Choosing fisheries harvest policies: when does uncertainty matter?

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Abstract: Fisheries harvest policies are formulated under uncertainty because estimates of stock abundance and biological parameters are imprecise. However, allowable harvests are often based solely on point estimates of these quantities, essentially ignoring uncertainty. Some fishery scientists have advocated adjusting harvest levels downward to account for uncertainty, but few formal methods have been developed to determine how large these “uncertainty adjustments” should be. We describe an age-structured simulation model that explicitly incorporated uncertainty in parameters of the stock-recruitment relationship, errors in abundance estimates, and year-to-year variability in recruitment and calculated which uncertainty adjustment was optimal in terms of expected discounted yield. The optimal adjustment varied considerably, depending on the stock and harvest policy simulated. The increase in expected value from incorporating the adjustment into the harvest policy was usually small, except when we modeled a biological “threshold”, where overharvests could lead to an irreversible stock collapse. Therefore, while our analysis suggests that basing harvest decisions solely on the best point estimates may often be an approximately optimal strategy, it also indicates that large adjustments may sometimes be appropriate. Consequently, fishery managers should avoid making arbitrary adjustments for uncertainty, and instead derive the optimal adjustment for each situation.

Introduction

Uncertainty is a widely recognized aspect of fisheries management, but it is often ignored in practice. Allowable harvest levels are frequently based solely on the best point estimates of stock abundance and stock-recruitment parameters, without accounting for the uncertainty in those estimates (Walters 1981; Gulland 1983; Lovejoy 1988; Sainsbury 1991). This practice conflicts with the common intuition that uncertainty should dictate more conservative...
harvesting strategies (Clark 1985) and is counter to the prevailing belief that uncertainty should be explicitly incorporated in the formation of management policy (e.g., Peterson and Smith 1981; Sissenwine 1984; Clark 1985; Brown and Patil 1986; Bergh and Butterworth 1987; Linder et al. 1987; Francis 1992; Thompson 1992).

Several approaches have been advanced to formally incorporate uncertainty into harvest policy decisions. One approach is to estimate the probabilities that various management options will cause the fish stock to fall below a predefined threshold in a specified period (Brown and Patil 1986; Linder et al. 1987; Francis 1992). A second approach is to measure the expected economic losses caused by errors in parameter estimates and to compare these losses for different harvest policies to help select one that is robust to such errors (Ruppert et al. 1985). A third approach is to use stochastic dynamic programming to directly compute the optimal harvest decision, given the management objectives, the current uncertainties, and the effect of management actions on future uncertainties (Walters 1975, 1981, 1986; Walters and Hilborn 1976; Silvert 1978; Parma and Deriso 1990).

Stochastic dynamic programming is the most sophisticated of these approaches because it explicitly calculates the decision that maximizes the chosen objective(s), and because it can formally incorporate the value of information of alternative management actions (the degree to which different harvest levels reduce future uncertainty). However, the computational demands of dynamic programming are formidable and it is often prohibitively difficult to calculate optimal policies for practical situations (Smith and Walters 1981; Walters 1981; Ludwig and Hilborn 1983; Mangel and Clark 1983; Parma and Deriso 1990).

The other approaches, which do not use formal optimization techniques, are much simpler to implement but do not provide the information the decision maker really needs; they usually focus more on describing the possible outcomes than on prescribing the optimal decision. Similarly, Clark (1985) and Walters (1986) noted that although confidence intervals often accompany the point estimates of parameters or state variables, the information they provide is rarely used, since, by themselves, they do not indicate how, or even if, the decision based on the point estimates should be modified. Without such quantitative guidelines for setting policy, managers sometimes make conservative allowances for uncertainty through arbitrary safety factors or uncertainty adjustments. For example, catch limits for groundfish stocks off the west coast of Canada are often set 25–50% lower than the estimated sustainable level when abundance estimates are highly uncertain (Tyler 1988).

Although such an approach may be intuitively appealing, it can only be justified if it improves management performance (i.e., if the benefit of reducing the risk of overfishing exceeds the cost of reducing harvests). Because there is little empirical evidence supporting this conclusion, it is worth asking whether such uncertainty adjustments are appropriate and, if so, exactly how large they should be.

In this paper, we propose a method for determining optimal uncertainty adjustments for harvest policies set in the presence of uncertainty in stock abundance estimates, uncertainty in the stock-recruitment parameters, and variability in recruitment. By computing the expected value of different uncertainty adjustments, we determined how much the harvest calculated from the best point estimates of stock abundance and stock-recruitment parameters should be adjusted to account for the uncertainty in those estimates. We first describe four concepts from decision theory that provide a useful framework for discussing harvest decisions made under uncertainty.

**Determining the optimal harvest under uncertainty: perspectives from decision theory**

**Deterministic and Bayes equivalent decisions**

A deterministic approach bases decisions solely on point estimates of parameters and state variables, with no formal consideration of uncertainty. In contrast, a "Bayes equivalent" approach (Walters 1981) explicitly accounts for parameter uncertainty by selecting the decision that maximizes expected value over the probability distributions of the uncertain quantities. An "active-adaptive" approach (Walters 1986) goes one step further — it calculates the optimal decision by considering not only the current uncertainties, but also the effect of the current decision on future uncertainties.

In this study, we consider the decision dictated by the Bayes equivalent approach to be the optimal decision. We recognize, however, that it may not always be truly "optimal" because it may impede movement toward the optimal harvest policy compared with an active-adaptive approach that explicitly includes the informative value of alternative decisions as a criterion for selecting management strategies (see Walters and Hilborn 1976; Smith and Walters 1981; Walters 1981, 1986; Parma and Deriso 1990; Hilborn and Walters 1992). Notwithstanding the fundamental conceptual differences between a Bayes equivalent and an active-adaptive approach, many stochastic dynamic programming studies in fisheries indicate that when the Bayes equivalent decision is not optimal, its expected value is usually very close to that of the true optimal strategy that involves adaptive “probing” (Walters and Hilborn 1976; Walters 1981; Parma and Deriso 1990). Furthermore, determining the Bayes equivalent solution is computationally tractable whereas obtaining the active-adaptive, optimal solution would be infeasible given the complexities included in our model — age structure, measurement errors, and multiple uncertain parameters. Thus, for the purpose of this study, ignoring the active-adaptive aspects of the problem is an appropriate and a necessary simplification.

**Calculating the optimal uncertainty adjustment**

The optimal harvest ($H^*$) in any given year is the harvest level that best achieves some chosen objective, such as long-term discounted yield. However, $H^*$ is unknown because it is a function of quantities that we are uncertain about, such as the stock abundance and the stock-recruitment parameters. Therefore, the harvest level that is actually set will typically deviate from $H^*$. The loss in value caused by this deviation is the opportunity loss (Raiffa 1968; Jones 1977) for that decision.
Fig. 1. (A) Loss function describing the costs of deviating from the optimal harvest level ($H^*$). (B) Distribution describing the probability of different deviations of $H$ from $H^*$, for a given harvest level ($H$). The distribution shown here corresponds to the deterministic harvest level ($H_D$). For harvests lower than $H_D$, the distribution would shift to the left; for harvests higher than $H_D$, it would shift to the right. (C) Expected opportunity loss (EOL) for each harvest level. The EOL is the expected value of eliminating the uncertainty in $H^*$, given the costs of deviating from $H^*$. $H_B$ is the Bayes equivalent harvest level.

The relationship between the size of this deviation and the magnitude of the opportunity loss can be plotted as a loss function (Figs. 1A and 2A). The opportunity loss is zero at $H^*$ by definition. It usually increases continuously as the harvest level that is set ($H$) deviates from $H^*$. The loss function can be linear or nonlinear, symmetric or asymmetric. For a symmetric loss function (Fig. 1A), the opportunity loss depends only on the size of the deviation — underharvests and overharvests are equally costly. But, for an asymmetric loss function (Fig. 2A), the opportunity loss depends on the direction of, as well as the size of, the deviation. In this example, overharvests are more costly than underharvests.

Because $H^*$ is unknown, the deviation of $H$ from $H^*$ is unknown. Therefore, the opportunity loss corresponding to a particular harvest level ($H$) cannot be calculated. Nevertheless, the expected opportunity loss (EOL) (Raiffa 1968) can be calculated by multiplying the probability of each possible deviation of $H$ from $H^*$ (Figs. 1B and 2B) by the opportunity loss corresponding to that deviation (Figs. 1A and 2A) and then summing the products. The EOL for different harvest levels is shown in Figs. 1C and 2C. (See Appendix A for a formal mathematical description of the calculations involved.)

The harvest level that minimizes EOL is the “Bayes equivalent” harvest ($H_B$). This may or may not correspond to the “deterministic” harvest ($H_D$) — the harvest level based solely on the best point estimate of $H^*$. For a symmetric loss function (Fig. 1A), where the costs of overharvest and underharvest are equal, $H_B$ will coincide with $H_D$, no matter how uncertain our estimate of $H^*$ is (provided that this uncertainty is described by a symmetric probability distribution — see the last part of the sensitivity analyses section). However, for an asymmetric loss function (Fig. 2A), $H_B$ will differ from $H_D$ because the EOL can be reduced by adjusting the allowable harvest in the direction that has lower opportunity losses. For example, in Fig. 2C, $H_B$ is lower than $H_D$ because it pays to “hedge” against the higher costs of overharvest.

The difference between $H_B$ and $H_D$ is the optimal uncertainty adjustment (“Adjust.” in Fig. 2C); it is the amount that the harvest level calculated from the best point estimates of the stock abundance and stock-recruitment parameters should be adjusted to account for the uncertainty in those estimates, given the difference in costs between overharvests and underharvests. The reduction in EOL resulting from the uncertainty adjustment is called the expected value of including uncertainty (EVIU) (Morgan and Henrion 1990). The EVIU must be differentiated from a more common term in decision theory — the expected value of perfect information (EVPI) (Raiffa 1968; Walters 1986). The EVPI is the EOL at $H_B$ (Figs. 1C and 2C) whereas the EVIU is the difference in EOL between $H_B$ and $H_D$ (Fig. 2C). In other words, the EVPI is the cost of being uncertain whereas the EVIU is the additional cost of ignoring uncertainty by basing the harvest decision solely on the best point estimates of the stock abundance and the stock-recruitment parameters (hereafter referred to just as the best point estimates). Equivalently, the EVPI is the value of eliminating uncertainty whereas the EVIU is the value of accounting for uncertainty by using the additional information contained...
in the probability distributions of the uncertain point estimates (Morgan and Henrion 1990).

A large EVPI does not necessarily imply a large EVIU. Although eliminating uncertainty is nearly always valuable, explicitly including uncertainty may not be. For instance, for a symmetric loss function and distribution (Figs. 1A and 1B), the decision dictated by the deterministic approach \(H_0\) is the same as that dictated by the Bayes equivalent approach \(H_b\), and therefore has the same EOL (Fig. 1C). Consequently, explicitly including uncertainty does not alter the decision in this case. Conversely, for an asymmetric loss function (Fig. 2A), explicitly including uncertainty is valuable because it leads to the selection of a superior decision — one with a lower EOL and thus a higher expected value (Fig. 2C).

The preceding hypothetical examples were only intended to illustrate why uncertainty adjustments may or may not be appropriate. In practice, neither the probability distribution describing the uncertainty in \(H^*\) nor the parameters of the loss function can be easily specified, and analytical solutions may require such extreme simplifying assumptions that they will be of little practical use. Therefore, we used a Monte Carlo simulation model to numerically estimate the optimal uncertainty adjustment.

**Methods**

This section illustrates how to calculate the optimal uncertainty adjustment for a harvest policy that confronts several sources of uncertainty simultaneously: errors in estimates of stock abundance, uncertainty in the underlying stock-recruitment parameters, and year-to-year variability in recruitment processes. We explored uncertainty adjustments for a constant escapement policy and a constant harvest rate policy for the Atlantic menhaden (*Brevoortia tyrannus*) and the Arcto-Norwegian stock of Atlantic cod (*Gadus morhua*). We chose these two stocks simply because they were examples of fish with widely differing life history characteristics (their biological parameter values are listed in Tables 1 and 2); this paper is not directed toward the management of these stocks specifically. This is particularly true because more recent data may alter estimates of the parameters that we used in our examples.

**General description of the model**

The nine basic steps of the simulation model (Fig. 3), which are elaborated upon below, were as follows. (1) The optimal escapement or optimal harvest rate was estimated from the point estimates of all biological parameters and the discount rate applied to future harvests. (2) An uncertainty adjustment was chosen, which modified the harvest level calculated from the deterministic analysis. (3) The abundance of the vulnerable stock was estimated. (4) The allowable harvest was set, according to the abundance estimate, the type of harvest policy simulated, and the specified uncertainty adjustment strategy for that loop in the model. (Note that henceforth we use "harvest policy" to refer to either the constant escapement or constant harvest rate policies whereas "strategy" will refer to the amount of adjustment for uncertainty in an escapement goal or harvest rate.) (5) The stock remaining after harvest produced
recruits according to a Shepherd (1982) stock–recruitment model. Uncertainty in the stock–recruitment parameters was simulated by bootstrapping the stock–recruitment data set and assigning the resulting parameter estimates to Monte Carlo trials. Interannual variability in recruitment was simulated by applying a lognormal, multiplicative random error term to the predicted recruitment. (6) The present value (PV) of the sum of harvests for each trial was calculated by discounting the dollar value of harvests to year 1, using a 5% discount rate in the baseline case. (To convert harvest in metric tons into dollar value, we assumed as a rough approximation that menhaden were worth $100 per metric ton and cod were worth $1000 per metric ton.) (7) 1000 trials were conducted for each uncertainty adjustment strategy; the expected PV for that strategy was the average PV of those 1000 trials. (8) A different uncertainty adjustment was set and the procedure was repeated. (9) The adjustment that maximized expected PV was the optimal uncertainty adjustment for that simulated stock and harvest policy. The EVIU was the increase in expected PV of the optimal adjustment strategy over the zero adjustment or deterministic strategy.

**Estimating the optimal escapement**

For an age-structured stock, the optimal escapement \((S^*)\) and optimal harvest rate \((F^*)\) are complicated functions of the recruitment parameters \((a, k, b\) in the Shepherd (1982) equation), natural mortality rate \((M)\), weight at age \((w)\), fecundity of females at age \((f)\), relative vulnerability at age \((q)\), and the discount rate applied to future harvests \((r)\). We estimated the optimal escapement or optimal harvest rate iteratively, by using the published estimates of the relevant biological parameters (or the best-fit parameters for the stock–recruitment relationship) and by incrementally varying the escapement or harvest rate until the PV of harvests was maximized. We assumed deterministic recruitment to simplify this calculation because we found that the optimal escapement or harvest rate was virtually identical whether deterministic or stochastic recruitment was simulated, which is consistent with the results of Reed (1978), Ludwig and Varah (1979), Ludwig and Walters (1982), and Clark (1985).

**Table 1. Parameter values used in the model for Atlantic menhaden (Brevoortia tyrannus) (taken from Ahrenholtz et al. 1987). Instantaneous natural mortality rate was 0.45.**

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Weight (kg)</th>
<th>Fecundity (1000's eggs per female)</th>
<th>Relative catchability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.015</td>
<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
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<td>2</td>
<td>0.300</td>
<td>41</td>
<td>1</td>
</tr>
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<td>3</td>
<td>0.480</td>
<td>176</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.600</td>
<td>269</td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0.720</td>
<td>525</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 2. Parameter values used for Arctic-Norwegian cod (Gadus morhua) estimated from graphs in Cushing (1966), Garrod (1967), and Schopka and Hempel (1973). Instantaneous natural mortality rate was 0.2.**

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Weight (kg)</th>
<th>Fecundity (1000's eggs per female)</th>
<th>Relative catchability</th>
</tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>1</td>
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<td>7.0</td>
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<td>1</td>
</tr>
<tr>
<td>11</td>
<td>8.2</td>
<td>2460</td>
<td>1</td>
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<tr>
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<td>1</td>
</tr>
<tr>
<td>15</td>
<td>12.9</td>
<td>6450</td>
<td>1</td>
</tr>
<tr>
<td>16</td>
<td>14.0</td>
<td>7000</td>
<td>1</td>
</tr>
</tbody>
</table>

The deterministic analysis was used solely in the first step of the model, to calculate the best point estimate of optimal escapement or optimal harvest rate and to obtain the equilibrium age structure for initializing each Monte Carlo trial. Uncertainty was still incorporated explicitly in the simulations because the stock–recruitment parameters for each trial were randomly drawn from a bootstrap distribution, the stock was estimated with error, and the stock–recruitment process was simulated with interannual variability.

**Estimating stock abundance**

For each year within a Monte Carlo trial, the estimate of the stock abundance that was vulnerable to the fishery \((S_{est})\) was calculated by adding a normally distributed error term to the true abundance of the vulnerable stock \((S)\). The error term was created by multiplying \(S\) by the coefficient of variation \((CV)\) of the estimation procedure and a normally distributed random variable \((w)\), with a mean of 0 and a variance of 1:

\[
S_{est} = S + (S \cdot CV \cdot w).
\]

The CV was varied from 0 to 0.5 in increments of 0.1 to represent a wide range of levels of uncertainty in estimates of abundance. (The average CV for trawl surveys in the Georges Bank area of the eastern United States is approximately 0.25 (Sissenwine 1984.).) \(CV = 0\) represents the special, hypothetical case where stock abundance was measured perfectly, which allowed us to dissociate the effects of uncertainty in the stock–recruitment parameters from the effects of uncertainty in abundance estimates.
Fig. 3. Flowchart showing the main components of the model.

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START

Optimal escapement is estimated from the point estimates of all biological parameters and discount rate

For each uncertainty adjustment

For Monte Carlo trial = 1 to 1000

Initialize stock abund.

For year = 1 to t

Estimate stock abund.

Harvest the stock and apply discount rate to calculate PV

Calculate remaining spawning stock and determine recruitment

Average PV for 1000 Monte Carlo trials is the expected PV for that uncertainty adjustment

Adjustment with highest expected PV is optimal

Loop over different adjustment strategies (in escapement or harvest rate)

Loop over Monte Carlo trials

Loop over years

(20 for menhaden, 50 for cod)
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### Calculating the harvest

When a constant escapement policy was simulated, the stock was not harvested when $S_{est}$ was lower than the target escapement ($S_T$). Otherwise, the harvest ($H$) was the difference between $S_{est}$ and $S_T$, provided that this difference was not larger than 99% of the vulnerable stock ($S$):

1. If $S_{est} < S_T$  
   \[ H = 0 \]
2. If $S_{est} - S_T \leq 0.99\cdot S$  
   \[ H = S_{est} - S_T \]
3. If $S_{est} - S_T > 0.99\cdot S$  
   \[ H = 0.99\cdot S \]

For a constant harvest rate policy, $H$ was determined by multiplying the estimated stock abundance ($S_{est}$) by the target harvest rate ($F_T$), although $H$ was again bounded at 99% of $S$:

If $F_T\cdot S_{est} \leq 0.99\cdot S$  
\[ H = F_T\cdot S_{est} \]

If $F_T\cdot S_{est} > 0.99\cdot S$  
\[ H = 0.99\cdot S \]

We bounded the harvest at 99% of the vulnerable stock for both policies because we assumed some limit to the efficiency of the fleet (that the entire vulnerable stock would not be harvested, no matter how high the allowable harvest was set). Because we modeled age-structured stocks, the numbers harvested at each age were determined on the basis of their relative abundance and catchability. We assumed equal catchability for all ages after first recruitment (see Tables 1 and 2).

### Calculating spawning stock biomass

The spawning stock ($S_{sp}$) was represented as egg potential, which was calculated by multiplying the numbers of female fish of each age class, $i$, remaining after harvest ($N_i$) by their average fecundity at age ($f_i$) and then summing over all age classes:

\[ S_{sp} = \sum(N_i \cdot f_i). \]

### Recruitment

We modeled recruitment with the Shepherd (1982) model:

\[ R = (a S_{sp} / (1 + (S_{sp}/k)^b))e^v \]

with the following definitions:

- $R$ is the recruitment at age 0 (in billions).
- $S_{sp}$ is the egg production (in billions).
- $a$, $k$, and $b$ are parameters estimated from nonlinear parameter estimation with $\log(R/S)$ as the response variable.
- $a$ is the slope of the recruitment curve at the origin, $k$ is the spawning stock level at which recruitment is reduced to half the level it would be under density-independent processes only (Shepherd 1982) (i.e., the spawning stock level at which compensatory processes reduce the recruitment to half the value that would be expected from extrapolating the slope of $R/S$ at the origin), and $b$ reflects the nature and intensity of compensatory processes that control the overall form or shape of the stock–recruitment relationship. When $b > 1$, the Shepherd model generates a domed stock–recruitment relationship, similar to a Ricker model. When $b = 1$, the Shepherd model is identical to a Beverton–Holt model. When $b < 1$, the Shepherd model generates a relationship where recruitment increases monotonically, without limit.

$v$ is a normal random variable distributed $N(0, \sigma^2_{env})$ where $\sigma^2_{env}$ is the variance of the residuals around the best fit curve to $\log(R/S)$. $\sigma^2_{env}$ was reestimated for each bootstrapped data set (it averaged 0.41 for menhaden and 0.75 for cod). Because $v$ is normally distributed, $e^v$ is lognormally distributed, with an expectation of $\exp(\sigma^2_{env}/2)$. Thus, including environmental variability in the simulations in this manner corrected for the fact that the stock–recruitment parameters, estimated by fitting $\log(R/S)$, represented the geometric, rather than the arithmetic, mean of the stock–recruitment relationship.

We chose to use the Shepherd model, rather than the more common Ricker or Beverton–Holt models, because the Shepherd model allows the uncertainty in the overall shape of the stock–recruitment relationship to be directly...
accounted for via the $b$ parameter. The Ricker and Beverton-Holt models do not have this flexibility. If the basic shape of the stock-recruitment relationship is itself uncertain, assuming one of these more common models may lead to misleading measurements of uncertainty.

To incorporate uncertainty about the parameters of the Shepherd model in our analysis, we performed a simple bootstrap (randomly sampled the stock-recruitment data points, with replacement, from the original data set) to generate 1000 "new" bootstrapped data sets. For each new data set, the stock-recruitment parameters and the environmental variability term were reestimated by nonlinear parameter estimation. Each parameter set was then used as the stock-recruitment parameters for a particular Monte Carlo trial.

Simulating the different uncertainty adjustment strategies
For the constant escapement policy, the different uncertainty adjustments consisted of either increasing or decreasing the escapement from the deterministic escapement — the one calculated as optimal from the best point estimates. To simulate harvest strategies ranging from extremely conservative to extremely permissive, the adjustments were varied in 10% intervals, from increasing the deterministic escapement by 100% to decreasing it by 100% (i.e., from doubling the escapement to setting the allowable harvest to be the same as the estimated stock abundance, $S_{est}$). Note in the latter case that escapement was $>0$ when the true abundance of the vulnerable stock, $S$, was $>S_{est}$ because $H$ was set to $S_{est}$ and escapement $= S - H$. Adjustments that increased escapement from the deterministic level were considered conservative because allowable harvests were set lower than the level that the point estimates dictated as optimal. Conversely, adjustments that decreased escapement were considered permissive because allowable harvests were set higher.

For the constant harvest rate policy, the different uncertainty adjustments consisted of either decreasing or increasing the harvest rate from the deterministic rate — the proportion of the stock that it would be optimal to harvest if the point estimates were, in fact, the true values. For menhaden, the deterministic harvest rate was 72%, and the different uncertainty adjustments consisted of simulating different harvest rates ranging from 52 to 92% in 2% increments. Exploitation rates lower than 72% were called conservative adjustments; those higher than 72% were called permissive. For cod, the deterministic harvest rate was 15% and the different adjustments were harvest rates ranging from 5 to 25% in 1% increments. Exploitation rates lower than 15% were conservative; higher ones were permissive.

Each Monte Carlo trial was run for 20 years for menhaden and 50 years for cod. Previous explorations indicated that these periods were sufficient to generate stable results because we used the equilibrium age structures to start each run, as noted above.

Determining the optimal adjustment strategy
The expected PV for each adjustment strategy was the average PV of the 1000 trials applying that adjustment. The optimal uncertainty adjustment was the one with the largest expected PV. The EVIU was the increase in expected PV of the optimal strategy over the deterministic strategy.

Results and discussion
The expected PV for each uncertainty adjustment for each simulated stock and harvest policy is shown in Fig. 4.

Constant escapement policy
For menhaden (Fig. 4A), the deterministic escapement was optimal for the CV = 0 case, despite the considerable uncertainty in the stock-recruitment parameters. However, when uncertainty in the stock abundance estimates was included (CVs of 0.1 through 0.5), the optimal escapement became increasingly conservative (in the CV = 0.5 case, it was 80% higher than the deterministic escapement). Nevertheless, note how flat the curves in Fig. 4A are over a wide range of escapements. Thus, the expected PV was so insensitive to escapement that the optimal strategy had only a slightly higher expected PV than the deterministic strategy — ranging from a 0.4% increase in the CV = 0.1 case to a 3.2% increase for the CV = 0.5 case.

For the CV = 0 case for cod (Fig. 4B), the optimal escapement was 20% lower than the deterministic escapement. As with menhaden, the optimal escapement became more conservative as the level of uncertainty in the abundance estimates was increased, but, again, the expected PV of the deterministic strategy was nearly as high as that of the optimal adjustment strategy — within 2% in all six cases.

The decrease in performance of conservative strategies was much more marked for cod than for menhaden. Because the average stock-recruitment relationship for cod was considerably more dome shaped (the point estimate of the $b$ parameter was 3.59 for cod, compared with 1.05 for menhaden), conservative harvest strategies not only reduced current harvests, but also often created high spawning stock abundances that reduced subsequent recruitment and, thus, future harvests. This phenomenon does not occur for stock-recruitment relationships that exhibit relatively constant or increasing recruitment with increases in stock abundance.

The performance of conservative strategies for cod was especially poor for low levels of uncertainty in the abundance estimates (low CVs). In fact, reducing the uncertainty in abundance estimates actually decreased the performance of conservative strategies (notice on the left side of Fig. 4B that performance was highest in the CV = 0.5 case, and lowest in the CV = 0 case). This counterintuitive result can easily be explained. For harvest strategies that are far too conservative or far too permissive, errors in abundance estimates may compensate for the inappropriateness of the strategy by causing the harvests to be set closer to the true optimal level. For instance, for conservative adjustments to the constant escapement policy for Arcto-Norwegian cod (Fig. 4B), high CVs in the stock estimation procedure frequently generated large overestimates of stock abundance, which increased the allowable harvests, often bringing escapements closer to the true optimal level. While the higher CVs also generated larger underestimates, the beneficial effect of the overestimates
Fig. 4. Expected present value (PV) (over 20 years for menhaden and 50 years for cod, with a 5% discount rate applied for both stocks) for each uncertainty adjustment for four different cases: (A and B) constant escapement policy for menhaden and cod; (C and D) constant harvest rate policy for menhaden and cod. "D" on the x-axes designates the deterministic strategy — the escapement or harvest rate that would be applied if only the point estimates were used to choose the management action. For Figs. 4A and 4B, the x-axis represents the amount of adjustment in the escapement goal from the deterministic strategy whereas for Figs. 4C and 4D, the x-axis is the constant harvest rate applied. In each graph, the six curves represent results for coefficients of variation (CV) in
the stock abundance estimation procedure ranging from 0 to 0.5. The optimal uncertainty adjustment for a given CV of the stock estimation procedure is the one that maximizes expected PV. The respective optima for each case of CV are shown explicitly in Figs. 4A and 4C as solid dots on the curves.

was not cancelled because underestimates did not affect the resulting management action as often. That is, whenever the abundance estimate was below the target escapement, the allowable harvest set to zero — no matter how large the underestimate.

The same phenomenon operated in reverse for menhaden. Notice on the right side of Fig. 4A that more accurate stock measurement (lower CVs) reduced the performance of extremely permissive adjustment policies. In this example, the benefits of the larger underestimates of stock abundance from the high CV cases outweighed the costs of the larger overestimates because the maximum possible harvest was bounded at 99% of the stock, regardless of how severely the stock was overestimated.

Thus, the value of improving abundance estimates depends on the context in which those estimates will be used. Reducing uncertainty in the abundance estimate is clearly beneficial when the adjustment strategy for escapement is approximately optimal (i.e., near the adjustment that generates the highest PV). The exact value of a given reduction in uncertainty can be assessed directly, by measuring the increase in performance as CV is reduced (compare the maximum PVs of each of the CV curves in Fig. 4A). However, as the adjustment strategy becomes increasingly suboptimal, the value of reducing uncertainty in the abundance estimates not only decreases, but may even become negative.

Constant harvest rate policy

The results above were for the constant escapement policy; results for a constant harvest rate policy differed. For the CV = 0 case, the optimal harvest rate for menhaden (Fig. 4C) was 68% — 4% lower than the deterministic rate of 72%. The optimal harvest rate became even more conservative as uncertainty in the stock estimates was increased: it was only 52% in the high uncertainty case (CV = 0.5). Although the increase in expected PV of the optimal adjustment strategy over the deterministic strategy was still modest, it was larger than for the constant escapement policy (for the CV = 0.5 case, the performance of the optimal adjustment strategy was 13.5% higher than the deterministic strategy).

For cod (Fig. 4D), the optimal harvest rate coincided with the deterministic rate for all levels of uncertainty in the stock estimates. More surprisingly, the expected PV was almost independent of the level of uncertainty in the stock estimates (the six curves in Fig. 4D nearly overlap one another). At the optimal harvest rate, the expected PV was only 2% higher in the CV = 0 case than in the CV = 0.5 case. This difference was much smaller than for the other three cases of Fig. 4, where the expected PV of the optimal harvest policy for the CV = 0 case was 5.7, 10.2, and 17.2% higher than it was for the CV = 0.5 case (see Figs. 4B, 4C, and 4A, respectively). In other words, in the particular case of cod managed with a constant harvest rate policy, not only was the value of including uncertainty (EVIU) negligibly small, but the value of reducing uncertainty (some fraction of the EVPI) was also very small.

This surprising result may be because we simulated cod recruiting to the fishery at age 4, although the optimal age at first entry was about 7. When the age at first capture is less than that which is required to produce maximum yield, periodic or “pulse-fishing” strategies will usually outperform strategies that aim at taking a constant harvest (see Walters 1969; Clark 1976; Botsford 1981; DeKlerk and Gatto 1981). In these cases, greater uncertainty in the abundance estimates creates more variable harvests from year to year, which may more closely simulate a pulse-fishing policy. The benefit of this system behavior may partially offset the costs of greater uncertainty, and may explain why, in this case, reducing uncertainty in the abundance estimates did not substantially improve performance.

Sensitivity analyses

Discount rate

For the analyses described above, the PV was calculated using a discount rate of 5%. We repeated these analyses using discount rates of zero and 10%, but the general conclusions were not altered. Neither the deterministic harvest level nor the optimal uncertainty adjustments were appreciably changed (the deterministic harvest level was slightly lower for a discount rate of zero, and slightly higher for a discount rate of 10%). Therefore, we would agree with Mendelssohn (1982) that for many fish stocks, the optimal harvest strategy is insensitive to the discount rate applied to future harvests. However, this generalization will not be valid for extremely unproductive stocks, whose intrinsic rates of growth are similar to, or below, the proposed discount rate (see Clark 1973), or for active-adaptive policies, since the value of future information will strongly depend on how heavily the future is discounted (Walters 1981). In those cases, the optimal strategy can be dramatically affected by the discount rate applied to future harvests.

Performance measure

To this point, we have measured the performance of alternative harvest policies solely in terms of the expected PV of the catches, but there are other measures of success. Reducing the between-year variability of harvests or the incidence of very low harvests are often important additional considerations. To address these issues, we recorded the percentage of annual harvests (over the 1000 trials) that fell below some minimum value ($H_{min}$), defined as 10% of the deterministic, equilibrium harvest.

The main results of this analysis are shown in Fig. 5. For a constant escapement policy (Figs. 5A and 5B), the incidence of low harvests was very high, especially for high CVs in the abundance estimates. Conservative adjustments did not reduce this high frequency because they created such a high target escapement that the allowable harvest
Fig. 5. Similar to Fig. 4, except the performance criterion is the percentage of time that the harvests fell below $H_{\text{min}}$ (defined as 10% of the deterministic equilibrium harvest).

**Constant Escapement Policy**

A constant escapement policy greatly reduced the incidence of low harvests because some harvests were always allowed, no matter how much the stock fluctuated (notice the difference in scale on the y-axes in Fig. 5). For the $CV = 0.5$ case of the deterministic strategy, switching from a constant escapement to a constant harvest rate policy reduced the incidence of low harvests from 24 to 10% for menhaden (Figs. 5A and 5C) and from 75% to 6% for cod (Figs. 5B and 5D). Furthermore, conservative adjustments drastically reduced the incidence of very low harvests often had to be set below $H_{\text{min}}$ just to account for natural stock fluctuations. Conversely, permissive adjustments resulted in such low target escapements that the stock was often driven below $H_{\text{min}}$ by overharvests.

**Constant Harvest Rate Policy**

For the $CV = 0.5$ case of the deterministic strategy, switching from a constant escapement to a constant harvest rate policy reduced the incidence of low harvests from 24 to 10% for menhaden (Figs. 5A and 5C) and from 75% to 6% for cod (Figs. 5B and 5D). Furthermore, conservative adjustments drastically reduced the incidence of very low harvests often had to be set below $H_{\text{min}}$ just to account for natural stock fluctuations. Conversely, permissive adjustments resulted in such low target escapements that the stock was often driven below $H_{\text{min}}$ by overharvests.
when a constant harvest rate policy was used (Figs. 5C and 5D).

Comparison of constant escapement and constant harvest rate policies

Many have discussed the apparent conflict between a constant escapement policy (usually characterized by a high average harvest but also a high interannual variability in harvests) and a constant harvest rate policy (usually associated with lower average harvests but also lower harvest variability) (e.g., Ricker 1958; Allen 1973; Gatto and Rinaldi 1976; Hall et al. 1988; Quinn et al. 1990). Although this is an accurate characterization of the two policies for stocks that are optimally managed with perfect information, it does not necessarily apply where there is observation error in stock abundance, or in the cases where pulse-fishing can improve yield. In such situations, a constant harvest rate policy may not only have a lower harvest variability than the constant escapement policy, but it may also have a higher expected discounted yield, depending on the type of fish life history and the deviation from the deterministic strategy (e.g., Fig. 6).

Broader comparisons of the harvest policies can be done by contrasting, through Fig. 4, the results of applying the optimal escapement strategy for a given CV case with the results of applying the optimal harvest rate strategy for that same CV. In this context, the optimal constant harvest rate policy for cod had a slightly higher expected yield than the optimal constant escapement policy for all cases of CV. Perhaps this is because the stock was allowed to vary more under the constant harvest rate policy, which may produce superior yields when pulse-fishing is optimal. For menhaden, the optimal constant harvest rate policy resulted in a higher expected yield in all cases where observation error was included (CV = 0.1 through CV = 0.5 cases).

When the stock abundance is observed with error, setting the allowable harvests as a constant proportion of the estimated abundance may be less likely to lead to a severe overharvest than a constant escapement policy, which sets the allowable harvest as the difference between the abundance estimate and the estimated optimal escapement. For example, consider a stock whose true, but unknown, abundance is 1000, whose optimal escapement is 500 (assume this is known), and whose harvest rate is set at 50%. If this stock is estimated at 1500, then strictly following the constant escapement = 500 policy would dictate an allowable harvest of 1000, which would, theoretically, harvest the stock to extinction. The allowable harvest dictated by the constant harvest rate = 50% policy, however, would only be 750, substantially reducing the severity of the overharvest.

Of course, when the stock estimate is reasonably accurate, the constant escapement policy is more consistent with the objective of maximizing expected yield because it protects the stock completely when it is below the optimal escapement and fully exploits it when it exceeds the optimal escapement. On average, however, our results suggest that substantial observation error may make the constant harvest rate policy superior to the constant escapement policy in terms of maximizing expected yield. This may be yet another reason to favor a constant harvest rate policy, in addition to its lower harvest variability noted by the authors above, its superior performance for risk averse performance criteria (Deriso 1985; Hilborn 1985; Walters and Ludwig 1987), and its superiority for mixed-stock fisheries whose component stocks exhibit uncorrelated (or...
**Biological “thresholds”**

Because the baseline simulations assumed that some age classes were invulnerable to harvest and that only 99% of the vulnerable stock could be harvested, the probability of extinction of the simulated stock was virtually zero. However, a stock may not have to be harvested to zero for permanent commercial extinction to occur. A biological “threshold” may exist—a nonzero stock abundance below which biological processes either drive the stock to extinction, maintain it near an unproductive lower equilibrium, or substantially delay its recovery. Although there is little direct evidence for such thresholds, some mechanisms are known that could cause them, including the “Allee effect” (difficulty of finding mates at low abundance) (Allee 1931), certain forms of interspecific competition (Gilpin and Case 1976), and depensatory predation mortality (Gulland 1977; Peterman 1977). Furthermore, some stocks have not recovered from overfishing or have taken longer than expected to do so, despite reduction or elimination of fishing pressure (Paulik 1971; Clark 1974), which suggests the operation of one or more of these phenomena.

To represent a biological threshold, we modeled a situation where the recruitment dropped to zero whenever the spawning stock fell below some arbitrarily low level (which we define as 5% of the unfished equilibrium spawning stock). The stock could fall below the critical threshold because of overharvests, many successive years of very poor recruitment, or some combination of these two factors. The stock did not necessarily go extinct once it fell below the threshold. The maturation of the immature cohorts sometimes provided enough additional spawning stock to bring the total spawning stock back above the threshold level and allow recovery. If not, the stock became extinct.

For cod, the optimal uncertainty adjustment for this threshold case was the same as for the nonthreshold case because the probability of stock extinction was extremely low, given that four age classes were invulnerable to the fishery. Thus, for cod, even following an extreme overharvest where 99% of the vulnerable stock was removed, there was still a high probability that the next four cohorts would, collectively, be large enough for the stock to recover.

For menhaden, however, only two age classes were invulnerable to the fishery, creating a much higher probability of stock extinction and making the effect of the simulated threshold very significant (Fig. 7). For the threshold case, the optimal adjustment strategy (leftmost solid dots in Fig. 7) was quite conservative for both the constant escapement and constant harvest rate policies; it dramatically increased the expected value compared with the deterministic strategy (indicated as “+” in Fig. 7). There was a 41% increase for the constant escapement policy, from $501 million to $708 million, and a 129% increase for...
the constant harvest rate policy from $334 million to $765 million.

Therefore, for some stocks, the possibility of a biological threshold may dictate large, conservative safety factors or uncertainty adjustments. In such cases, conservative adjustments may increase expected PV of yields even if the probability that a threshold exists is fairly low because the cost of acting as if a threshold does exist when it actually does not is much lower than the cost of acting as if it does not exist when it actually does. To illustrate this, notice in the bottom panel of Fig. 7 that the expected PV is reduced by only $36 million (from $815 million to $779 million — Loss 1) if the optimal harvest rate for the threshold case (48%) is applied when no threshold exists whereas it is reduced by $186 million (from $765 million to $579 million — Loss 2) if the “no-threshold” optimal harvest rate (60%) is applied when a threshold actually exists.

In decision theoretic terms, these results indicate that the presence of a threshold may create such a highly asymmetric loss function (large differences in the costs between overharvesting and underharvesting) that extremely conservative uncertainty adjustments are appropriate. We emphasize that uncertainty adjustments should be justified on the basis of such asymmetries — the mere presence of uncertainty and the demonstration that estimation errors lead to losses in value are typically not sufficient justifications for acting conservatively, contrary to what Clark (1985, p. 241) implied. Thus, those who have advocated acting conservatively in the face of uncertainty (e.g., Kirkwood 1981; Kimura 1988; Thompson 1992) presuppose an asymmetric loss function for harvests or an asymmetric probability distribution of an uncertain quantity, although these issues are rarely addressed explicitly.

Our results presented in previous sections indicate that asymmetries may frequently exist and that they can be large in certain cases. However, they also suggest that in many situations, the loss function will be fairly symmetric, in which case a deterministic strategy will perform almost as well as the optimal adjustment strategy.

Two major exceptions to this apparent inadequacy of the deterministic strategy are when declines in stock abundance resulting from overharvesting are irreversible (when some critical, nonzero abundance threshold exists) or when the policy is nonfeedback, such that there is little opportunity to change the allowable harvests between years. Most managed fisheries have some feedback mechanism because the status of the stock is estimated annually (Kirkwood 1981). Therefore, the appropriateness of a deterministic harvest strategy may often rest on the likelihood and particular characteristics of possible nonzero thresholds or multiple stable states. While there is little direct evidence for such biological thresholds, there is strong indirect evidence that the component processes that can create them are common (e.g., nonlinear functional and numerical responses of predators (including humans) or density-dependent competitive interactions among species). Therefore, it is reasonable for managers to use conservative harvest adjustments to hedge against this possibility, but the justification for such an approach should rest on the asymmetric nature of the loss function or an asymmetric probability distribution and not simply on the presence of uncertainty.

Some of our results for the performance of the deterministic strategy might seem to contradict the message to be drawn from the many stock declines that have resulted from overfishing. But such collapses were not generally the result of adhering to a deterministic strategy in the face of uncertainty. Rather, they were the result of failing to keep harvests below levels even approximating those that the fishery models (deterministic or otherwise) specified as optimal. If anything, focusing excessively on uncertainty in these situations diminished the extent to which apparent stock declines motivated actions to control harvests (e.g., Saetersdal 1980).

Risk aversion
By using expected value as the primary criterion to compare the performance of alternative strategies, we have assumed a “risk-neutral” decision stance. That is, we have assumed that the strategy with the highest expected PV of harvests is preferable (has the highest utility), regardless of the particular distribution of harvests among years. For a “risk averse” manager, however, utility does not increase linearly with harvest, so maximizing expected PV may not be the appropriate performance criterion. A fisheries manager who is risk averse may often prefer a management policy with more stable harvests, even if that policy reduces the long-term average harvests.

Therefore, in some circumstances it may be better to use a risk averse performance criterion, such as the sum of the natural logarithm of harvests, rather than the sum of harvests (see Mendelssohn 1982; Deriso 1985; Ruppert et al. 1985; Walters and Ludwig 1987). We should clarify here that we use the term risk aversion in its strictest sense — to refer to a concave utility function, where the value placed on an additional unit of harvest decreases with increasing harvest. Confusion can result if the term is loosely used to mean “a strong desire to avoid bad outcomes.” For example, a constant harvest rate policy is typically viewed as more risk averse than a constant escapement policy because it avoids the low or zero harvests that a constant escapement policy would often dictate. However, if risk aversion is used to imply “a strong desire to avoid a biologically disastrous outcome,” a constant harvest rate policy may be less risk averse than a constant escapement policy because it may provide less protection to the stock when it is at dangerously low levels. Therefore, when using terms like risk aversion, it is important to be clear about exactly what one is “risk averse” to.

When we repeated our analyses using the natural logarithm of harvests as the performance criterion, the constant harvest rate policy outperformed the constant escapement policy more than previously indicated because it more frequently avoided the very low harvests that this performance criterion heavily penalizes. The optimal adjustment was virtually unchanged for the constant harvest rate policy, but was shifted to be more permissive for the constant escapement policy because avoiding zero harvests became so critical.

Asymmetric probability distributions
Until now we have implied that the symmetry of the loss function is the fundamental attribute determining how
much the harvests calculated from the best point estimates of stock abundance and stock-recruitment parameters should be adjusted to account for the uncertainty in these estimates. We have suggested that highly asymmetric loss functions dictate large adjustments to the deterministic strategy, that relatively symmetric loss functions dictate smaller ones, and that perfectly symmetric loss functions dictate no adjustment at all. However, if the uncertainty in the optimal harvest level (H*) is best described by an asymmetric probability distribution, the optimal strategy may deviate from the deterministic strategy, even if the loss function describing the costs of underharvests and overharvests is perfectly symmetric.

Thus, any assumptions made about the probability distribution describing the uncertainty in H* (and, thus, in the deviation of H from H*) should be carefully scrutinized. Because H* depends on the true stock abundance and the true stock-recruitment parameters, its probability distribution depends on shapes of the constituent distributions describing the uncertainty in these quantities. We made no direct assumption about the shape of the multivariate distribution of the recruitment parameters, because that was generated nonparametrically via bootstrapping. Moreover, whatever the shape of this distribution, it entered directly into the Monte Carlo simulations, so any asymmetry was accounted for. However, possible asymmetry in the distribution of errors in abundance estimates was not accounted for, since we assumed that they were normally distributed. If this assumption is strongly violated in practice, then the conclusions we drew would have to be reevaluated. However, some partial sensitivity analyses suggested that our conclusions were robust to moderate departures from this assumption.

**Conclusions**

Explicit representations of uncertainty, such as probability distributions, contain more information than point estimates, such as the mean. In some situations, formally including this additional information in policy analyses can reveal harvest strategies whose expected performance is much better than a deterministic strategy, which ignores uncertainty. However, uncertainty by itself is not typically a sufficient condition to warrant a departure from the deterministic strategy unless it can also be shown that the costs of overharvest and underharvest are asymmetric or that the probability density function for an uncertain component is asymmetric. The degree of asymmetry in costs will depend on the biological parameters of the stock’s population dynamics and on the criteria by which performance is measured. Although it is likely that some degree of asymmetry will always be present, our results suggest that such loss functions are not always highly asymmetric, since the deterministic strategy frequently performed well, even under high levels of uncertainty.

The situation where the deterministic strategy is most likely to perform poorly is when a biological threshold exists, such that large overharvests could lead to an irreversible stock decline. In this situation, overharvests can be much more costly than underharvests, which creates a highly asymmetric loss function and, correspondingly, dictates a large conservative uncertainty adjustment.

Therefore, our results demonstrate that uncertainty “adjustments” can improve management performance under certain circumstances, but also indicate that there are no general rules for determining the appropriate adjustment in a particular situation. Our methodology does, however, provide a rational basis for determining the best approach for each situation, which may prevent uncertainty from being improperly used to justify overly permissive or overly conservative harvest policies.

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**References**


**Appendix A**

$H^*$ is the true, but unknown, optimal harvest level.

$H_0$ is the best point estimate of $H^*$, as determined from the point estimates of stock abundance and the stock–recruitment parameters (hence called the deterministic harvest).

$H$ is the allowable harvest level that is set by the management agency.

$L(H,H^*)$ is the opportunity loss for a particular harvest $H$, given the true optimal harvest, $H^*$ (i.e., the opportunity loss corresponding to a particular deviation).

$f(H^*)$ is the probability distribution describing the uncertainty in $H^*$.

$EOL[H,f(H^*)]$ is the expected opportunity loss (EOL) for a given harvest level, $H$, and a given level of uncertainty in $H^*$. It is calculated by multiplying the opportunity loss corresponding to a particular deviation, $L(H,H^*)$, by the probability of that deviation, $f(H - H^*)$, integrated over all possible deviations $H - H^*$:

$$EOL[H,f(H^*)] = \int L(H,H^*)f(H - H^*) \, dH^*.$$ 

The harvest level that minimizes expected opportunity loss is the “Bayes equivalent” harvest ($H_B$):

$$EOL[H_B,f(H^*)] = \min \{EOL[H,f(H^*)] \}.$$ 

The optimal uncertainty adjustment is $H_B - H_D$.

The expected value of perfect informative (EVPI) is the expected opportunity loss at $H_B$:

$$EVPI = EOL[H_B,f(H^*)].$$

The expected value of including uncertainty (EVIU) is the difference between the expected opportunity loss at $H_D$ and the expected opportunity loss at $H_B$:

$$EVIU = EOL[H_D,f(H^*)] - EOL[H_B,f(H^*)].$$