BUNDLING AS AN ENTRY BARRIER*

Barry Nalebuff

In this paper we look at the case for bundling in an oligopolistic environment. We show that bundling is a particularly effective entry-deterrent strategy. A company that has market power in two goods, A and B, can, by bundling them together, make it harder for a rival with only one of these goods to enter the market. Bundling allows an incumbent to credibly defend both products without having to price low in each. The traditional explanation for bundling that economists have given is that it serves as an effective tool of price discrimination by a monopolist. Although price discrimination provides a reason to bundle, the gains are small compared with the gains from the entry-deterrent effect.

I. Introduction

In this paper we look at the case for bundling in an oligopolistic environment. We show that bundling is a particularly effective entry-deterrent strategy. A company that has market power in two goods, A and B, can, by bundling them together, make it harder for a rival with only one of these goods to enter the market. Bundling allows an incumbent to defend both products without having to price low in each. While it is still possible to compete by offering a rival bundle, a monopolist can significantly lower the potential profits of a one-product entrant without having to engage in limit pricing prior to entry.

We also show that bundling continues to be an effective pricing tool even if entry deterrence fails (or if there is already an existing one-product rival). A company with a monopoly in product A and a duopoly in product B makes higher profits by selling an A–B bundle than by selling A and B independently. Leveraging market power from A into B and accepting some one-product competition against the bundle is better than using the monopoly power in good A all by itself. Since bundling mitigates the impact of competition on the incumbent, an entrant can expect the bundling strategy to persist, even without any commitment.

The traditional explanation for bundling that economists have given is that it serves as an effective tool of price discrimination by a monopolist (see Stigler [1968], Adams and Yellen

* My heartfelt thanks go to Kyle Bagwell, Adam Brandenburger, Meghan Busse, Jeremy Bulow, Aaron Edlin, Rena Henderson, Roger Howe, Michael Riordan, Marius Schwartz, Philip Williams, an anonymous referee, and seminar participants at the Federal Trade Commission, Massachusetts Institute of Technology, Melbourne Business School, and Yale University.

The Quarterly Journal of Economics, February 2004

[1976]), Schmalensee [1982, 1984], McAfee, McMillan, and Whinston [1989], and Bakos and Brynjolfsson [1999]). Typically, a firm has to charge one price to all consumers. In these cases, variability in customer valuations frustrates the seller's ability to capture consumer surplus. Thus, a tool that helps reduce heterogeneity in valuations will help a monopolist earn greater profits. This advantage of bundling is especially apparent when the values of A and B are perfectly negatively correlated: offering an A–B bundle leads to homogeneous valuations among consumers, and thus the monopolist can capture 100 percent of the consumer surplus. Even if A and B have independent valuations, McAfee, McMillan, and Whinston [1989] show that a monopolist still does better by selling A and B as a bundle rather than independently.

Although price discrimination provides a reason to bundle, the gains are small compared with the gains from the entry-deterrent effect. Our baseline model has two goods, where consumer valuations are independent and uniformly distributed over [0,1]. In this environment, price discrimination through bundling raises a monopolist's profits from 0.50 to approximately 0.544, a gain of about 9 percent. Certainly worthwhile. But the same act of bundling cuts an entrant's potential profits by 60 percent. If this deters entry, profits are more than doubled. Even if this is not enough to deter entry, bundling is still of great value—postentry profits to the incumbent are more than 50 percent higher with a bundle offering compared with when the goods are sold independently.

The literature on bundling as a price discrimination tool emphasizes that it works best when the bundled goods have a negative correlation in value. This is when bundling most reduces the dispersion in valuations and allows the monopolist to capture the lion's share of consumer surplus. Bundling still works when the valuations are independent, but the gain from bundling disappears with perfect positive correlation.

The opposite is true when bundling is used as an entry deterrent or monopoly extension strategy. It is most effective when the bundled goods are positively correlated in value. Even with independent valuations, bundling is still an effective tool, but it loses its effectiveness when the goods are perfectly negatively correlated in value. The reason is that a one-product entrant has everything its consumers want when the valuations for A and B are negatively correlated. The markets for A and B are essentially different groups of consumers. In contrast, when A and B are positively correlated, the same group of consumers is buying both A and B and, thus, a one-product entrant cannot satisfy its customers.

Price discrimination and entry deterrence are only two of many reasons to offer a bundled product. Creating cost savings is another (see Salinger [1995]) as is creating a more valuable product.[1] In a larger sense, almost everything is a bundled product. A car is a bundle of seats, engine, steering wheel, gas pedal, cup holders, and much more. An obvious explanation for many bundles is that the company can integrate the products better than its customers can. For simplicity, we start with the case in which there is no complementarity in creating the bundle, either in consumption or in production. On the consumption side, that means Value(A + B together) = Value(A alone) + Value(B alone). Similarly, there is no complementarity in producing the bundle: Cost(A + B together) = Cost(A alone) + Cost(B alone).[2]

The motivating example for this paper is Microsoft Office, in which Word, Excel, PowerPoint, and Exchange are bundled together into a software suite. The consumer could just as well assemble his or her own bundle. In fact, many consumers may feel that they could do an even better job of assembling a bundle using “best-of-breed” components, perhaps combining Microsoft’s PowerPoint with Corel’s Word Perfect, IBM’s Lotus 123, and Qualcomm’s Eudora for email. Since not all of its products are best-of-breed, how does Microsoft gain an advantage by selling its office products as a bundle?

Certainly there are synergies between the software applications in Microsoft Office. The commonality of commands and the ability to create links between applications make the products easier to use. A single telephone number to call for help also makes the package more attractive. On the supply side, it is cheaper to include multiple products on a single CD disc than to package each one individually. We will show that even absent these synergistic gains, a monopolist concerned about competi-

1. Salinger [1995] recognizes that cost synergies from bundling are most valuable when consumer valuations are positively correlated. Thus, if most consumers would buy both (or neither) A and B when sold separately, then any cost savings from selling them together will create an incentive for a monopolist to sell bundled products when valuations are positively correlated.

2. This assumption rules out such topical bundles as Microsoft Windows and Explorer as Value(Windows + Explorer) > Value(Windows) + Value(Explorer). This is because the value of Explorer by itself is essentially zero since it needs an operating system on which to run. The case of bundling complements is considered later in the extensions section and in Cournot [1838], Matutes and Regibeau [1992], Spiller and Zelner [1997], Nalebuff [2000], Economides [1998], Carlton and Waldman [2002], and Heeb [2003].

tion would have a strong incentive to sell these products as a bundle rather than individually.

While bundled products are common in many sectors of the economy, the software suite is a particularly good example of the power of bundling because the marginal cost of software is zero. As marginal costs rise, bundling becomes less attractive. Bundling creates an inefficiency in that some consumers are "forced" to buy the bundle even though they value one of its components at below production cost (see Adams and Yellen [1976]). Thus, it is not surprising that software is especially conducive to bundling since this countervailing force does not arise. The potential for very large scale bundling of information goods is demonstrated in Bakos and Brynjolfsson [1999] who consider the limiting case of bundling together an infinite number of information goods.[3]

Remarkably few papers examine the role of bundling as an entry deterrent device or as a competitive tool to use against a rival with a limited product line. One explanation for this is that the Chicago School has largely succeeded in discrediting the idea of leveraging monopoly power (see, for example, Director and Levi [1956] and Schmalensee [1982] for a more formal argument). A company with a monopoly in good A gains no advantage by only selling A as part of a bundle with a competitively supplied product B.[4] The reason is that good B is freely available at its marginal cost. Thus, consumers evaluate an A–B bundle by whether or not A is worth more than the incremental cost of the bundle over B alone. Anyone who buys the bundle would also be willing to buy A alone, at the same profit margin for the monopolist. In fact, the monopolist would typically do even better by selling A alone as some customers who would buy A alone would not choose to buy the bundle.[5]

Whinston [1990] was the first to reexamine and resurrect the role of tying as an entry deterrent. He recognized that the Chicago School's criticism of leveraging monopoly power from market A to market B applies only if market B is perfectly competitive. Whinston demonstrates the advantages of tying when one firm has a monopoly in A and faces a competitor in B with a differen-

3. Bakos and Brynjolfsson [2000] apply this result to show how large bundles make entry especially difficult.

4. In this paper we will focus on creating an A–B bundle of fixed proportions. More generally, as the Chicago School has recognized, tying the sale of A to B but allowing for variable proportions creates the possibility of profitable price discrimination or metering.

5. A customer who has a small surplus for A but who values B below its production cost might prefer to purchase nothing than purchase an A–B bundle.

tiated product. In his model, tying commits the monopolist to being more aggressive against an entrant, and this commitment discourages entry. Of course, if entry occurs, the incumbent would then prefer to abandon his bundling strategy. We, too, find a reason to leverage monopoly power, although the model and the mechanism of action are quite different. For example, we find that bundling reduces the entrant's potential profits while mitigating the profit loss to an incumbent if entry occurs. Thus, bundling is credible even without any commitment device.

Choi and Stefanadis [2001] also rely on a commitment to bundle as a way to deter entry. In their model, A and B are only of any value when consumed together. Thus, an incumbent bundle completely forecloses a one-product entry. Innovation has a lower expected payoff as success in both the A and B good is required to gain access to the market.

Carlton and Waldman [2002] show how bundling can be used to deny an entrant scale and thus deter entry. One case they consider involves an entrant with a superior complementary product that might subsequently enter the original product market. If this firm can only enter the complements market, then its entry will increase industry profits, some of which will go to the original producer and entry will be welcomed. The problem arises if the rival might later enter the original market, in which case the incumbent's profits fall. Forcing a bundle product on consumers denies the entrant sales when it only has the complement product and this reduction in sales can be enough to make entry unprofitable. However, note that if entry were to occur, forcing a bundle would depress profits, and thus there is a commitment issue (which might be resolved through technical bundling). In our paper bundling arises without commitment. Even without complementarity, we find that bundling leaves little opportunity for a one-product firm to enter the market.

The plan of the paper is to first present the main results in as simple a model as possible. We begin with the case of independent valuations, a neutral ground on which to compare the price-discrimination effect with the entry deterrent or market-power effect. Conveniently, this is also the most mathematically tractable case. Then, in an extensions section, we show that the results are quite general. The extensions include nonuniform value distributions, three or more good bundles, nonzero production costs, positive and negative correlation in valuations, and complementarity among the products. We also demonstrate that the power of bundling extends to a game against an existing one-product rival or to the game where an incumbent responds to entry. The proofs for most propositions are in an Appendix.

II. A Model

Consider a market with two goods, labeled A and B. Consumers are interested in purchasing exactly one unit of A or one unit of B or both. A consumer of type α values A at α_a and B at α_b. We normalize the total market to be of size 1 and assume that budget constraints are not an issue. We further assume that the distribution of consumers is uniform over the unit square.[6] This implies that the valuations of A and B are independent and uniform over [0,1].

The two strategic players in the game are an incumbent and a potential challenger. The incumbent produces both goods A and B, each at zero marginal cost. The challenger is assumed to have a perfect substitute product for one of A or B, also produced at zero marginal cost. Whether the challenger will have a rival A product or a rival B product is random, and each outcome is equally likely. The incumbent's challenge is to prepare against possible entry in either A or B without knowing which flank the entrant will attack.[7]

Even though a challenger has a rival product, it need not enter the game. The entry decision will be based on whether the expected profits in the game cover its costs of entry. The entry costs are determined by the environment and known to all players.

It is reasonable to ask what an entrant will expect prices to be following entry. In the subgame perfect approach, the preentry price is irrelevant, except, perhaps, as an indication of the incumbent's costs.[8] But as Bain [1949] argued "the supposition that the potential entrant's judgment of industry demand and rivalry he

6. Note that the assumption of a uniform density is not just a mathematical simplification. As shown in the extensions section, over the entire class of symmetric quasi-concave densities—such as the multivariate normal—the uniform density minimizes the pure bundling effect. Thus, we can put aside issues of realism as the uniform density is chosen to be conservative rather than for its simplicity.

7. The model also applies to the more traditional setting where there is potential entry into only one market. In fact, the use of bundling as an entry deterrent strategy is unchanged whether the incumbent is defending market A or B or both. One of the attractive features of bundling is this very feature, that it allows an incumbent to defend both flanks at once.

8. The status quo price of the incumbent is relevant to an entrant when it can be taken as a signal of the incumbent's cost structure. Here, all the costs are known, and there is no private information and, hence, nothing to signal.

will meet is entirely unrelated to current price or profit in the industry, however, probably goes too far. Even if he does not believe the observed price will remain there for him to exploit, he may nevertheless regard this price as an indicator both of the character of industry demand and of the probable character of rival policy after his entry." We do not imagine that an entrant expects the incumbent not to react at all to entry—rather that the postentry price will be somewhere between the incumbent's preentry price and the result of postentry competition. If an entrant cannot justify entry costs at the prevailing preentry prices, then this is a persuasive argument not to enter the market.

Moreover, the entrant may employ a judo strategy by entering into the market with a limited capacity. If the incumbent is constrained to charge one price to all its customers (perhaps by a most-favored customer clause), it may not be worthwhile to reduce price in order to regain the limited number of customers that were stolen away.[9]

For these reasons, we first assume that the incumbent sets its prices prior to the challenger's entry decision. The incumbent's prices are then fixed for the rest of the game. This approach is generally favorable toward an entrant.[10] If an incumbent can deter entry without being able to lower prices post entry, then even a myopic entrant would be deterred from coming into the market.

The result that entry is made difficult via bundling does not rely on holding prices fixed. Later in the paper, we extend the results to include a Bertrand-Nash pricing equilibrium post entry. An entrant will also expect to make low profits when the incumbent responds. The surprise is how low an entrant's potential profits are without any price response. Profits are low primarily because a one-product entrant has difficulty capturing market share against a bundle, even when charging half the bundle price. By first employing a Stackelberg approach, we are able to illustrate this effect with simple mathematics.

9. In a similar vein, the entrant might be able to approach some customers and offer them a better deal before the incumbent can react. Once the incumbent reacts, the ensuing price war might destroy subsequent profitability; however, the short-term profits that arise during the period prior to discovery could be enough to cover entry costs. Here, we are not considering the repeated game and the incentives this might create to engage in a price war to deter future entry.

10. Over a small range of parameters, a sophisticated entrant might expect the incumbent to raise price post entry (to the Nash equilibrium level), as a commitment to a low price is used to deter entry.

As an entrant considers the incumbent's potential response, the most relevant question is not whether the incumbent will respond immediately in terms of pricing. The most relevant question is whether the incumbent will continue to bundle in response to entry. If the incumbent can revert to selling its goods individually, then Bertrand-Nash postentry pricing is a very powerful entry deterrent tool. (Pricing and profits on the good with entry would fall to zero, and entry would be costlessly deterred.) In fact, akin to a nuclear bomb, unbundled pricing is too powerful. We show that it is not credible to respond to entry by pricing goods individually when bundling is an option. Thus, an entrant can expect that an incumbent will bundle absent entry and will continue to bundle post entry.[11] This underscores the need to understand the nature of bundling.

II.A. *Independent Pricing*

As a baseline, we consider the case in which A and B are only sold separately. If the incumbent were alone in the market and priced the two goods independently, profits would be maximized at $p_a = p_b = 0.5$. The incumbent's profits would be $0.25 + 0.25 = 0.50$.

This pricing makes it particularly easy for a challenger to enter the market. The challenger comes in with a price of $0.5 - \epsilon$ and steals all the market in whichever product he has. If this happens, the entrant earns 0.25, and the incumbent's profits are reduced by half to 0.25.

The incumbent can also engage in limit pricing. For example, if the incumbent lowers the price of A, this would reduce a challenger's potential profits. Symmetrically, if it made sense for the incumbent to lower the price of A, it would also make sense for it to lower the price of B. Since the challenger can make essentially all of the profits of an incumbent in whichever market it enters, if an incumbent has preentry profits of Z, the entrant can make $Z/2$. Thus, instead of choosing a price directly, it is easier to think of the incumbent's strategy as choosing a profit level, which translates back into a price.

To deter a challenger with entry costs of E requires the incumbent to price at a point p such that its profits are no more than $2E$. This is because the entrant can take half of the incumbent's market and profits. If the incumbent chooses not to deter

11. This is the reverse of the Whinston [1990] result where the firm needs to commit to bundle in order to deter entry. We compare these two approaches and the different assumptions behind them in our section on the Nash pricing game.

entry, it should charge (½, ½), the optimal price absent entry. This is because the firm will lose one of two markets. In the remaining market, the firm does best to maximize profits, which occurs when price is ½.

Thus, the incumbent's choice is between deterring entry and earning profits of $2E$ or accepting entry and earning profits of ¼. The incumbent will deter entry if $E > 1/8$ and accept entry otherwise.

II.B. Pure Bundling

Here we assume that the incumbent can sell A and B only as a bundle. If the incumbent prices the bundle at 1, the sum of the monopoly prices for A and B, it would sell to half the market, and its profits would remain unchanged at ½. However, the marginal consumers are now more valuable (as price is 1, not ½), and this creates an incentive to cut price.[12] Absent entry, the profits of an incumbent with bundle price x are $x * (1 - x^2/2)$, for $x \leq 1$, which is maximized at $x = \sqrt{2/3}$. (Since the optimal monopoly bundled price is below 1, henceforth we will restrict our attention to the case of $x \leq 1$.)

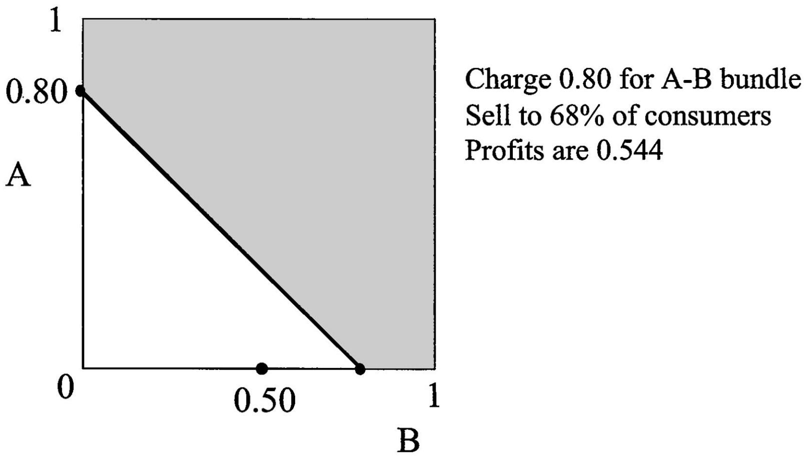

FIGURE I

12. The intuition of bundling as a more cost-effective way to discount comes from McAfee, McMillan, and Whinston [1989].

Having presented the basic mathematics of monopoly bundling, we are now ready to explain how bundling makes entry much less profitable without lowering the profits of the incumbent if entry is deterred. The advantages come from two channels, what we call (i) the pure bundling effect and (ii) the bundle discount effect.

The pure bundle effect examines the case in which the incumbent and challenger simply translate their independent pricing strategy into the bundled case without reoptimizing. Call this "equivalent prices." If the bundle is priced at 1, the incumbent makes the exact same amount as it would selling the two products independently, assuming no entry. The entrant considers coming into the market with a price of 0.50, essentially the same price it would charge against an incumbent selling A and B each for 0.50. (Of course, neither the incumbent's bundled price of 1 nor the entrant's contemplation of 0.50 is an optimal strategy—these simple translations from the independent case are used to demonstrate the pure bundling effect.)

Pure Bundling Effect. At equivalent prices, the act of putting two independent products into a bundle reduces the entrant's profits by 50 percent.

Assume that the entrant has product B. An entrant who comes in to the B market with independent pricing can steal the entire market with a marginal discount and thereby earn 0.25. Consider, in contrast, what happens when an entrant comes into a market against an incumbent with a bundle priced at 1. If the entrant prices at 0.50, it sells only to people who value B at above 0.50 and A at less than 0.50. A consumer who values A at more than 0.50 would be better served by buying the bundle. Consequently, the entrant only sells to 25 percent of the market, and its profits are cut by 50 percent. Figure II illustrates this effect. Instead of capturing all of the area to the right of p_e, the entrant is limited to a box with height $x - p_e$. Thus, if $x = 1$ and $p_e = 0.5$, the entrant's market is reduced by 50 percent—the entrant only gets the bottom half of its potential market.

The second channel that provides additional entry deterrence is the "bundle discount effect." As the bundle price is reduced and the bundle is sold at a discount relative to the original component prices, entry becomes even less profitable. This is low-cost or even costless deterrence, as absent considerations of entry an incumbent would choose to price its bundle at below 1.

Bundling Discount Effect. Selling the bundle at a discount to the optimal independent pricing provides an opportunity to raise

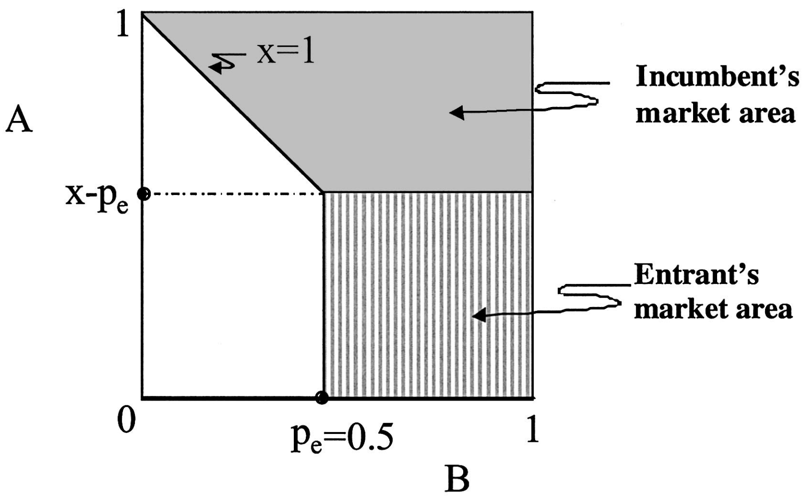

FIGURE II

the incumbent's profits absent entry, while making entry even less profitable. Reductions below the profit-maximizing bundle price have a second-order loss to the incumbent and a first-order loss to the entrant.

Recall that the optimal bundled price for an uncontested monopoly is $\sqrt{2/3} \approx 0.8$. Lowering the incumbent's bundle price from 1 to 0.8 reduces the potential profits of an entrant, while raising profits if entry is deterred. Reducing the bundled price below 0.8 further reduces the potential profits of an entrant, while also lowering profits if entry is deterred. However, the incumbent's price is near an optimum, and so profits fall slowly while the entrant's profits continue to fall rapidly.

Table I shows the incumbent's profits alongside the opportunities presented to an entrant. These calculations are all based on

TABLE I

Incumbent's price	Incumbent profits|no entry	Entrant's potential profits
1.00	0.500	0.148
0.80	**0.544**	0.105
0.68	0.523	0.081
0.41	0.375	0.034

the optimal price response by an entrant, as shown by Proposition 1 in the Appendix.[13]

If the incumbent charges 0.8, its profits absent entry rise from 0.50 to 0.544. Meanwhile, the entrant's potential profits fall by 30 percent, from 0.148 to 0.105.

The incumbent can push things even further at little cost. For example, a move down to a bundled price of 0.68 reduces the incumbent's profits absent entry to 0.523, or by about 2 percent. The entrant's potential profits are reduced by another 23 percent, down to 0.081.

Further prices begin to impose first-order costs on the incumbent. For example, if the bundle price falls to 0.41, the incumbent's profits fall to 0.375, about a 25 percent reduction. The entrant's potential profits fall almost 60 percent, all the way down to 0.034.[14]

The cumulative effect of these price cuts is impressive. Taking the bundled price down from 1 to 0.68 results in an inconsequential loss to the incumbent, while reducing the entrant's profits by 45 percent. Going all the way down to a bundled price of 0.41 reduces the incumbent's profits by 25 percent, but leaves the entrant with potential profits of only 0.034, a 77 percent reduction from its profits against a bundle priced at 1 and a fall of 86 percent compared with its potential profits of 0.25 when the incumbent sells the goods independently.[15]

II.C. Postentry Protection

Up until this point, we have focused on the reduction in potential profits to an entrant. A further benefit to the incumbent is that, if entry should occur, the loss to the incumbent is significantly reduced relative to the independent pricing case. Going back to the pure bundling effect and Figure II helps illustrate the point.

Take the case in which the incumbent charges $x = 1$ for the bundle and the entrant comes in selling B at $p_e = 0.5$. Instead of losing all of its B sales, the incumbent loses sales only to its

13. In response to an incumbent price of 1, the optimal entry price is ⅓; in response to an incumbent price of 0.8, the optimal entry price is approximately 0.29. Thus, the entrant's potential profits fall from $(1/3)(2/3)(2/3) = 0.148$ at $x = 1$ to $(0.29)(0.71)(0.51) = 0.105$ at $x = 0.8$ or a further 30 percent at no cost, and even some gain, to the incumbent.

14. As we show later on, the optimal bundle price in the subgame following entry is 0.59. If prices below 0.59 are not considered credible, then an entrant's potential profits can be held to 0.064.

15. At $x = 0.68$, $p_e = 0.27$, and $\Pi_e = (0.27)(0.73)(0.41) = 0.081$. At $x = 0.41$, $p_e = 0.18$, and $\Pi_e = (0.18)(0.82)(0.23) = 0.034$, a fall of 86 percent.

customers who value the bundle at above 1, but value A at less than 0.50. In fact, this is only one-quarter of the incumbent's market—but, since it loses the bundled sale, this is like losing half the sales on one of the products.

This, of course, understates the cost of entry. Against a bundle price of 1, the entrant would charge ⅓, not ½, and capture (⅔)(⅔) = 4⁄9 of the market. When $x = 1$, it is always the case that the incumbent loses half of what the entrant captures. Thus, the incumbent loses 2⁄9 ths of the market. Since the incumbent was charging a price of 1 for the bundle, this also represents its lost profits. In comparison, when the challenger came into the market with independent pricing, the incumbent lost all of the B market and half of its profits, or 0.25.

Anticipating entry, the incumbent can do much better by charging a price below 1. $\Pi_I|entry$ is maximized at $x = 0.68$, which leads to profits of 0.374. In response to this price, the entrant would charge 0.27 and earn 0.081.

Since the incumbent can earn 0.374 by allowing entry, the incumbent should attempt to deter entry only if doing so leads to profits above 0.374. Recall that the incumbent's profits without entry are given by $x(1 - x^2/2)$. As seen in Table II, a bundle price of 0.41 translates to incumbent profits of 0.375, absent entry. Facing a bundle price of 0.41, an entrant can earn only 0.034.

Putting this all together, the incumbent should charge 0.68 and accept entry if the rival's entry costs are below 0.034. Otherwise, the incumbent should engage in "limit pricing" and set a bundled price just low enough to deter the rival from entering. The incumbent follows this policy from a bundle price of 0.41 all the way up to a price of $\sqrt{2/3} \approx 0.8$, at which point there is no further gain from raising price.

The results about bundling can all be summarized in Figures III and IV. In Figure III the top curve shows the incumbent's profits as a monopolist. The middle curve traces the incumbent's profits if entry occurs, while the bottom curve illustrates the

TABLE II

Incumbent's price	Incumbent profits|no entry	Incumbent profits|entry
1.00	0.500	0.278
0.80	0.544	0.361
0.68	0.523	**0.374**
0.41	0.375	0.309

entrant's potential profits for each bundle price. The incumbent can always achieve the maximum of its profits-with-entry curve (profits of 0.374). Therefore, it deters entry only when the top curve lies above 0.374.

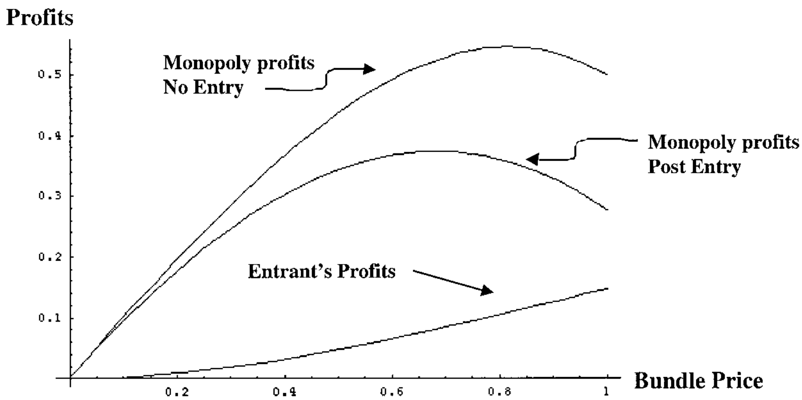

FIGURE III

The results of Figure III can be incorporated into a new graph (Figure IV) in which the horizontal axis now reflects entry costs.

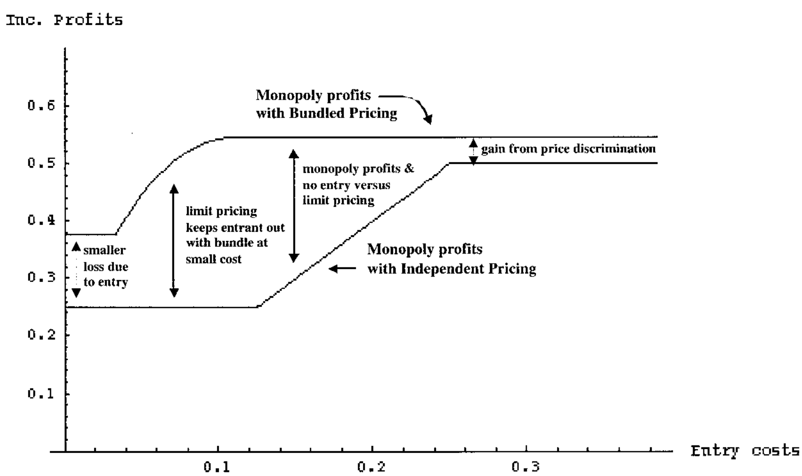

FIGURE IV

The price-discrimination effect on which previous authors have focused is evidenced by the fact that the bundle line reaches profits of 0.54 as opposed to 0.50. The ability of bundling to deter entry is reflected in the fact that entry occurs only when entry costs are below 0.034, compared with 0.25 in the case with independent pricing. It is remarkable that once entry costs are above 0.10, the incumbent can pick its unconstrained monopoly price and not be concerned about entry.

The result that entry is less costly to the incumbent is reflected in the fact that when entry occurs, the incumbent's profits are 0.374 versus 0.25, a 50 percent improvement. Thus, bundling mitigates the cost of entry. For this reason, it is in the incumbent's self-interest to maintain the bundle post entry. There is no issue of credibility or commitment.[16]

The rapidly rising profits in the curved section, where entry costs are between 0.034 to 0.1, demonstrate that bundled pricing is an efficient way to deter entry. When entry costs are 0.1, an incumbent that bundles earns 0.54, more than double the profits of an incumbent that sells its products independently. The takeaway message demonstrated by Figure IV is that the price discrimination effect offered by bundling is valuable, but the largest gains come from the entry-mitigation effect and the efficient entry-deterrence.

There is a side benefit to bundling that is hard to quantify. This benefit arises when entry costs are so low (below 0.034) that entry is not profitably deterred. When the entrant comes into the market, its profits are only 0.08, which is 68 percent less than when it enters against unbundled pricing.[17] Once the decision to accommodate entry has been made, there is no reason in our model for the incumbent to be concerned with a competitor's profits. Even so, we recognize that most firms would feel less threatened when their rivals are less profitable.

III. Extensions

In this section we first develop the basic model along several lines. We begin with some simple extensions to include mixed

16. In contrast, in Whinston [1990] bundling is used as a way to commit to *greater* postentry competition. This lowers the entrant's profits along with the incumbent's. Thus, the credibility of this commitment will be an issue.

17. In the case of independent pricing, firm 1 charges 0.5 for both A and B. Firm 2 comes in at a penny less for its product, say B, and earns ¼ (minus ϵ). When firm 1 employs bundling, the low-cost entrant faces a bundle price of 0.68 against which it would charge 0.265 and earn 0.08.

bundling, uncertain entry costs, nonzero production costs, and nonuniform distribution of valuations.[18]

We then explore in more depth the advantages of bundling more than two items together and the effect of positive and negative correlations in value. Positive correlation amplifies the gains to bundling while negative correlation makes it less valuable. The intuition for positive and negative correlation is often better captured by the existence of complementarity between the bundled goods, and we turn to this topic next. When the bundled products are complements, bundling is an even more powerful tool. The value does not disappear with substitutes, although it is diminished.

Finally, we consider the issue of commitment in the post-entry Bertrand-Nash pricing game. We show that the incumbent has an incentive to persist with a bundled offering following entry. Even if the incumbent can adjust prices post entry, we demonstrate there are still significant advantages to competing with a bundled offering. Bundling mitigates the impact of entry. Although an entrant's potential profits would be even lower if the incumbent could commit not to bundle following entry, we find there is little room for an entrant to earn profits against a bundle offering.

III.A. Simple Extensions

An uncontested monopolist, selling a mixed bundle—A, B, and an A–B bundle—could always achieve higher profits. However, mixed bundling is less effective in the presence of a rival than in the pure monopoly model. The reason is that the incumbent has to be concerned that a rival with one product, say B, will use the incumbent's other product, A, to create a rival bundle and thereby steal all of the incumbent's bundle sales. Thus, the individual items need to be priced very high relative to the bundle, and so the individual items in the mixed bundle generate relatively few additional sales. Given the limited potential for this approach to increase profits, we do not pursue it further.

Our analysis was made under the assumption that the incumbent knows the precise entry cost of the challenger and thus can fine-tune its entry deterrent strategy. The same analysis can

18. We do not consider entry by multiple firms as once there is a rival A and B good available, then we have bundle versus bundle competition and in the present model that would result in zero profits.

be used to illustrate the advantages of bundling when the potential challenger's entry costs are uncertain. In this case the incumbent deters the entrant with some probability. With bundling, the optimal probability of entry deterrence is much higher and is done at a lower cost [Nalebuff 1999].

Selling two or more goods together will create inefficiencies when production costs are no longer zero at the margin. As shown in the working paper version, significant gains from bundling persist, even for moderate costs [Nalebuff 1999].

Extending the model to nonuniform distribution of valuations strengthens the results. Over the entire class of symmetric, single-peaked densities, the uniform density is the *least* favorable to the pure bundling effect.

Pure Bundling Effect with nonuniform densities. If the density of consumer types is symmetric and unimodal, then at equivalent prices, the act of putting two products into a bundle reduces the entrant's profits by at least 50 percent.

Proof of Pure Bundling Effect with Nonuniform Densities. See the Appendix.

The intuition for the result is that the entrant only sells to customers who value its one good, say B, at more than p_e and good A at less than its monopoly price, $\hat{p}_a$. With a uniform distribution, $\hat{p}_a = 1/2$, and the entrant loses half of its potential market. With a more general symmetric unimodal density of preferences, $\hat{p}_a \leq 1/2$, and this makes it harder for the entrant to attract customers for its single B product.

III.B. Bundling Three or More Goods

It might be tempting to extrapolate and predict that if bundling two items together is a good idea, then putting three together would be even better. It turns out that there are advantages of adding more goods to the bundle, but they are smaller than one might expect given our preceding analysis. The discrete gains that come from the act of bundling two goods do not continue to grow when more goods are added to the bundle. There is a gain from creating a very large bundle, but that gain is primarily due to a third effect, the law of large numbers.

More precisely, Proposition 2 below shows that the pure bundling effect remains constant for any number of products in the bundle. Assume that a one-product entrant prices at $1/2$. Its sales will be $1/2$ against a monopolist with independent pricing, but only $1/4$ against a two-product bundle priced at 1. In fact, the entrant's sales *remain* at ¼ against a three-product bundle priced at 1.5, a four-product bundle priced at 2, a one-hundred-product bundle priced at 50, and so on.

PROPOSITION 2. For all $n \geq 2$, with $x = n/2$ and $p_e = ½$, the challenger has sales of ¼ units and revenue of ⅛.

Proof of Proposition 2. See the Appendix.

The intuition for this result is as follows: if a one-product entrant offers its product at ½ against a 100-product bundle priced at 50, whether consumers will buy the single product or the bundle depends on whether they value the other 99 products above or below 49.50. Half do, and half do not. Thus, the entrant's demand is cut by 50 percent, just as in the two-product bundle.

The advantage of adding more products to the bundle is the incremental gain from the bundle discount effect. The price-discount effect gets stronger as the dimensionality of the bundle increases. By the time there are ten goods in the bundle, the entrant gets only one-third as many customers at its optimal entry price compared with its optimal entry against a two-good bundle (see Nalebuff [1999]).

In a recent series of papers, Bakos and Brynjolfsson [1999, 2000] examine the effect of bundling at a very large scale. Their first paper [1999] demonstrates that bundling an infinite number of goods together allows a monopolist to achieve perfect price discrimination. Along with being a super-effective price-discrimination tool, bundling also creates a significant barrier to entry [2000]. Because the monopolist is able to sell its bundle to 100 percent of consumers, an entrant can sell its individual products only for their incremental value. When all the entrant's potential customers have already purchased the incumbent's B product, the entrant's price is thus limited by the extent to which its product is differentiated and viewed as superior by a subset of consumers.

Our focus lies at the other end of the spectrum, namely small bundles, mostly two goods. Although two-good bundles are not that effective at price discrimination, they are surprisingly effective at entry deterrence.

When it is feasible to create super-large bundles, this will clearly be an effective strategy, both for price-discrimination and entry deterrence reasons. There are several constraints that may prevent this. First, as the size of the bundle grows, the i.i.d. assumption implies that the bundle will also become increasingly valuable—and expensive. Although AOL might bundle thousands of information goods together in its package, the value of the bundle does not seem to grow proportionately. Nor does the distribution in valuations appear to converge. One likely reason is that valuations across goods are typically positively correlated, so that dispersion remains. This would be the case if valuations depend on some common factors, such as personal or business use. As we will observe below, bundling as an entry deterrent continues to work—is even enhanced—with positive correlation while the price discrimination effect is diminished.

Upon reflection, when the scope of bundling is modest, the strongest argument for adding another good to a bundle is to keep one step ahead of a potential entrant. An incumbent with a three-good bundle has less to fear from a potential entrant who can put together a two-good package. The third good prevents the incumbent from competing head-to-head and makes it harder for the entrant to gain share.

III.C. Positive and Negative Correlation in Values

Looking at most bundles, especially software bundles, it appears that the items are positively correlated in value. At first glance, this is somewhat of a puzzle. Positive correlation in values lowers a monopolist's profits as it makes price discrimination more difficult. The gain from bundling must lie elsewhere. With positive correlation, entry against a bundle is even less profitable. However, if entry should occur, the entrant is forced to come in with a very low price, and profits are correspondingly very low. Thus, positive correlation in values is helpful to the incumbent to the extent that it makes entry deterrence more effective. But if entry cannot be prevented, then positive correlation in values ends up hurting the incumbent as the resulting entry is more costly. Conversely, a negative correlation in values makes entry easier, but also less costly to the incumbent.

We begin by considering the pure bundling effect with correlated values. For comparison purposes we want to keep the marginal density for each individual good unchanged and thus the optimal independent price at ½.

When the two values are perfectly correlated, all of the individuals lie along the 45-degree line, $\alpha_a = \alpha_b$. Similarly, when the two values are perfectly negatively correlated, all of the individuals lie along the opposite diagonal, $\alpha_a + \alpha_b = 1$. The way we model intermediate levels of correlation is to take a weighted average of the uniform density and either the diagonal density or the off-diagonal density. The correlation between A and B is denoted by $\rho \in [-1,1]$.

Pure Bundling Effect with Correlation. At equivalent prices, the act of putting two independent products into a bundle reduces the entrant's profits by $(1 + \rho)/2$.

At perfect negative correlation there is no pure bundling effect, while at perfect positive correlation the pure bundling effect reduces the entrant's profits to zero. Consider what happens when an entrant comes into a market against an incumbent with a bundle priced at 1. If the entrant prices at 0.50, it sells only to people who value B at above 0.50 and A at less than 0.50. Consequently, the entrant only sells to 25 percent of the uniformly distributed market. There is no overlap with the 45-degree line so none of the perfectly positively correlated consumers will go to the entrant. Conversely, half of the off-diagonal consumers lie in the lower right quadrant. Thus, with $\rho = 1$, the entrant's sales are zero while with $\rho = -1$ sales are 0.50. Intermediate cases lie linearly in between.

The full analysis is more complicated as the optimal response of the entrant is influenced by the correlation; details can be found in Nalebuff [1999].

III.D. Bundling Complements

Often the products in a bundle complement each other. This is separate from and potentially in addition to the existence of positive correlation in the value of the bundled products. Thus, a high valuation for Excel may indicate a high income or a business use, and this correlation would suggest a high value for PowerPoint. With complementarity, the value of PowerPoint plus Excel is higher than PowerPoint alone plus Excel alone. In the case of the components of Microsoft Office, we suspect that both positive correlation and complementarities are present.

We say that A and B are complements if $V_{a+b} = (1 + \delta)(V_a + V_b)$, $\delta > 0$. If $\delta < 0$, then A and B are substitutes.

Complementarities, similar to positive correlation, amplify the advantages of bundling. Intuitively, consumers are less attracted to a one-product offering since that means giving up on the bundle bonus. Nalebuff [1999] demonstrates the following.

Pure Bundling Effect with Complementarity. At equivalent prices, the act of putting two complementary products into a bundle reduces the entrant's profits from 1/4 to $1/8(1/(1 + \delta))[1 - 3\delta/2]$, where δ is the measure of complementarity.

Thus, if $\delta = 0$, the reduction in profits is 50 percent, from 0.25 to 0.125. This is our original pure bundling result. If $\delta = 0.10$, then profits are reduced to 40 percent of their previous level, from 0.25 to 0.10. If $\delta = 0.25$, then profits are reduced to 25 percent of their previous level, from 0.25 to 0.0625.

Conversely, if the two products are substitutes, bundling is less effective. If $\delta = -0.10$, then profits are reduced by 36 percent, from 0.25 to 0.16. If $\delta = -0.25$, then profits are only reduced by 12 percent, from 0.25 to 0.22.

An anonymous referee suggested the possibility that the complementarity might not be symmetric. Here we consider the case where $V_{a+b} = (V_a + V_b)$ if the two goods are purchased together, but $V_b = 0$ if good B is purchased alone. To say this another way, the only way to get the value from good B is to also buy good A. One might think of B as software that runs on an operating system and thus has no value on its own but serves to enhance the value of the operating system.

In that case, the optimal pricing strategy for an incumbent is the same as first presented. Because the incumbent sells a bundle, consumer valuations are still $(V_a + V_b)$.

Entry is a bit different. Since good A is not sold on its own, there is no value in entering the good B market by itself. No consumers will buy B without A. As for entering with good A, that follows as in the original analysis. A more sophisticated analysis of this case is considered in Heeb [2003], who adds an element of product differentiation between the incumbent's and entrant's B goods. He provides conditions that lead all consumers to purchase of one of the B goods, in which case bundling is an optimal strategy.

III.E. Bertrand-Nash Pricing or Competition against an Existing Rival

In this subsection we show how the earlier results generalize to the case of a Nash pricing game post entry. This can also be understood as the expected outcome of competition between two companies, each established in the market, where one has two products and the other has only one.

Company 1 sells both products A and B, and company 2 sells only product B. The question is whether company 1 (with its monopoly in good A) should extract all of its monopoly rents in good A directly, or, would it do better by leveraging that monopoly and selling an A–B bundle?

We demonstrate that the benefits to bundling continue to hold when we consider the Nash equilibrium in the pricing game. Bundling is a credible strategy. Firm 1 becomes more aggressive in its bundle price, since raising price no longer leads firm 2 to follow suit. Yet, firm 1's profits are still nearly 50 percent higher than what it can extract by simply exploiting its monopoly in good A and competing head-to-head on good B. Bundling prevents the price of good B from being driven to marginal cost, and firm 1 is able to capture most of the demand for good B as part of its bundle.

The case in which the two goods are sold independently is straightforward. The two B goods are perfect substitutes, and so the Bertrand-Nash equilibrium price is 0. As for good A, the monopoly price is ½, and profits are ¼. Thus, total profits of firm 1 are ¼, and profits of firm 2 are 0.

Next, we consider the Nash equilibrium when firm 1 sells only an A–B bundle and firm 2 sells B. Firm 2's optimal response to a bundle price of x is the same as in the Stackelberg case. As demonstrated in Proposition 1 in the Appendix, its profits are maximized when it charges p_b^*:

$$p_b^* = \frac{1+x}{3} - \frac{1}{3}\sqrt{1 - x + x^2}.$$

Firm 1's profits given a price of p_b for good B are

$$\Pi_1|entry = x * (1 - x + p_b - p_b^2/2).$$

Profits are maximized at

$$1 - 2x + p_b - p_b^2 = 0 \qquad \text{or} \qquad x^* = (1 + p_b - p_b^2)/2.$$

The Nash equilibrium is $x^* = 0.59$, $p_b^* = 0.24$. Equilibrium profits are 0.366 for firm 1 and 0.064 for firm 2. With two goods, firm 1 makes nearly six times the profits of its one-good rival. In that sense, bundling triples the advantage a two-good firm has over a one-good rival while protecting most of the incumbent's profits in the process.

PROPOSITION 3. In the postentry Nash equilibrium pricing game between a two-product incumbent and a one-product rival, bundled pricing is the subgame perfect equilibrium.

Proof of Proposition 3. As just demonstrated, the incumbent makes profits of 0.366 in the event it continues to bundle and 0.25 in the event it sells its two products separately.

When compared with independent pricing, the gain in profits is 46.8 percent for firm 1 and an infinite percent for firm 2 (which previously made 0). In absolute terms, it is the incumbent who has the greater gain, by almost two to one.[19]

In this setting, the gains from bundling come from mitigating the effects of competition. Thus, it is interesting to note that the result is more efficient than the outcome with independent pricing.

PROPOSITION 4. In the postentry Nash equilibrium pricing game between two-product incumbent and a one-product rival, total surplus is greater under bundling than when A and B are sold independently.

Proof of Proposition 4. See the Appendix.

The intuition for this result is as follows: bundling leads to an increase in the price of good B from 0 to 0.24. Initially, this price increase only results in a second-order welfare loss. With a bundle priced at 0.59 and B priced at 0.24, the marginal price of A is 0.35. Thus, the price of A falls from 0.50 to 0.35 to those customers who are prepared to buy B, and this increase in sales leads to a first-order welfare gain. While the gain is 60 percent bigger than the loss in this case, the net result may still depend on the specific distribution of preferences.

We note that the results in the Nash equilibrium game are sensitive to the details in the model setup. For example, Whinston [1990] considers a similar problem, a monopoly in product A and a duopoly in product B. In his initial setup the two B goods are differentiated, but all customers have identical valuations for A. As a result, if the rival were to get any B customers, the incumbent would lose *all* of its A customers. This is because all customers have the same extra value from getting a bundle with A. Thus, in a second-stage pricing game, the incumbent has a great incentive to price the bundle low so as to preserve the value it creates in the A market. The end result is that a potential rival finds it hard to beat the A–B bundle and chooses not to enter the market.

In Whinston's model, the pure bundling effect not only disappears, it is reversed. Assume that the common value of A is ½ and that the value of B is uniformly distributed on [0,1]. Against a bundle price of 1, an entrant who comes in at ½ (minus epsilon) sells to half the market, up from ¼ . And, instead of taking away

19. In this context, bundling might be viewed as a facilitating device in that it raises the postentry profits of both the incumbent and the entrant. This perspective is explored further in Carbajo, De Meza, and Seidman [1990] and Chen [1997].

only ¼ of the incumbent's sales, it takes away 100 percent of the incumbent's sales.

With a common value for good A, bundling does not help the incumbent defend itself once the entrant is in the market—quite the contrary. However, this weakness is turned into strength as an entry deterrent.[20] The only way that the incumbent can earn any money is to ensure that the entrant makes zero sales. Even an entrant with a cost advantage (or superior product) in B is deterred, as the incumbent uses its value in good A to cross-subsidize the B good in the bundle and thereby deny the entrant any sales.[21] This approach requires the incumbent to make a credible commitment to selling its products only as part of a bundle; if entry occurs, the incumbent would prefer not to bundle.[22]

Credibility is not an issue in our approach, even in the Nash game, because the incumbent's postentry profits are higher with a bundle than without. This is because we allow for heterogeneity in consumer values of A.[23] As a result, the entrant can take a small number of customers without threatening all of the incumbent's sales. Second, the entrant does not have a cost advantage in B, and so the entrant is not forced to cross-subsidize the bundle price. Of course, in Whinston's model if the incumbent does not have a cost disadvantage in B, then it can fight the entrant better when the goods are sold independently, and so there is no gain from bundling. We saw this in the previous section. When all costs are zero, the entrant's profits rise from 0 with independent pricing to 0.064 when the incumbent bundles.

While bundling may be less effective at deterring entry than an independent pricing strategy, it has the advantage of being ex

20. There is the old saying: my enemy's enemy is my friend. In a two-stage game, weakness in the second stage often translates into strength in the first. When the postentry market is more competitive, then there is less incentive to enter the market (see, for example, Bernheim [1984]).

21. For example, imagine that the incumbent can produce A at a cost of 0.20 while all consumers value it at ½ . The rival can only produce good B, but can do so at a cost advantage—the rival's unit cost is 0.10 while the incumbent's unit cost is 0.30. With independent sales, the price of B would be 0.30, and the rival would expect to earn a margin of 0.20 on sales of 0.70. If the incumbent only sold A as part of a bundle, it would be forced to sell the bundle at a price of 0.60 in order to make any sales. It would use 0.20 of the surplus it generates in sales of A to subsidize its cost disadvantage in B. The result is that the entrant would not be able to attract any customers and, anticipating this, would not enter the market.

22. A commitment to bundling also changes an incumbent's incentives to innovate, as shown by Choi [2003]. Bundling gives the monopolist a greater incentive to engage in cost-cutting R&D and thus helps preserve and extend its advantageous position.

23. Whinston also considers the case in which there is heterogeneity in the valuation of A. He finds that a high dispersion in the value of good A and a low differentiation in B goods are necessary for bundling to raise the entrant's profits.

post credible. In that sense it may be useful to have a powerful, if imperfect, tool that does not require any commitment. It is to be expected that the Nash pricing game post entry will lead to lower profits for both the incumbent (0.375 down to 0.366) and the entrant (0.081 down to 0.064) compared to the Stackelberg game.[24] But, the difference in profits is relatively small. This suggests that most of the effect is due to the *pure bundle effect* and the *bundle discount effect* identified earlier in the paper. A potential entrant cannot expect to make much profit against an incumbent who can bundle, whether the incumbent responds or even if it does not.

IV. Conclusions

Bundling is a credible tool to protect a multigood monopolist against entry. In this case, bundling is not used to leverage a monopoly from one market to another. The incumbent is starting from a position of two monopolies. By bundling these two goods together, the incumbent is able to use each of the monopolies to protect the other one. What makes this strategy remarkable is that unlike most entry deterrent strategy (such as limit pricing) it can actually raise profits absent entry.

A firm that has only some components of a bundle will find it hard to enter against an incumbent who sells a package solution at a discount. This will be especially true when the consumers have positively correlated values for the components of the package or when the components are complements. Bundling also softens the harm done by a one-product (or limited line) competitor. The rival takes fewer customers away, and prices do not fall as far.

A monopolist, even without fear of entry, has incentives to bundle, either as a way to achieve better price discrimination (when values have a negative correlation) or to help save costs (when valuations are positively correlated). But most important to a firm with market power is preserving that power, by deterring a potential entrant or reducing the impact of a one-product rival. It is in this role that bundling truly shines. Entry is more easily deterred, in which case profits are more than doubled. And when entry deterrence fails, postprofits are still more than 50 percent higher when products are sold as a bundle.

24. The Nash pricing can sometimes lead to higher entrant profits. Potential entrants with costs between 0.034 and 0.064 will enter in the Nash game and earn 0.064 but will not enter in the Stackelberg game. The difference arises as the incumbent is able to commit to a limit price below 0.59 in order to deter entry.

Appendix

Pure Bundling Effect with nonuniform densities. If the density of consumer types is symmetric and unimodal, then at equivalent prices, the act of putting two products into a bundle reduces the entrant's profits by at least 50 percent.

Proof of Pure Bundling Effect with Nonuniform Densities. Let $f(\cdot)$ represent the symmetric and unimodal density of consumer types α over the unit square. Let $f_a(\cdot)$ and $f_b(\cdot)$ represent the marginal density functions for the valuations of goods A and B, and $F_a(\cdot)$ and $F_b(\cdot)$ represent the marginal cumulative densities. Without loss of generality, we assume that the entrant comes into the B market.

Absent bundling, we denote the optimal monopoly prices of incumbent firm 1 as $\hat{p}_a$ and $\hat{p}_b$. We then compare a potential entrant's profits at a price of $p_e = \hat{p}_b - \epsilon$ when the incumbent does not bundle with the entrant's profits at a price of $\hat{p}_b$ against a bundle at price $x = \hat{p}_a + \hat{p}_b$.

If the incumbent does not bundle, firm 2 can earn $\hat{p}_b[1 - F_b(\hat{p}_b)]$.

If the incumbent does bundle, then at an entry price of $\hat{p}_b$ the marginal cost of buying the bundle (and getting product A) over just buying B from the entrant is $\hat{p}_a$. Thus, the entrant's demand, denoted by D_b, is

$$D_b(\hat{p}_b) = \int_0^{\hat{p}_a} \int_{\hat{p}_b}^{1} f(\alpha_a,\alpha_b)\, d\alpha.$$

The entrant only attracts customers who value B at above $\hat{p}_b$ and A at below $\hat{p}_a$.

Assume that $\hat{p}_a \leq 1/2$. Then

$$D_b(\hat{p}_b) \leq \int_0^{1/2} \int_{\hat{p}_b}^{1} f(\alpha_a,\alpha_b)\, d\alpha.$$

$$= \left(\frac{1}{2}\right) \int_0^{1} \int_{\hat{p}_b}^{1} f(\alpha_a,\alpha_b)\, d\alpha$$

$$= \left(\frac{1}{2}\right)[1 - F_b(\hat{p}_b)],$$

where the middle step follows as $f(a,b) = f(1 - a,b)$. This would complete the proof as the entrant's demand (and hence profits) is reduced by at least ½.

Thus, we need only show that $\hat{p}_a \leq$ ½. In choosing its optimal individual prices, firm A maximizes

$$\hat{p}_a[1 - F_a(\hat{p}_a)].$$

Since the density is symmetric on [0,1], we know that at $p =$ ½, $F_a(½) =$ ½ and the first-order condition is $[1 - F_a(½)] - (½) f_a(½) = (½)[1 - f_a(½)]$. Since the density is maximized at ½, it must be the case that $f_a(½) \geq 1$ (as otherwise the cumulative density could not integrate up to 1). Thus, the first-order condition is weakly negative at $p =$ ½. Moreover, for $p \geq$ ½, the second-order condition is $-2f_a(p) + pf'_a(p)$. Since $f(p)$ is single-peaked and maximal at $p =$ ½, for $p \geq$ ½, $f'_a(p) \leq 0$, and the profit function is clearly concave for $p \geq$ ½. Thus, the maximum must arise at some $\hat{p}_a \leq$ ½.

PROPOSITION 1. The entrant maximizes profits at

$$p_e^* = \frac{1+x}{3} - \frac{1}{3}\sqrt{1 - x + x^2}.$$

Proof of Proposition 1. The entrant's profits are $\Pi_e = p_e * (1 - p_e) * (x - p_e)$. Differentiating the profit function, the first-order condition is $(1 - 2p)(x - p) - p(1 - p) = 0$. Rearranging and applying the quadratic formula leads to the result.

COROLLARY 1. $x/3 \leq p_e^* < x/3 + 0.045$.

The first inequality, $p_e^* \geq x/3$, follows from the fact that $(1 - x + x^2) \leq 1$ for $0 \leq x \leq 1$; the second inequality, $p_e^* < x/3 + 0.045$, follows as $1 - x + x^2$ is minimized at $x = 0.5$.

Corollary 1 implies that the elasticity of the entrant's profits with respect to the incumbent's price is at least 150 percent.

COROLLARY 2. $\partial \log(\Pi_e)/\partial \log(x) \geq 3/2$.

This follows as $(\partial \log(\Pi_e))/(\partial \log(x)) = x/(x - p_e)$ and $x/(x - p_e) \geq 3/2$ as $p_e \geq x/3$.

PROPOSITION 2. For all $n \geq 2$, with $x = n/2$ and $p_e =$ ½, the challenger has sales of ¼ units and revenue of ⅛.

Proof of Proposition 2. Without loss of generality, assume that the entrant offers product 1. A consumer buys from the entrant if and only if $\alpha_1 - 1/2 \geq \alpha \cdot 1 - n/2$. Rearranging this inequality leads to $\Sigma_2^n \alpha_i \leq (n - 1)/2$. It follows from this directly that the challenger's sales always equal ¼. Among the population that values good 1 at more than ½, the challenger sells to half the population, and the incumbent sells to the other half.

PROPOSITION 4. In the postentry Nash equilibrium pricing game between two-product incumbent and a one-product rival, total surplus is greater under bundling than when A and B are sold independently.

Proof of Proposition 4. With independent pricing, all consumers purchase good B. The surplus from this transaction is 0.5, as this is the mean valuation of good B. As for the surplus from the sale of A, only half the market, namely those consumers with valuations above 0.5, purchase A. Their mean value for A is 0.75. Thus, 0.5 purchases with a mean value of 0.75 adds 0.375 to the 0.5 surplus from the B sales for a total of 0.875.

With bundling, the mean valuation for B among the consumers who buy only B is $(1 + 0.24)/2 = 0.62$. Sales are 0.266, which generates a surplus of $0.62 * 0.266 = 0.165$. The average value of the bundle among those who purchase the bundle from firm 1 is 1.21. Bundle sales are 0.62, which leads to a total surplus of 0.75 on bundled sales. Combining this with the 0.165 surplus from B sales results in aggregate surplus of 0.915 which exceeds 0.875 without bundling.

YALE SCHOOL OF MANAGEMENT

REFERENCES

Adams, Williams J., and Janet L. Yellen, "Commodity Bundling and the Burden of Monopoly," *Quarterly Journal of Economics,* XC (1976), 475–498.
Bain, Joe, "A Note on Pricing in Monopoly and Oligopoly," *American Economic Review,* XXXIX (1949), 448–464.
Bakos, Yannis, and Eric Brynjolfsson, "Bundling Information Goods: Pricing, Profits, and Efficiency," *Management Science,* VL (1999), 1613–1630.
Bakos, Yannis, and Eric Brynjolfsson, "Bundling and Competition on the Internet: Aggregation Strategies for Information Goods," *Marketing Science,* XIX (2000), 63–82.
Bernheim, B. Douglas, "Strategic Deterrence of Sequential Entry into an Industry," *Rand Journal of Economics,* XV (1984), 1–11.
Carbajo, Jose, David De Meza, and Daniel J. Seidman, "A Strategic Motivation for Commodity Bundling," *Journal of Industrial Economics,* XXXVIII (1990), 283–298.
Carlton, Dennis W., and Michael Waldman, "The Strategic Use of Tying to Preserve and Create Market Power in Evolving Industries," *RAND Journal of Economics,* XXXIII (2002), 194–220.
Chen, Yongmin, "Equilibrium Product Bundling," *Journal of Business,* LXX (1997), 85–103.
Choi, Jay Pil, "Tying and Innovation: A Dynamic Analysis of Tying Arrangements," *Economic Journal* (2003), forthcoming.

Choi, Jay Pil, and Christodoulos Stefanadis, "Tying, Investment, and the Dynamic Leverage Theory," *RAND Journal of Economics,* XXXII (2001), 52–71.
Cournot, Augustin, *Recherches sur les principes mathématiques de la théorie des richesses* (Paris: Hachette, 1838), English translation: (N. Bacon, trans.), *Research into the Mathematical Principles of the Theory of Wealth* (Mountain Center, CA: James and Gordon, 1995).
Director, Aaron, and Edward Levi, "Law and the Future: Trade Regulation," *Northwestern University Law Review,* LI (1956), 281–296.
Economides, Nicholas, "Raising Rival's Costs in Complementary Goods Markets: LECs Entering into Long Distance and Microsoft Bundling Internet Explorer," Discussion Paper EC-98-03 NYU Stern School, 1998.
Heeb, Randal, "Innovation and Vertical Integration in Complementary Markets," *Journal of Economics and Management Strategies,* XII (2003), 387–417.
Matutes, Carmen, and Pierre Regibeau, "Compatibility and Bundling of Complementary Goods in a Duopoly," *Journal of Industrial Economics,* XL (1992), 37–54.
McAfee, R. Preston, John McMillan, and Michael D. Whinston, "Multiproduct Monopoly, Commodity Bundling, and Correlation of Values," *Quarterly Journal of Economics,* CIV (1989), 371–384.
Nalebuff, Barry, "Bundling," Working paper, available online at Social Science Research Network http://papers.ssrn.com, 1999.
——, "Competing against Bundles," in *Incentives, Organization, and Public Economics,* Peter Hammond and Gareth Myles, eds. (New York, NY: Oxford University Press, 2000).
Salinger, Michael A., "A Graphical Analysis of Bundling," *Journal of Business,* LXVIII (1995), 85–98.
Schmalensee, Richard, "Commodity Bundling by Single-Product Monopolies," *Journal of Law and Economics,* XXV (1982), 67–71.
——, "Gaussian Demand and Commodity Bundling," *Journal of Business,* LVII (1984), 58–73.
Spiller, Pablo T., and Bennet A. Zelner, "Product Complementarities, Capabilities and Governance: A Dynamic Transaction Cost Perspective," *Industrial and Corporate Change,* VI (1997), 561–594.
Stigler, George J., "A Note on Block Booking," in G. J. Stigler, ed., *The Organization of Industries* (Homewood, IL: Irwin, 1968), pp. 165–170.
Whinston, Michael D., "Tying Foreclosure, and Exclusion," *American Economic Review,* LXXX (1990), 837–859.