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17 Competing against Bundles

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1. Introduction

One of the distinguishing features of our modern economy is the competitive success achieved by product bundles, Microsoft Office being the case in point. It has achieved a commanding market share over previously dominant firms selling individual software applications, such as WordPerfect, Quattro or Lotus, Adobe PageMill, and Harvard Graphics. While no one single factor explains Microsoft's success, one part of the explanation can be found in the writings of Cournot (1838).[1] But this is not the classic Cournot oligopoly model that has become the textbook standard. Instead, it is Cournot's dual.

The classic Cournot oligopoly model has a limited number of commodity sellers each putting a quantity out in the market. With linear demand, price is determined by

$$p = A - b * (q_1 + q_2 + \ldots + q_n). \tag{1}$$

Although the standard textbook model is Cournot quantity competition, there are few real markets—perhaps the Fulton fish market—which satisfy this characterization. It is rare to find a firm that simply dispenses some quantity of goods on the market and then accepts the market price.[2] Instead, firms set prices, taking into account the expected prices set by other firms and the anticipated demand at those prices.

As Cournot himself realized, it is entirely possible to flip the ps and qs. In this case, the goods are complements rather than substitutes and the strategic variable becomes price, not quantity:

$$q = A - b * (p_1 + p_2 + \ldots + p_n). \tag{2}$$

In this interpretation, consumers are interested in buying a collection of n complementary products, for example, hardware and software. Each of the products is sold separately at price p_i. When determining whether or not to purchase this bundle, the

One of the many things I learned from Professor Mirrlees in the Nuffield College classrooms was the value of taking the dual perspective. As for how well these lessons were learned, the pudding will be in the proofs. My heartfelt thanks go Adam Brandenburger, Erik Brynjolfsson, Jeremy Bulow, Richard Zeckhauser, and seminar participants at Stanford and UCLA whose perceptive comments led me to tackle this problem.

[1] Other explanations include Novell and other's delay in updating their products to be compatible with Windows in its migration from DOS.

[2] Kreps and Scheinkman (1983) provide an elegant defence of the Cournot model. They demonstrate that Cournot quantity competition is equivalent to Bertrand price competition in a two-stage game where firms first pick capacity levels and then choose quantities in a capacity-constrained second period. This greatly expands the applicability of the Cournot model. However, in an economy increasingly dominated by knowledge goods and where increasing returns to scale are the order of the day, firms rarely face capacity constraints.

consumer takes into account the aggregate cost. Thus a computer-user examines the cost of hardware and software. A student looks at the cost of tuition, room, and board. A skier considers the price of lodging, transportation, lift tickets, equipment, and lessons.

In Cournot's own words:

> We imagine two commodities, (a) and (b), which have no other use beyond that of being jointly consumed in the production of the composite commodity (ab). . . . Simply for convenience of expression we can take for examples copper, zinc, and brass under the fictitious hypothesis that copper and zinc have no other use than that of being jointly used to form brass by their alloy. (1838: ch. ix)

In Cournot's analysis, each component that goes into the bundle is unique—one type of hardware and one type of software. There is a monopoly in each component. How then will the components be priced? Cournot shows that if the two monopolists get together they would price the bundle of their goods *lower* than they would acting individually.

The two-firm case illustrates the general result. One firm sets the price of good 1 and the other sets the price of good 2. The equilibrium has each firm setting a price of $1/3$. The bundle price is then $2/3$ and total sales are then $1/3$. This result should seem familiar. As Sonnenschein (1968) observed, the mathematics are exactly the same as with the standard Cournot–Nash equilibrium, only here we have switched prices for quantities and complements for substitutes. Consider what happens if the two firms get together and coordinate their pricing decision. Now they would choose $p_1 + p_2 = 0.5$ and joint profits would rise from 0.22 to 0.25.

While it is not surprising that coordinated pricing leads to higher profits, what might be surprising is that coordinated pricing leads to a reduction in prices. Both consumers and firms are better off.[3] The reason is that in the case of complementors, when one firm lowers its price, the other firm's sales increase, an externality that is not taken into account with uncoordinated pricing. Thus there is an advantage to bundling when two firms each have market power, but each is missing one of the complementary products.[4]

In this paper, we take the next step in this dual to the Cournot model. We examine what happens when there is imperfect competition between the component products that go into the bundle. There are three cases to consider, component against component, bundle against bundle, and bundle against components.

In the first case, each component is sold separately—hardware against hardware and software against software. The second scenario considers bundle against bundle—a

[3] The fact that coordination leads to higher profits suggests that a company that sells two complementary products will have a higher incentive to innovate than when the products are sold separately, see Heeb (1998).

[4] It is interesting to note that Posner (1979) looked at the case for bundling pure complements. He concluded that as consumers care only about the price of the bundle, there would be no point in trying to leverage a monopoly in A to B so as to raise the price of B above its otherwise competitive level. Raising the price of A would do just as well. The surprise is that this argument no longer holds when B is sold by an oligopoly. Because its price lies above the competitive level, the A monopoly wants to use its leverage to *lower* the price of B.

hardware-software package competes against a rival package.[5] The third case presents bundle against uncoordinated component sellers. Here we have a hardware-software package competing against independent sellers of hardware and software.

Our interest is in the third case, bundle against components. We use the first two scenarios to form our basis for comparison. This allows us to better understand what happens when a player in the market aggregates a collection of complements and sells them as a bundle while the competition remains independent or uncoordinated.

Following the intuition of Cournot, it will not be a surprise that the bundler does better than the collection of independent competitors. But the scale of the advantage is remarkable. Once there are four or more items to the bundle, the bundle aggregator does better than the sum of its previous parts. And this outcome is stable as the disadvantaged independent sellers do not have an incentive to form a rival bundle. This is because the resulting 'ruinous' competition of bundle against bundle would leave the independent sellers even worse off than their present disadvantaged position.

Thus the results of this paper suggest that a firm who creates or simply aggregates a bundle of complementary software applications would have a substantial pricing advantage over its rivals and thereby achieve a leadership position in the market. This is especially true as the bundle grows in scale. Thus Microsoft's taking the lead in creating a software application bundle—putting together word-processing, spreadsheet, presentation, HTML editing, and e-mail applications—may help explain the stunning market success of Microsoft Office suite.

This paper also contributes to an explanation of how Freeserve gained the lead over AOL as the dominant Internet service provider in the UK. Consumers in this market pay a metered charge for local telephone service. In this case, the model suggests that Freeserve's success was in part due to the fact that Freeserve provided a bundle of internet connectivity and phone service while AOL customers paid separately for phone connection and Internet connection.

2. The Model

Our model of imperfect competition is duopoly. We assume that for each component there are two competing alternatives in the market, A and B. The A and B components are only imperfect substitutes. We imagine that the 'A' components are all located at 0 while the 'B' components are all located at 1.

We further assume that the consumer only gets the value of the products if all the components are purchased. Thus, in the case of a two-good bundle, each consumer will buy one of (A_1, A_2), (A_1, B_2), (B_1, A_2), or (B_1, B_2). The value of the package is

[5] The first two scenarios are closely related to the work of Matutes and Regibeau (1989, 1992). Their papers ask the question whether competing firms would make their products compatible or not and whether or not it would be advantageous to sell them in bundles or not. After we have presented the model and our results, we return to explain the relationship between our papers.

sufficiently large (relative to the equilibrium prices charged) so that all consumers will purchase one or the other bundle. Each consumer is interested in exactly one unit of the bundle.

Consumers assemble the package that best suits their preferences. Each consumer purchases the bundle with the smallest total cost, where total cost is comprised of price plus a linear transportation cost. The cost of component i to consumer of type α is $-\alpha_i - p_{A_i}$ for the A_i product and $-(1 - \alpha_i) - p_{B_i}$ for the B_i product. We assume that α is uniformly distributed over the unit hypercube.

We further assume that production cost is zero, although with constant marginal costs everything perfectly translates into mark-up over cost.

In a series of models we will examine the pricing equilibrium that results for various bundle sizes and market structures. We begin with the simplest case, two firms each selling a one-component product. A one-product bundle is the limiting case of a bundle.

For firm A, demand and profits are:

$$\begin{aligned} D_A &= 0.5 + 0.5 * (p_B - p_A). \\ \Pi_A &= p_A * D_A. \end{aligned} \tag{3}$$

For firm B, the functions are symmetric. This leads to the following first-order conditions

$$\begin{aligned} D_A - (0.5) * p_A &= 0; \\ D_B - (0.5) * p_B &= 0. \end{aligned} \tag{4}$$

In equilibrium, prices equal 1 and the market is evenly split between firms A and B.

$$p_A = p_B; \quad D_A = D_B = 1/2; \quad p_A = p_B = 1. \tag{5}$$

More generally, this result holds true for any number of components. The only subtlety is how to interpret the nature of the competition when there are multiple components. For example, assume that there are two A components (A_1, A_2) and two B components (B_1, B_2). In this case there would be four firms, two A firms, each selling one of the A components and two B firms, each selling one of the B components.

Each consumer will purchase two components (A_1 or B_1, A_2 or B_2). He evaluates how far he is from each of the items on a component-by-component basis. For example, a customer located at $\alpha = (0.1, 0.5)$ has a strong preference for A_1 and is indifferent between A_2 and B_2. Customers are allowed to mix-and-match in forming their own bundles. Consequently, consumers optimize their purchase decision component-by-component. With a uniform distribution of α (on the unit hypercube) this leads us back to the one-component product case:

$$P_{A1} = P_{A2} = P_{B1} = P_{B2} = 1. \quad \Pi_{A1} = \Pi_{A2} = \Pi_{B1} = \Pi_{B2} = 1/2.$$

This case is the baseline from which we can evaluate the impact of coordinated pricing decisions. In this baseline case, consumers mix-and-match their preferred components and pay a price of n for their n-good customized bundle.

3. Bundle against Bundle

Next we consider the case where all the A firms coordinate their pricing and sell their product as a bundle against the B firms, who have also coordinated their pricing decisions. We assume that consumers buy only one of the two bundles.[6]

Let bundle A sell for an amount $P_A = P_{A1} + P_{A2} + \dots + P_{An}$, and bundle B sell for P_B defined similarly. We assign Δ to represent the price premium of bundle B over bundle A, $\Delta = P_B - P_A$.

A consumer of type α will prefer to purchase the A bundle over the B bundle if

$$\alpha \cdot 1 \leqslant \frac{n + \Delta}{2}. \tag{6}$$

Recall that our consumers are uniformly distributed over the unit hypercube. With n goods in the bundle and a price difference of Δ, the demand for bundle A is thus

$$D(\Delta, n) = \frac{1}{n!}\sum_{k=0}^{n}(-1)^k\binom{n}{k}\max[0, (n + \Delta)/2 - k]^n. \tag{7}$$

For the two-good case, $n = 2$, and demand simplifies to

$$D(\Delta, 2) = (1/2)[(1 + \Delta/2)^2 - 2\max(0, \Delta/2)^2]. \tag{7$'$}$$

As expected at $\Delta = 0$, demand is 0.5. At $\Delta = 2$ even the consumer located at (1, 1) is just willing to 'travel' the extra distance of 2 to purchase the A bundle at (0, 0) in order to save an extra \$2 in price; thus all of the consumers will go to A and demand for bundle A will be 1.

We now proceed to calculate the symmetric duopoly equilibrium.[7] Firm A maximizes Π_A:

$$\Pi_A = P_A * D(\Delta, n), \quad \Delta = P_B - P_A. \tag{8}$$

The first-order conditions are

$$D(\Delta, n) + P_A D'(\Delta, n) = 0. \tag{8$'$}$$

[6] Even if the two technologies are compatible, equilibrium prices are too high to justify buying both bundles until $n \geqslant 4$. At $n = 4$, some consumers would buy both bundles in order to mix-and-match. In the $n = 4$ equilibrium we calculate below, bundle prices are 1.5; thus, the consumer of type $\alpha = (0, 0, 1, 1)$ would buy both bundles in order to consume A_1, A_2, B_3, B_4. We simplify our analysis by assuming that the A and B products are incompatible so that consumers must buy only one or the other.

[7] We would normally call this a Bertrand complements duopoly. Although Cournot was the first to consider a pricing game among complementary products, calling this a Cournot equilibrium would be too confusing.

At the symmetric equilibrium, $\Delta = 0$ and $D(0, n) = 1/2$. Thus, $P_A = P_B = -1/2D'(0, n)$ where

$$D'(0, n) = -(1/2) * \left[\frac{1}{(n-1)!}\sum_{k=0}^{n}(-1)^k\binom{n}{k}\max[0, n/2 - k]^{n-1}\right] = 0. \tag{9}$$

For $n = 2$, this simplifies to

$$D'(0.2) = -1/2, \quad \Rightarrow \quad P_A = 1, \quad \Pi_A = 1/2. \tag{10}$$

Profits fall by 50 per cent. We repeat, profits fall by 50 per cent. Profits fall by 50 per cent because the aggregate bundle price has fallen by 50 per cent. The price of the entire bundle is reduced to the prior price of each of the single components. In hindsight, the intuition is relatively straightforward. Cutting price brings the same number of incremental customers as when selling individual components. So the bundle price must equal the individual price in a symmetric equilibrium.

As the number of elements in the bundle increases, the equilibrium price rises, but slowly.

$$n = 2 \Rightarrow P_A = P_B = 1.00, \quad \Pi_A = \Pi_B = 0.50.$$
$$n = 3 \Rightarrow P_A = P_B = 1.33, \quad \Pi_A = \Pi_B = 0.66.$$
$$n = 4 \Rightarrow P_A = P_B = 1.50, \quad \Pi_A = \Pi_B = 0.75.$$
$$n = 5 \Rightarrow P_A = P_B = 1.67, \quad \Pi_A = \Pi_B = 0.83.$$
$$n = 6 \Rightarrow P_A = P_B = 1.82, \quad \Pi_A = \Pi_B = 0.91.$$
$$n = 7 \Rightarrow P_A = P_B = 1.95, \quad \Pi_A = \Pi_B = 0.97.$$

Bundle against bundle is ferocious competition. To put this in perspective, in the seven-good case, if each of the goods were sold separately the price would add up to 7, rather than 1.95, and industry profits would be 3.5 times bigger. From the firms' perspective, the problem with bundle-versus-bundle competition is that the stakes are too high. Lowering the price of any one component increases the sale of all n components. The result is that the component prices fall down to such a low level that those incremental sales, all combined, are just enough to offset the loss in margin.

We can use the normal approximation to calculate the limiting result as n gets large. We know that the sum of any number of independent distributions approaches the normal. In this case, the value of each component has a uniform distribution over [0, 1] with mean 1/2 and variance 1/12. Thus the density approaches a normal with mean $n/2$ and variance $n/12$. The density at the mean $f(\mu) = 1/\sqrt{(2\pi\sigma^2)} = 1/\sqrt{(2\pi(n/12))} = \sqrt{(6/\pi n)}$.

At the symmetric equilibrium, price is simply $1/f(\mu)$, which implies

$$P_A(n) = P_B(n) \approx \sqrt{(\pi n/6)}. \tag{11}$$

The equilibrium bundle price rises with the square root of n when competition is over bundles, while the bundle price rises with n when components are sold individually.

4. Bundle against Components

We are now ready to consider the case of interest, bundle against components. The pricing externality suggests that the bundler will have an advantage over the component sellers. But the results of the previous section suggest that this gain may be offset by an increase in competition induced by the A firms only selling their products as bundle. Which effect dominates?

We start with the case where the A and B components are incompatible. This simplifies the analysis as consumers must choose between buying the A bundle or buying all the B components and thereby assembling a B bundle. In the Appendix, we allow a consumer to buy B components along with the A bundle (since A components are not sold individually, the reverse case isn't applicable).

Just as before, a consumer of type α will prefer to purchase the A bundle over the B bundle if

$$\alpha \cdot 1 \leqslant \frac{n + \Delta}{2}. \tag{12}$$

With n goods in the bundle and a price difference of Δ, again the demand for bundle A is

$$D(\Delta, n) = \frac{1}{n!} \sum_{k=0}^{n} (-1)^k \binom{n}{k} \max[0, (n + \Delta)/2 - k]^n. \tag{13}$$

The mathematics of bundle against components is very similar to bundle against bundle. The significant difference from the previous section is that there is no one firm B making a coordinated pricing decision. Instead, each of the B component firms sets its own price. Hence there is no longer a symmetric equilibrium. The first-order conditions for firm A and the n firm Bs are:

$$\begin{aligned} P_A D' &= -D, \\ P_{B_i} D' &= -(1 - D), \quad \{i = [1, n]\}. \end{aligned} \tag{14}$$

This implies:

$$\Delta = [D(\Delta, n) - n(1 - D(\Delta, n))]/D'(\Delta, n). \tag{15}$$

Where $D(\Delta, n)$ is defined in (13) and

$$D'(\Delta, n) = -1/2\left[\frac{1}{(n-1)!} \sum_{k=0}^{n} (-1)^k \binom{n}{k} \max[0, (n + \Delta)/2 - k]^{n-1}\right]. \tag{16}$$

The Δ that solves (15) is the equilibrium price gap between the A bundle and the B bundle. Plugging this back into (14) reveals the equilibrium values of P_A and P_{B_i}.

In general, (15) is best solved via computer. But when $n = 2$ the solution can be found directly.[8]

[8] Note, in calculating the solution below, we assume that $\Delta \leqslant 2$ and show this assumption is justified.

$$\Delta = [D - 2(1 - D)]/D'$$
$$= [3D - 2]/D', \tag{17}$$

$$D(\Delta, 2) = 1/2\left[\left(1 + \frac{\Delta}{2}\right)^2 - 2\left(\frac{\Delta}{2}\right)^2\right]$$
$$= 1/2[1 + \Delta - \Delta^2/4], \tag{18}$$

$$D'(\Delta, 2) = -1/2\left[\left(1 - \frac{\Delta}{2}\right)\right]. \tag{19}$$

$$\Delta = [1 - 3\Delta + 3\Delta^2/4]/[1 - \Delta/2] \Rightarrow$$
$$0 = 1 - 4\Delta + 5\Delta^2/4 \Rightarrow$$
$$\Delta = [4 \pm \surd(16 - 5)]/(5/2)$$
$$= [8 - 2\surd(11)]/5 = 0.273. \tag{20}$$

$$P_A = 2 * P_{B_i} - \Delta = 1.45,$$
$$P_{B_i} = 1 - \Delta/2 = 0.86. \tag{21}$$

Although the solution is harder to calculate for larger n, computer simulation is quite tractable. The results for $n = [2, 8]$ are reported in the table below:

$$n = 2 \Rightarrow P_A = 1.45,\ P_{B_i} = 0.86,\quad \Pi_A = 0.91,\ \Pi_{B_i} = 0.32.$$
$$n = 3 \Rightarrow P_A = 2.09,\ P_{B_i} = 0.88,\quad \Pi_A = 1.47,\ \Pi_{B_i} = 0.26.$$
$$n = 4 \Rightarrow P_A = 2.84,\ P_{B_i} = 0.92,\quad \Pi_A = 2.15,\ \Pi_{B_i} = 0.22.$$
$$n = 5 \Rightarrow P_A = 3.63,\ P_{B_i} = 0.94,\quad \Pi_A = 2.88,\ \Pi_{B_i} = 0.19.$$
$$n = 6 \Rightarrow P_A = 4.48,\ P_{B_i} = 0.96,\quad \Pi_A = 3.69,\ \Pi_{B_i} = 0.17.$$
$$n = 7 \Rightarrow P_A = 5.40,\ P_{B_i} = 0.99,\quad \Pi_A = 4.56,\ \Pi_{B_i} = 0.15.$$
$$n = 8 \Rightarrow P_A = 6.36,\ P_{B_i} = 1.02,\quad \Pi_A = 5.48,\ \Pi_{B_i} = 0.14.$$

There are several interesting things to note. First, as expected, the bundler does much better that the sum of the uncoordinated B firms. When the bundles are small ($n = 2, 3$) the bundler does a bit worse than the first baseline case where each component is sold in an uncoordinated fashion.[9] The explanation is that the success of the bundle takes away so much market share from the B firms that their resulting equilibrium prices are so low that the A firm is worse off. But this setback is only temporary. By the time the bundle has four or more items, the bundler is doing better than previously. The gap in prices continues to grow and consequently, so does the bundler's market share.[10]

[9] The $n = 3$ case is a very close call. The bundling firm has a profit of 1.47 compared to an aggregate profit of 1.5 in the component-versus-component baseline case.

[10] We note that once the bundle has eight or more items the price of the B items begins to exceed 1. Hence consumers will not find it worthwhile to engage in mixed bundling.

It is also interesting that this game is not a prisoner's dilemma. While it is true that the profits of the B firms are very seriously depressed, they would not improve their lot by getting together and forming a rival bundle. This is because the intensity of the bundle-against-bundle competition so much reduces profits that the B firms are better off accepting what they get as individual players in the above equilibrium.

This suggests that the first firm to create a bundle wins. It wins in terms of profits, once the bundle has four or more items. It always wins in terms of market share. And, the victory is long-lasting. The rival firms will only hurt themselves more by forming a competitive bundle.[11]

5. Literature Review and Conclusions

Firms have many incentives to bundle, even absent competition. Bundling can help a multi-product monopolist achieve better price discrimination (Adams and Yellen 1976; McAfee, McMillan, and Whinston 1989; Bakos and Brynjolfsson 1999).[12] It can also help lower costs and lead to superior products (Salinger 1995).

Bundling can also be used as an entry-deterrence device, as first recognized by Whinston (1990). The ability to leverage a monopoly had previously been in doubt as there is little gain from bundling a monopolized good along with one sold in a competitive market.[13] The gain arises when the incumbent has market power in two (or more) goods. In Whinston's model and in related work by Aron and Wildman (1998), a monopolist's commitment to bundling makes it a tougher competitor to a one-good entrant. By committing to a bundle, the firm uses its surplus in good A to cross-subsidize good B, the one under attack. This denies market share to the entrant, often enough to deter it from entering. One qualification to these results is that if entry does occur, bundling ends up hurting the incumbent and thus there must exist some way to credibly commit to a bundling strategy.

A companion work to this paper (Nalebuff 1999) re-examines bundling as an entry deterrent using a Stackelberg pricing game. Here, too, an incumbent firm with a monopoly in several components can help protect these multi-monopolies from entry by bundling them together. This is because a one-product entrant faces a very restricted market compounded by a low incumbent price all of which makes entry much less profitable. And now credibility is not an issue as the incumbent does better using a bundle against an entrant compared to losing a head-to-head competition on one of the components.

[11] This, of course, doesn't take into account the fact that forming a competing bundle would also destroy the rival A firm's profits. Misery loves company. Or, more to the point, firms may prefer not to be in such an asymmetric position relative to a rival when there are issues of R&D financing or similar dynamic issues in a repeated game.

[12] Note that bundling as a price discrimination device works best when the items have a negative correlation in value. If the items are highly complementary and hence always bought as a package, then there is no gain from superior price discrimination.

[13] Some limited gains are possible when consumers purchase variable quantities of the two goods.

These models leave open the issue of multi-product entry (or sequential entry by independent firms). This paper helps us better to understand how bundling can change the nature of competition, post-entry. Of course it is difficult to come up with two or more products so as to have simultaneous entry. But having done so, there are much lower profits available to an entrant if the incumbent has a bundle in the market. If the challenger comes in with a bundle, everyone's profits are very depressed. If two firms come in at the same time (or there is one latent firm who becomes active with the second firm's entry), here, too, the entrants' profits are greatly reduced compared to when the incumbent sells its product as components or the incumbent is in fact two separate monopolies, each selling its one component.

The results of this paper are perhaps closest to the work of Matutes and Regibeau (1989, 1992). They consider a model with two firms each selling two goods. They ask whether these two firms would choose to make their products compatible or not and whether the firms would prefer to sell their products as a bundle or not. In our first two baseline cases the two approaches essentially converge. Consider first component-versus-component competition. If all the components are compatible, then there is no externality created by lowering the price of one. Hence there is no gain from getting the two A firms (or the two B firms) to coordinate their pricing.[14] Thus our four-firm oligopoly and their two-firm duopoly yield the same outcome. In the case of bundle-versus-bundle competition, compatibility is irrelevant as consumers cannot mix and match. The bundle-versus-bundle competition reduces the number of firms from four to two and thus is a model of duopoly. We, too, observe that bundle-versus-bundle competition leads to the lowest profits—and the result only gets worse as the bundle size grows beyond two goods.

Our third scenario, the model of a bundler versus component sellers, is where the two approaches truly diverge. We are able to focus on the coordination problem of component sellers against a bundler.[15] This is the imperfect competition extension to Cournot's multi-product monopoly model. We also see that the results from two-good bundles may turn around as the bundle grows in scope. The disadvantage of creating a two-good bundle essentially disappears when there are three goods and even becomes an advantage once there are four or more items together.

Putting all of these results together a fuller picture of bundling emerges. As powerful as bundling is to a monopolist, the advantages are even larger in the face of actual competition or potential competition. Selling products as a bundle can raise profits absent entry, raise profits even against established but uncoordinated firms, all the

[14] Matutes and Regibeau do not assume that valuations are all sufficiently high so that every consumer makes a purchase. When the market is not all served, there would be a small advantage as lower prices expand the market. Because the whole market may not be served, their paper also is better designed to examine consumer welfare implications. In our case, component selling is unambiguously the best since all consumers, by mixing and matching, will end up with their most preferred package. The bundle-versus-bundle option restricts choice and therefore reduces social welfare. The bundle versus components is even worse for social welfare. This is because prices are no longer symmetric. Consequently, consumers who should naturally prefer the B firm products are induced to travel inefficiently far in order to get the lower price on the A bundle.

[15] This coordination problem doesn't arise in Matutes and Regibeau's duopoly model.

while lowering profits of existing or potential entrants, and putting these rivals in the no-win position of not wanting to form a competing bundle. The only real disadvantage of bundling is the potential cost of inefficiently including items consumers don't desire. This is less important when the items are complementary and when the marginal cost is essentially zero, as with information goods. Thus we can expect bundling to be one of the more powerful and prevalent tools, perhaps we should say weapons, in our information economy.

APPENDIX: BUNDLE AGAINST COMPONENTS

Mixed Bundles

In this Appendix we extend the results in Section 4 to consider the slightly more complicated equilibrium when consumers can purchase B items along with their A bundle. For example, a customer who is particularly well matched with WordPerfect might buy this product along with the Office bundle. (Note that in our model it is not possible to mix the other way. Since the A products are only sold as part of a bundle, any customer who wants any of the A components must buy all of them.)

Consider the choice of a consumer who is already buying the A bundle. It is worth purchasing a B component in addition to the bundle if

$$1 - \alpha_i + P_{B_i} \leqslant \alpha_i \Leftrightarrow \alpha_i \geqslant (1 + P_{B_i})/2. \qquad (A1)$$

Conversely, consider the case of a consumer who plans to purchase B_1 because he who prefers B_1 to A_1. It is still worthwhile to buy the A bundle and throw away A_1 provided

$$\alpha_2 + P_A \leqslant (1 - \alpha_2) + P_{B_2} \quad \alpha_2 \leqslant (1 + P_{B_2} - P_A)/2. \qquad (A2)$$

These two regions are depicted in Figure 17.1.

In this environment, demand adds up to more than one. For example, B_2 gets an extra demand from the upper-left-corner trapezoid (from people who buy B_2 along with the bundle) but loses a smaller triangle in the lower right from people who choose to buy B_1 along with an A bundle rather than consume B_1 along with B_2.

We calculate the equilibrium for $n = 2$. The result is quite similar to the previous case. This is not surprising as the component prices are relatively close to 1 and hence few consumers are interested in buying a bundle plus components. (Once n reaches 8 the equilibria are identical as the individual components are priced at above 1 so that none are bought in addition to a bundle.)

Since the two demands don't add up to 1, we need to calculate each separately. The notation is a bit simpler if we define $L = 1 - \Delta/2$. For the B_1 firm,

$$D_{B_1} = (1/2)L^2 + (1/8)[1 + 2 * P_{B_2} + P_{B_1} - 2P_A](1 - P_{B_1}) - (1/8)(1 - P_{B_2}). \qquad (A3)$$

The equation for B_2 is symmetric. For the bundle provider

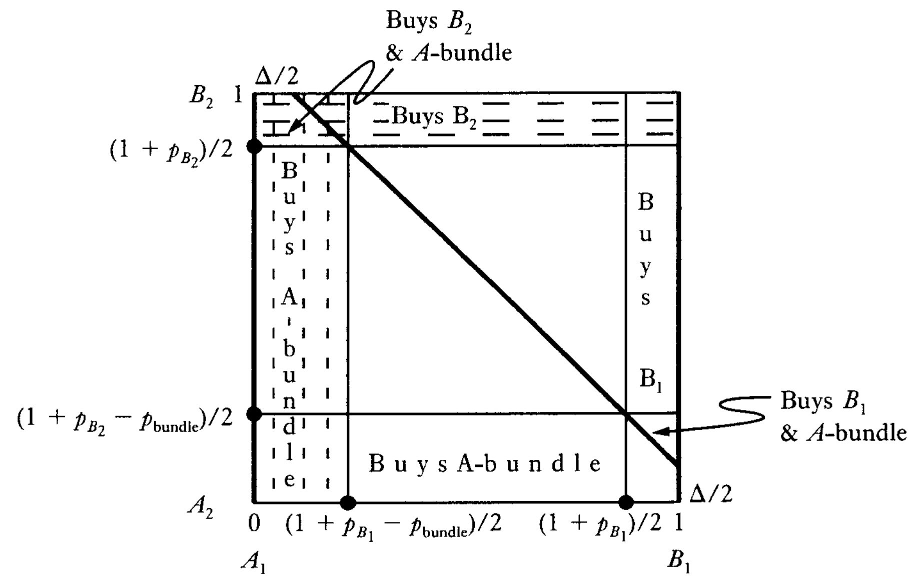

FIG. 17.1. *Division of Market*

$$D_A = 1 - 1/2L^2 + 1/8[(1 - P_{B_1})^2 + (1 - P_{B_2})^2]. \tag{A4}$$

One simplification in all this is that it turns out that D' is very simple for both types of firms

$$D'_{B_i} = -1/2$$
$$D'_A = L/2. \tag{A5}$$

In equilibrium, $P_{B_1} = P_{B_2}$ so that

$$D_{B_i} = (1/2)L^2 + (1/4)(1 - P_{B_i})\Delta. \tag{A6}$$
$$D_A = 1 - (1/2)L^2 + (1/4)[(1 - P_{B_i})^2].$$

The first-order conditions then reduce to

$$P_A * L/2 = 1 - L^2 + (1/4)(1 - P_A^2).$$
$$P_{B_i}/2 = L^2/2 + \Delta/4 * (1 - P_{B_i}). \tag{A7}$$

Note that now both P_A and P_{B_i} are functions only of L, which in turn is a function of Δ. And since $\Delta = 2P_{B_i} - P_A$ we have an implicit equation for Δ, the solution to which is

$$\Delta = 0.23, \quad P_A = 1.39, \quad P_{B_i} = 0.81.$$
$$D_A = 0.616, \quad D_{B_i} = 0.404.$$
$$\Pi_A = 0.856, \quad \Pi_{B_i} = 0.327. \tag{A8}$$

Once again the bundler dominates the individual component sellers. Prices for both firms are a little lower (1.39 versus 1.45 for firm A and 0.81 versus 0.86 for the B firms) as each firm can now expand the market with lower prices. The lower prices are almost exactly offset by the increase in total demand so that profits are almost unchanged. Thus, although the mathematics are more complicated, the intuition and the results are very similar to those in Section 4.

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