_i^{CPA} = m_i - \rho_i t_a$ and $e_p^{CPA} = E(\rho_i)t_a$, advertiser i's payoff, if it wins the auction, is $y_i^{CPA} = \frac{1}{2}\left(1 - r\sigma^2\right)(m_i - \rho_i t_a)^2 + (m_i - \rho_i t_a)E(e_p)$. Because advertisers bid their true values in the second-price Vickery auction, $b(m_i, \rho_i) = \frac{m_i}{\rho_i}$.

Let $\frac{m_D}{\alpha} = n_D$ and $\frac{m_B}{\beta} = n_B$ equal the bids from the direct selling and branding advertisers, respectively. Then $n_D \sim U\,[0,\,\frac{1}{\alpha}]$ and $n_B \sim U\,[0,\,\frac{1}{\beta}]$. The probability that the branding advertiser wins the auction $(E\,[\Pr(w = B)]^{CPA})$ is

$$\begin{aligned}
\Pr[n_B \geq n_D] &= \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} f(n_B) f(n_D)\, dn_B\, dn_D + \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\alpha}} f(n_B) f(n_D)\, dn_B\, dn_D \qquad (7)\\
&= \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \alpha\beta\, dn_B\, dn_D + \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\alpha}} \alpha\beta\, dn_B\, dn_D\\
&= 1 - \frac{\beta}{2\alpha}. \qquad \text{Q.E.D.}
\end{aligned}$$

Derivation of the payoffs to the advertisers and the publisher in the CPA model.

Let $\frac{m_D}{\alpha} = n_D$ and $\frac{m_B}{\beta} = n_B$ be the bids from the direct selling and branding advertiser, respectively. Then, $n_D \sim U\,[0,\,\frac{1}{\alpha}]$ and $n_B \sim U\,[0,\,\frac{1}{\beta}]$, where $\frac{1}{\alpha} \leq \frac{1}{\beta}$. There are two different regions which lead to different inferences for the publisher and different analysis (see Figure 3).

1. First, when the winning bid is greater than $\frac{1}{\alpha}$ (region A in Figure 3), it must be a bid from branding advertiser n_B since the direct selling advertiser never bids more than $\frac{1}{\alpha}$: Type B advertiser wins the auction, and $t_a = n_D$ because $n_D < n_B$ always holds in this case. Thus,

the publisher's expectation of the winning advertiser is $E(\rho_i) = \beta$, and the publisher undertakes effort $E(e_p) = \beta n_D$. The expected payoff for the winning advertiser and the publisher are,

$$E(y_i^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \left\{ \frac{1}{2}\left(1 - r\sigma^2\right)\left(\beta\, n_B - \beta\, n_D\right)^2 + \left(\beta\, n_B - \beta\, n_D\right)\beta\, n_D \right\} \alpha\beta\, dn_B\, dn_D \tag{8}$$

$$= \frac{1}{2}\left(1 - r\sigma^2\right)\left(\frac{1}{3} - \frac{\beta}{2\alpha} + \frac{\beta^2}{3\alpha^2} - \frac{\beta^3}{6\alpha^3}\right) + \left(\frac{\beta}{4\alpha} - \frac{\beta^2}{3\alpha^2} + \frac{\beta^3}{12\alpha^3}\right).$$

$$E(y_p^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \left\{ \beta\, n_D\left(\beta\, n_D + \beta\, n_B - \beta\, n_D\right) - \frac{1}{2}\left(\beta\, n_D\right)^2 \right\} \alpha\beta\, dn_2\, dn_1 \tag{9}$$

$$= \frac{\beta}{4\alpha} - \frac{\beta^2}{6\alpha^2} - \frac{\beta^3}{12\alpha^3}.$$

2. Second, when the winning advertiser's bid is less than $\frac{1}{\alpha}$ (region B and C in Figure 3), the winning advertiser can be from either type of advertiser. In this case, the publisher can consider the probability with which each event arises. The probability that the winning bid is from the branding advertiser ($n_D < n_B \leq \frac{1}{\alpha}$: region B in Figure 3) is $\int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\alpha}} \alpha\beta dn_B dn_D = \frac{\beta}{2\alpha}$. Also, the probability that the winning bid is from the direct selling type ($n_B \leq n_D \leq \frac{1}{\alpha}$: region C in Figure 3) is $\int_0^{\frac{1}{\alpha}} \int_{n_B}^{\frac{1}{\alpha}} \beta\alpha dn_D dn_B = \frac{\beta}{2\alpha}$. Hence, the winning advertiser can be either type with equal probability ($\frac{\beta}{2\alpha}$). Therefore, in this case, the posterior is $E(\rho_i) = \frac{\alpha+\beta}{2}$. Now, we consider two different sub-cases when the winning advertiser's bid is less than $\frac{1}{\alpha}$.

   (a) When $n_D < n_B \leq \frac{1}{\alpha}$ (region B in Figure 3): The type B advertiser wins the auction. Thus, $t_a = n_D$. Because $E(\rho_i) = \frac{\alpha+\beta}{2}$, and the publisher's effort is $E(e_p) = \frac{\alpha+\beta}{2} n_D$. The expected payoff of the winning advertiser and the publisher are,

$$E(y_i^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\beta}} \left\{ \frac{1}{2}\left(1 - r\sigma^2\right)\left(\beta\, n_B - \beta\, n_D\right)^2 + \left(\beta\, n_B - \beta\, n_D\right)\frac{\alpha+\beta}{2} n_D \right\} \alpha\beta\, dn_B\, dn_D \tag{10}$$

$$= \frac{1}{2}\left(1 - r\sigma^2\right)\left(\frac{\beta^3}{12\alpha^3}\right) + \frac{(\alpha+\beta)\beta^2}{48\alpha^3}.$$

$$E(y_p^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\beta}} \left\{ \beta\, n_D\left(\frac{\alpha+\beta}{2} n_D + \beta\, n_B - \beta\, n_D\right) - \frac{1}{2}\left(\frac{\alpha+\beta}{2} n_D\right)^2 \right\} \alpha\beta\, dn_B\, dn_D \tag{11}$$

$$= -\frac{\beta}{96\alpha} + \frac{\beta^2}{48\alpha^2} + \frac{7\beta^3}{96\alpha^3}.$$

   (b) When $n_B \leq n_D \leq \frac{1}{\alpha}$ (region C in Figure 3): The type D advertiser wins the auction. Thus, $t_a = n_B$. Because $E(\rho_i) = \frac{\alpha+\beta}{2}$, and the publisher's effort is $E(e_p) = \frac{\alpha+\beta}{2} n_B$. The expected payoff of the winning advertiser and the publisher are,

$$E(y_i^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_0^{n_D} \left\{ \frac{1}{2}\left(1 - r\sigma^2\right)\left(\alpha\, n_D - \alpha\, n_B\right)^2 + \left(\alpha\, n_D - \alpha\, n_B\right)\frac{\alpha+\beta}{2} \cdot n_B \right\} \alpha\beta\, dn_B\, dn_D \tag{12}$$

$$= \frac{1}{2}\left(1 - r\sigma^2\right)\left(\frac{\beta}{12\alpha}\right) + \frac{(\alpha+\beta)\beta}{48\alpha^2}.$$

$$E(y_p^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_0^{n_D} \left\{ \alpha\, n_B \left(\frac{\alpha+\beta}{2} n_B + \alpha\, n_D - \alpha\, n_B\right) - \frac{1}{2}\left(\frac{\alpha+\beta}{2} n_B\right)^2 \right\} \alpha\beta\, dn_B\, dn_D \tag{13}$$

$$= \frac{7\beta}{96\alpha} + \frac{\beta^2}{48\alpha^2} - \frac{\beta^3}{96\alpha^3}.$$

Hence, in the CPA , the winning advertiser's expected payoff is the sum of the advertiser's payoffs in regions A, B, and C in Figure 3. Thus, $E(y_i^{CPA}) = \frac{1}{2}\left(1 - r\sigma^2\right)\left(\frac{\beta}{3\alpha} - \frac{\beta^2}{3\alpha^2} + \frac{\beta^3}{6\alpha^3}\right) + \frac{(\alpha+\beta)^2\beta}{48\alpha^3}$.

Similarly, the publisher's expected payoff is the sum of the advertiser's payoffs in regions A, B, and C in Figure 3. Hence, $E(y_p^{CPA}) = \frac{15\beta}{48\alpha} - \frac{\beta^2}{8\alpha^2} - \frac{\beta^3}{48\alpha^3}$. **Q.E.D.**

Proof of Proposition 1.

In the CPC pricing model, the expected profit margin of the winning advertiser is

$$E(m_i)^{CPC} = \int_0^1 \int_0^{m_D} m_D\, dm_B\, dm_D + \int_0^1 \int_0^{m_B} m_B\, dm_B\, dm_D = \frac{2}{3}.$$

In the CPA, let $\frac{m_D}{\alpha} = n_D$ and $\frac{m_B}{\beta} = n_B$ be the bids from the direct selling and branding advertisers, respectively. Then, $n_D \sim U\,[0,\, \frac{1}{\alpha}]$ and $n_B \sim U\,[0,\, \frac{1}{\beta}]$. The expected profit margin of the winning advertiser $E(m_i)^{CPA}$ is the sum of the expected profit margin for the three regions.

(a) When $\frac{1}{\alpha} < n_B$: $E(m_i) = \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \alpha\beta \cdot \beta\, n_B\, dn_B\, dn_D = \frac{1}{2}\left(1 - \frac{\beta^2}{\alpha^2}\right)$.

(b) When $n_D < n_B \le \frac{1}{\alpha}$: $E(m_i) = \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\alpha}} \alpha\beta \cdot \beta\, n_B\, dn_B\, dn_D = \frac{\beta^2}{3\alpha^2}$.

(c) When $n_B \le n_D$: $E(m_i) = \int_0^{\frac{1}{\alpha}} \int_0^{n_D} \alpha\beta \cdot \alpha n_D\, dn_B\, dn_D = \frac{\beta}{3\alpha}$.

Thus, the expected profit margin of the winning advertiser is $E(m_i)^{CPA} = \frac{1}{2} + \frac{\beta}{3\alpha} - \frac{\beta^2}{6\alpha^2}$.

The function $f(x) = \frac{1}{2} + \frac{1}{3}x - \frac{1}{6}x^2$ is increasing on [0,1] and reaches its maximum of $f(1) = \frac{2}{3}$ in this region. Therefore, we have proven $E(m_i)^{CPA} = \frac{1}{2} + \frac{\beta}{3\alpha} - \frac{\beta^2}{6\alpha^2} \le \frac{2}{3} = E(m_i)^{CPC}$. **Q.E.D.**

Proof of Proposition 2.

Note that $E(\theta)=E(e_i)+E(e_p)$. From the results in the proofs of Propositions 2, we know $E(\theta^{CPC})=E(e_i^{CPC})+E(e_p^{CPC})=\frac{2}{3}$ and $E(\theta^{CPA})=E(e_i^{CPA})+E(e_p^{CPA})=\left(\frac{1}{2}-\frac{\beta}{3\alpha}+\frac{\beta^2}{6\alpha^2}\right)+\left(\frac{2\beta}{3\alpha}-\frac{\beta^2}{3\alpha^2}\right)=\frac{1}{2}+\frac{\beta}{3\alpha}-\frac{\beta^2}{6\alpha^2}$. The function $f(x)=\frac{1}{2}+\frac{1}{3}x-\frac{1}{6}x^2$ is a increasing function on [0,1] that reaches its maximum of $f(1)=\frac{2}{3}$ in this region. Thus, we have proven $E\left(\theta^{CPA}\right)=\frac{1}{2}+\frac{\beta}{3\alpha}-\frac{\beta^2}{6\alpha^2}\leq\frac{2}{3}=E\left(\theta^{CPC}\right)$. **Q.E.D.**

Proof of Proposition 3.

First, $E\left(y_p^{CPC}\right)=\frac{1}{12}\left(1-r\sigma^2\right)$ and $E\left(y_p^{CPA}\right)=\frac{15k-6k^2-k^3}{48}$. Thus, $\frac{\partial\left(E\left(y_p^{CPA}\right)-E\left(y_p^{CPC}\right)\right)}{\partial r}=\frac{1}{12}\sigma^2\geq 0$ and $\frac{\partial\left(E\left(y_p^{CPA}\right)-E\left(y_p^{CPC}\right)\right)}{\partial\sigma^2}=\frac{1}{12}r\geq 0$. Also, $E\left(y_i^{CPC}\right)=\frac{1}{6}\left(1-r\sigma^2\right)$ and $E\left(y_i^{CPA}\right)=\frac{1}{2}\left(1-r\sigma^2\right)\cdot\left(\frac{2k-2k^2+k^3}{6}\right)+\left(\frac{k+2k^2+k^3}{48}\right)$. Thus, $\frac{\partial\left(E\left(y_i^{CPA}\right)-E\left(y_i^{CPC}\right)\right)}{\partial r}=-\frac{1}{2}\sigma^2\left(\frac{2k-2k^2+k^3}{6}\right)+\frac{1}{6}\sigma^2$, and $\frac{\partial\left(E\left(y_i^{CPA}\right)-E\left(y_i^{CPC}\right)\right)}{\partial\sigma^2}=-\frac{1}{2}r\left(\frac{2k-2k^2+k^3}{6}\right)+\frac{1}{6}r$. It is easy to prove that $\frac{2k-2k^2+k^3}{6}<\frac{1}{3}$, for $\forall k\in[0,1]$. That is, $\frac{\partial\left(E\left(y_i^{CPA}\right)-E\left(y_i^{CPC}\right)\right)}{\partial r}\geq 0$, and $\frac{\partial\left(E\left(y_i^{CPA}\right)-E\left(y_i^{CPC}\right)\right)}{\partial\sigma^2}\geq 0$. **Q.E.D.**

Proof of Proposition 4.

From $E\left(y_i^{CPC}+y_p^{CPC}\right)=\frac{1}{4}(1-r\sigma^2)$ and $E\left(y_i^{CPA}+y_p^{CPA}\right)=\frac{1}{2}\left(1-r\sigma^2\right)\left(\frac{2k-2k^2+k^3}{6}\right)+\left(\frac{4k-k^2}{12}\right)$, we get $E\left(y_i^{CPA}+y_p^{CPA}\right)-E\left(y_i^{CPC}+y_p^{CPC}\right)=\frac{1}{2}\left(1-r\sigma^2\right)\left(\frac{2k-2k^2+k^3}{6}-\frac{1}{2}\right)+\left(\frac{4k-k^2}{12}\right)$. When $k=0$, $E\left(y_i^{CPA}+y_p^{CPA}\right)-E\left(y_i^{CPC}+y_p^{CPC}\right)<0$; when $k=1$, $E\left(y_i^{CPA}+y_p^{CPA}\right)-E\left(y_i^{CPC}+y_p^{CPC}\right)>0$. Moreover, $\frac{\partial\left(E\left(y_i^{CPA}+y_p^{CPA}\right)-E\left(y_i^{CPC}+y_p^{CPC}\right)\right)}{\partial k}=\frac{1}{2}\left(1-r\sigma^2\right)\left(\frac{(3k-2)^2+2}{18}\right)+\frac{2-k}{6}>0$, for $\forall k\in[0,1]$. Therefore, there exists a threshold value of $k_1\in(0,1)$, such that when $k\geq k_1$, $E\left(y_i^{CPC}+y_p^{CPC}\right)\leq E\left(y_i^{CPA}+y_p^{CPA}\right)$ but when $k<k_1$, $E\left(y_i^{CPC}+y_p^{CPC}\right)>E\left(y_i^{CPA}+y_p^{CPA}\right)$. **Q.E.D.**

Proof of Corollary.

From Lemma 1 and 2, $E\left(y_p^{CPC}\right)=\frac{1}{12}\left(1-r\sigma^2\right)$, $E\left(y_p^{CPA}\right)=\frac{15k-6k^2-k^3}{48}$. When $k=0$, $E\left(y_p^{CPA}\right)-E\left(y_p^{CPC}\right)<0$; when $k=1$, $E\left(y_p^{CPA}\right)-E\left(y_p^{CPC}\right)>0$. Moreover, $\frac{\partial\left(E\left(y_p^{CPA}\right)-E\left(y_p^{CPC}\right)\right)}{\partial k}=\frac{15-12k-3k^2}{48}>0$, for $\forall k\in[0,1]$. This proves that there exists$k_2\in(0,1)$, such that when $k\geq k_2$, $E\left(y_p^{CPC}\right)\leq E\left(y_p^{CPA}\right)$ but when $k<k_2$, $E\left(y_p^{CPC}\right)>E\left(y_p^{CPA}\right)$. Similarly, we can see the existence of k_3 since when $k=0$, $E\left(y_i^{CPA}\right)-E\left(y_i^{CPC}\right)<0$; when $k=1$, $E\left(y_i^{CPA}\right)-E\left(y_i^{CPC}\right)>0$. Moreover, $\frac{\partial\left(E\left(y_i^{CPA}\right)-E\left(y_i^{CPC}\right)\right)}{\partial k}=\frac{1}{2}\left(1-r\sigma^2\right)\left(\frac{2-4k+3k^2}{6}\right)+\frac{1+4k+3k^2}{48}>0$, for $\forall k\in[0,1]$.

Finally, We define $\Delta E\left(y_p\right)=E\left(y_p^{CPA}\right)-E\left(y_p^{CPC}\right)$ and $\Delta E\left(y_i\right)=E\left(y_i^{CPA}\right)-E\left(y_i^{CPC}\right)$. Then, $\Delta E\left(y_p\right)-\Delta E\left(y_i\right)=\frac{1}{12}\left(1-r\sigma^2\right)\left(1-2k+2k^2-k^3\right)+\frac{1}{48}\left(14k-8k^2-2k^3\right)$. In turn, it is easy to see that for $\forall k\in[0,1]$, $1-2k+2k^2-k^3\geq 0$ and $14k-8k^2-2k^3>0$. Therefore, we have proven that $\Delta E\left(y_p\right)-\Delta E\left(y_i\right)\geq 0$, for $\forall k\in[0,1]$. We have already shown that $\Delta E\left(y_p\right)$ and $\Delta E\left(y_i\right)$ are both increasing functions for $\forall k \in [0,1]$. Therefore, $k_2 < k_3$. **Q.E.D.**

Proof of Proposition 5.

First note that advertiser $i's$ payoff, if it wins the auction is unchanged. Using the result from Lemma 2, advertisers bid $b(m_i, \rho_i) = \frac{m_i}{\rho_i}$. Let $\frac{m_D}{\alpha} = n_D$ and $\frac{m_B}{\beta} = n_B$ equal the bids from the direct selling and branding advertisers, respectively.

Case (i): The publishers has correct signals of both advertisers' types. It uses this to adjust each advertiser's bid to calculate the expected revenue by multiplying α to type D advertiser's bid n_D and multiplying β to type B advertiser's bid n_B. The probability that type B advertiser wins the auction $(E\left[\Pr(w = B)\right]^{CPA})$ is

$$\Pr[\beta n_B \geq \alpha n_D] = \int_0^{\frac{1}{\alpha}} \int_{\frac{\alpha}{\beta} n_D}^{\frac{1}{\beta}} \alpha\beta \, dn_B \, dn_D = \frac{1}{2}.$$

Case (ii): The publishers has incorrect signals of both advertisers' types. It uses this to adjust each advertiser's bid to calculate the expected revenue by multiplying β to the type D advertiser's bid n_D and multiplying α to the type B advertiser's bid n_B. The probability that type B advertiser wins the auction $(E\left[\Pr(w = B)\right]^{CPA})$ is

$$\Pr[\alpha n_B \geq \beta n_D] = \int_0^{\frac{1}{\alpha}} \int_{\frac{\beta}{\alpha^2}}^{\frac{1}{\beta}} \alpha\beta \, dn_B \, dn_D + \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{\beta}{\alpha^2}} \alpha\beta \, dn_B \, dn_D = 1 - \frac{\beta^2}{2\alpha^2}.$$

Thus, the expected probability that type B advertiser wins the auction in the CPA model is

$$E\left[\Pr(w = B)\right]^{CPA} = \frac{1}{2}\phi + (1 - \frac{\beta^2}{2\alpha^2})(1 - \phi) = 1 - \frac{1}{2}\left[\phi + (1-\phi)\left(\frac{\beta}{\alpha}\right)^2\right].$$

Because $\frac{\beta}{\alpha} \leq 1$, we have $\phi + (1-\phi)\left(\frac{\beta}{\alpha}\right)^2 \leq 1$. Therefore, $E\left[\Pr(w = B)\right]^{CPC} = \frac{1}{2} \leq 1 - \frac{1}{2}\left[\phi + (1-\phi)\left(\frac{\beta}{\alpha}\right)^2\right] = E\left[\Pr(w = B)\right]^{CPA}$. **Q.E.D.**

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