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Abstract

In Cournot's model of complements, the producers of A and B are both monopolists. This paper extends Cournot's model to allow for competition between complements on one side of the market. Consider two complements, A and B, where the $A + B$ bundle is valuable only when purchased together. Good A is supplied by a monopolist (e.g., Microsoft) and there is competition in the B goods from vertically differentiated suppliers (e.g., Intel and AMD). In this simple game, there may not be a pure-strategy equilibria. In the standard case where marginal costs are weakly positive, there is no pure strategy where the lower quality B firm obtains positive market share. We also consider the case where A has negative marginal costs, as would arise when A can expect to make upgrade sales to an installed base. When profits from the installed base are sufficiently large, a pure strategy equilibrium exists with two B firms active in the market. Although there is competition in the complement market, the monopoly Firm A may earn lower profits in this environment. Consequently, A may prefer to accept lower future profits in order to interact with a monopolist complement in B.

Keywords: AMD, complementors, complements, co-opetition, equilibrium non-existence, installed base, Intel, Microsoft, pricing.

JEL classification numbers: C72, D43, K21, L13, L15, M21.

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1 Introduction

Over the last two decades, an increasing number of industries have evolved from vertical integration to more horizontal structures where firms design and manufacture components which are later assembled by third parties for the final customer. In these horizontal industries, firms may be 'complementors,' rather than customers, suppliers, or competitors. The classic pair of complementors is Intel and Microsoft. Similar complementor relationships arise in industries ranging from communications to consumer electronics, and automobiles to healthcare. In these industries, complementor analysis may be as important as competitor analysis.

In his seminal book, Augustin Cournot [7, Chapter 9] introduced a model of competition between producers of complementary goods. Using the example of copper and zinc that is combined to make brass, Cournot showed that monopolists in each industry will divide the profits evenly, regardless of cost differences. The applicability of Cournot's model is limited by the assumption that the two suppliers of complements are each monopolists. In many horizontal industries there is competition both *between* complementors (Microsoft and Intel) and also *within* rival complements (Intel and AMD). While price competition between complementors and price competition between vertically differentiated goods are each well-understood, there is no previous work on the combined case which describes the 'competing complements' phenomenon.

In this paper we introduce competition into one side of the complements game. As in Cournot [7], we consider two strictly complementary goods, A and B: the bundle $A + B$ is valuable, though neither A or B alone are of any value. The A good is supplied by a monopolist while the B market is a duopoly. There is a high-quality and a low-quality supplier of B (B_H and B_L, respectively). Both bundles (A, B_H) and (A, B_L) are valuable, but the bundle with B_H is preferred by all customers.

We illustrate this model using the pc industry. Following IBM's decision to set up an open standard for its Personal Computer in 1980, the microcomputer industry became gradually more horizontal, which led to specialized players increasingly dominating each component layer. The microprocessor and the operating system (OS) are strictly complementary in that Intel architecture microprocessors are worthless without Windows and Windows is of no value without microprocessors. Windows is monopolistically supplied by Microsoft; in contrast, there is competition between Intel and AMD in the supply of microprocessors.[1] For simplicity, we assume that customers all view Intel's chips as superior.

While the model is a simple extension of Cournot, the introduction of competition from

[1] The model can also be flipped. We can think of Microsoft and Linux supplying rival operating systems and Intel being the monopoly chip supplier.

rival complements has a dramatic effect on market outcomes. Competition can create instability and surprisingly complicated interactions. When the two complements are sufficiently similar, competition within complements breaks the existence of any pure-strategy equilibria. Specifically, we show that with zero (or positive) marginal costs, there is no equilibrium in pure strategies where B_L gets positive demand. Intuitively, and in the context of our motivating example, if AMD active in the market, Intel wants to decrease its price to recover demand lost to AMD. But if AMD is not active in the market, Microsoft is willing to raise price, as the reduction in demand is low. With high prices from Microsoft, AMD is not a threat to Intel, which gives Intel an incentive to raise price. But with high prices from Intel, Microsoft then has an incentive to lower prices and bring AMD back into the market. Thus the waltz begins once again. It is this dance between Microsoft, Intel and AMD that blocks any pure strategy solution.

Although there is no pure strategy, we can eliminate strictly dominated strategies to derive bounds on prices and thereby evaluate the effects of competition within complements on profits. We confirm the intuition that competition between the B complements is good for A when B_H and B_L are close in quality. This suggests that Microsoft will take action to ensure that Intel and AMD's products are perceived as similar by, for example, supporting microprocessor enhancements only when offered by both Intel and AMD.

We then extend the model to consider negative marginal costs and demonstrate conditions under which there is a pure-strategy equilibrium where B_L gets positive demand (and profit). Monopolist A could have negative marginal costs if a sale also leads to future revenue in addition to the current price. For example, Microsoft enjoys negative marginal costs because, in addition to making money from selling operating systems to the flow of new customers, it anticipates future revenue from selling upgrades and applications to the installed base. Thus each new customer creates an annuity, which is reflected in the negative cost. In contrast, Intel and AMD's revenue derives only from the sale of microprocessors to new customers, not from the installed base. With negative marginal costs Microsoft is willing to set very low prices to increase the installed base. The low prices of Microsoft induce Intel to raise its price and this allows AMD to come in at a low price and enjoy positive demand.

Finally, we use the model to gain insight into the desirability of competition in B from the perspective of A and from the maker of B_H. Microsoft prefers the world with AMD when customers see Intel and AMD's microprocessors as close in quality. In this case, the strong substitutability leads to low microprocessor prices. With low processor prices, the installed base grows fast and Microsoft earns more money from the initial sales. The surprise arises when Intel and AMD's products are sufficiently vertically differentiated. Then if Microsoft has sufficiently negative costs, Intel is better off and Microsoft worse off with AMD present.

In the equilibrium with AMD, Microsoft sets very low prices and Intel captures more of the pie.

1.1 Related Literature

The paper contributes to literature on 'Co-opetition' (Brandenburger and Nalebuff [1]) which presents Intel and Microsoft as a motivating example on the tension between cooperation and competition that characterizes relationships between complementors. Casadesus-Masanell and [3] present a formal model to study the incentives of Microsoft and Intel to cooperate and compete and show that there are significant misalignments of incentives. In particular, Intel sets prices 'too high,' taking advantage of Microsoft's willingness to price low to build the installed base. Here, we evaluate an approach that Microsoft could follow to induce Intel to set lower prices: encourage the development of competition in microprocessors.

Farrell and Katz [8] study innovation in a setting with a monopoly in A and a competitive market in B. They analyze A's incentives to enter B's market to force suppliers of B to set lower prices. Consistent with the motivating example, we do not allow A to enter B's turf (and vice versa). In addition, our setting is one with heterogeneous consumers and we focus on pricing, not on incentives to innovate.

Cheng and Nahm [5] consider the issue of double marginalization in a pricing game with heterogeneous consumers where products A and B need not be strict complements. In their model, a customer can enjoy A alone, but gets an additional utility from A with B. This is mathematically equivalent to imagining that A comes packaged with a free low-quality B; the consumer would prefer to enjoy A with the higher quality B, but the upgrade may not be worth the additional price. The Cheng and Nahm paper has two strategic players, the A monopolist (who can be thought to sell an integrated product that includes a low-quality B complement) and the high-quality B monopolist. We consider the strategic interaction between three players, as we allow the low-quality B complement to set price and maximize its profits. This allows us to examine the potential profits of the competing complementors and when such competition ends up being detrimental to the A monopolist.

Another difference between our approaches is that Cheng and Nahm emphasize the Stackelberg pricing game, while we focus on the Nash pricing game. Furthermore, we consider the results in the case with negative costs, which turn out to be quite different. The possibility of negative costs naturally arises in our context due to the potential for follow-on sales to the installed base which means that the firm can expect to make more than the current price with the initial sale.

Chen and Nalebuff [4] study competitive interaction in markets with one-way essential complements (A is essential to the use of B, but can be enjoyed without B). Their setting gives insight into competitive interactions between Microsoft and independent software vendors (because the OS is essential to applications, but not the other way around) rather than into the competition between Microsoft, Intel and AMD (because in this case both the OS and the microprocessor are essential to one another). Just as in Farrell and Katz [8] and Cheng and Nahm [5], Chen and Nalebuff [4] allow A to also compete in B's market. They show that A has an incentive to produce a competing version of B and sell it at zero price. Moreover, when the value of B is small relative to A, then giving away B leads to the joint monopoly outcome.

Our paper is also related to aspects of the literature on bundling (see Stigler [13] and McAfee et al. [11]). In our setting, independent firms offer separate components but the final customer can only enjoy the bundled product. Our model can be interpreted as one of competition for customers by two vertically differentiated bundles with a common component. In contrast to the question of entry deterrence, which has been the focus of much of this literature (see Whinston [14], Choi and Stefanadis [6], Carlton and Waldman [2], Gilbert and Riordan [10], and Nalebuff [12]), our focus is on how the pie is split between the independent suppliers of bundle components.

2 Benchmark: Competition *between* Complementors

We begin by presenting the standard complementor game. Two firms m and i produce perfect complements: the products are worthless unless used together. For most of the paper, we will refer to firm m as Microsoft and firm i as Intel as they are the archetype of complementary firms.[2] Let $q = D(p_m + p_i)$ be the demand for the bundle. Let $C_j(q) = F_j + c_j q$, $j \in \{m, i\}$ be the cost to firm j of producing quantity q. Players choose prices simultaneously.

In this case, the pie is split 50:50. Profit margins are equal *independent* of marginal cost differences. While this result may be surprising, the proof below follows directly from Cournot [7].[3] Formally, the result is as follows:

[2]In 2007, more than 80% of the personal computers worldwide shipped with an Intel microprocessor running Microsoft's Windows operating system. Customers to Microsoft and Intel are original equipment manufacturers (OEMs). Pcs are commodities and OEMs have negligible pricing power. Moreover, all other components in a pc (disk drives, monitors, memory and other chipsets) are also commodities. It is therefore appropriate to assume that the main two strategic players affecting quantity sold of pcs are Microsoft and Intel ($q = D(p_m + p_i)$). In fact, the combined profit of Intel and Microsoft during most years in the 1990s exceeded the total profit of the entire world pc industry. In 2004, for example, Intel and Microsoft earned over \$15 billion in net profits while the three largest OEMs (Dell, HP and IBM) made roughly \$2.5 billion in profits from their pc operations. IBM, alone, lost over \$1 billion in pcs in 1998, and another \$965 million between 2001 and 2004. Only Dell made material profits in the pc industry. For more background information on Microsoft and Intel and a detailed account of their interactions over the years see Yoffie, Casadesus-Masanell and Mattu [15].

[3]As Cournot wrote: "... by the purely abstract hypothesis under consideration, the profits would be equally divided between the two monopolists (p. 102)." He generalized the result to the case of constant

Proposition 1 *Given demand $D(p)$ and players m and i with linear cost functions, profit margins and variable profit (profits after accounting for fixed costs) are equal for both players.*

Proof. Firm j maximizes $\pi_j = (p_j - c_j)D(p_i + p_m) - F_j$, for $j \in \{m, i\}$. First-order conditions are $(p_j - c_j)D(p_i + p_m)' + D(p_i + p_m) = 0$. Therefore $p_m - c_m = p_i - c_i$ and $(p_m - c_m)D = (p_i - c_i)D$, as demand is the same for both firms. ■

It is surprising that a firm with high costs does just as well as one with low costs. The intuition is that the gain from existing customers from raising price is the same for both firms as, by definition, the demand is the same. Also, the incremental loss or gain of customers from a price change must be the same (as customers only look at the combined price). Thus profit margins must be the same.

At present, Intel faces significant competition from AMD and thus fits into the more advanced case of competition between complements. But for much of the early pc period, Intel and Microsoft were dual monopolists. Proposition 1 predicts that during this period, the two firms would have made comparable profits on the sale of a pc. Does that accord with the evidence?

Here, we have to be careful in interpreting the model. When a customer purchases a pc with an Intel chip and a Microsoft OS, Intel makes all of its profits at that juncture. In contrast, Microsoft creates an annuity where it will continue to earn money from that customer with upgrades to a new OS and with the sale of Office. The theory predicts that the two complementors will make the same profits when measured over the lifetime of each pc.

Another way of seeing the equal profit result is that it is as if the two firms are engaged in joint production where their combined costs are $c_i + c_m$. Because customers must buy the two goods together, the firms effectively share the cost of making the two complementary goods.

The fact that costs are split has two strategic implications. First, firms do not have sufficient incentives to reduce their individual variable costs. The reason is that half of the costs are effectively paid by the other complementor.

A second implication of this result is that a firm would like to treat its fixed costs as if they were variable. While this inefficiency reduces output, that loss is second order. The reason the firm gains is because it is able to split the "artificial" cost with its complementor. Thus it can raise its price while the complementor lowers price. This may explain why Intel seems to act as if some of the costs of its fab plant are variable.

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costs on page 106.

Corollary 1 *To the extent that a monopoly complementor can act as if it has higher variable costs, with log-concave demand this strategy will initially lead to increased profits.*

The proof is in Appendix A.

3 Adding Competition *within* Complements

3.1 Motivation

We now investigate the effect on market outcomes when there is competition in one of the complements markets. In particular, we study the game where A is a monopolist and there are two differentiated firms in the B market.

Our motivating example is the interaction between Microsoft, Intel, and AMD. This is just one example of a competing complements framework. We use this example as it illustrates how the model applies to a specific industry. The application to hardware and software also motivates the relevance of negative marginal costs (due to follow-on sales).

Microsoft is a monopolist in operating systems and there is a duopoly in microprocessors (Intel vs. AMD).[4] As it will become apparent, the game with competition in one side is sufficiently complex that we do not attempt to solve the case with competition in both sides.

AMD was founded in 1971 with the primary mission of being a second source to Intel's innovations. At the time, most customers (original equipment manufacturers) required innovators to license their products to second sources as a condition for winning designs. By 1976, AMD and Intel signed a broad cross-license: the deal required Intel license its newest technology in exchange for AMD delivering technology of 'comparable value.' In 1985, however, Intel believed that AMD could not offer comparable value, and it abrigated the cross-license. After a decade of litigation, Intel and AMD settled, with AMD gaining rights to Intel's 386 microcode in exchange for monetary compensation.

We will assume that customers view Intel's microprocessors as offering higher performance.[5] This vertical differentiation between Intel and AMD comes from Intel's history as a more reliable supplier, its deeper balance sheet which offers customers a greater sense of safety in buying from a secure supplier, and Intel's ability to supply complementary technologies, such as chips sets, which are necessary to deliver a complete system. Yet if the price differential is large enough, some customers will buy AMD. Because AMD is perceived as an inferior supplier to Intel, AMD needs to set prices below Intel's in order to generate demand for its microprocessors.

[4] Alternatively, Intel could be considered a monopolist in microprocessors and Microsoft and Linux duopolists in operating systems. However, being open source, it is questionable that Linux prices strategically. Therefore, the analysis of OS competition is simpler, but less general.

[5] We note that David Yoffie is a director of Intel.

3.2 Demand

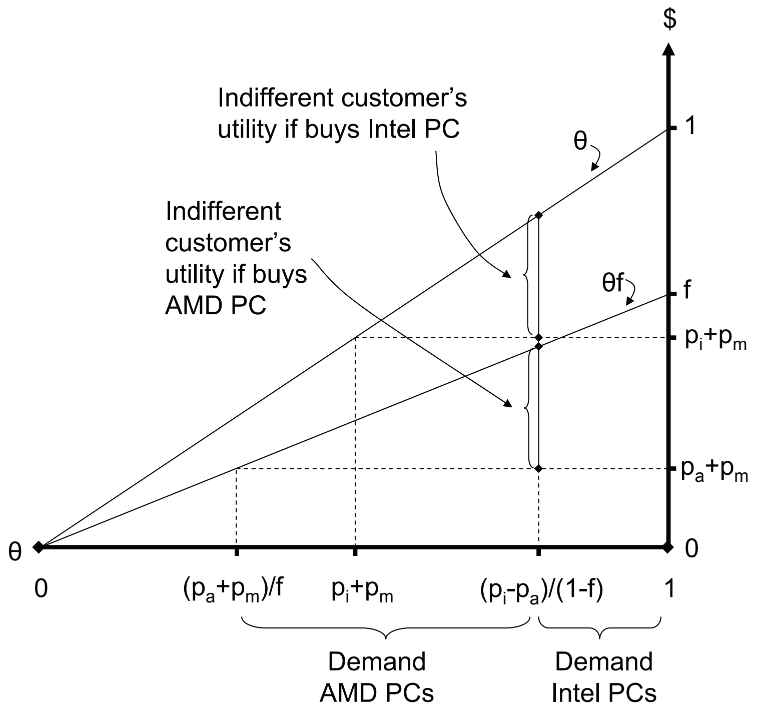
Customers are indexed by $\theta \sim U[0,1]$. A customer of type θ values $A + B_h$ at θ and values $A + B_l$ at $f\theta$ where $0 < f < 1$.

In our context, that means the value of Microsoft Windows with AMD is a fraction f of its value with Intel. As before, A and B are essential complements so that the value of Microsoft alone or Intel/AMD alone is zero. Every pc has a Microsoft OS but the microprocessor may be Intel or AMD.

Initially, we solve the model where all three firms have zero costs and then generalize the result to include positive (and negative) costs in Section 4. We first derive demand for firms A, B_h, and B_l, which for exposition purposes we will refer to as Microsoft, Intel and AMD. Each firm will be denoted by its initial. Thus p_a and q_a refer to the price and quantity for AMD; p_i and q_i apply to Intel; p_m and q_m apply to Microsoft.

Lemma 1 *Given p_m, p_i, and p_a, demand for AMD is the interval of line from $\frac{p_i - p_a}{1-f}$ down to $\frac{p_a + p_m}{f}$, assuming the interval is positive; else demand is zero. Demand for Intel is the interval from $\frac{p_i - p_a}{1-f}$ up to 1, assuming the interval is positive; else demand is zero. Demand for Microsoft is the sum of the demand for AMD and Intel.*

The following diagram plots the utility of customer of type θ when using an Intel+MS bundle and an AMD+MS bundle.



The indifferent customer is $\hat{\theta} = \frac{p_i - p_a}{1-f}$. Note that the diagram is for the case where $\frac{p_i - p_a}{1-f} > \frac{p_a + p_m}{f}$. Else, Intel's demand is based on $1 - (p_i + p_m)$ and AMD gets zero demand.

In what follows we say that AMD is 'active' if AMD earns positive profit or is on the margin of earning positive profit. Being on the margin of earning profits arises when AMD is just pushed down to charging marginal cost (here 0) and the lowest value customer in the market is just indifferent between Intel and AMD. More formally, $p_a = 0$ and $q_a = 0$, but $dq_a/dp_i > 0$, so that were Intel to raise its price AMD would have positive demand. In contrast, when AMD is not active, then at $p_a = 0$ all customers in the market strictly prefer Intel to AMD.

When AMD is active demand functions are:

$$q_m = 1 - \frac{p_a + p_m}{f}, \qquad q_i = 1 - \frac{p_i - p_a}{1-f} \qquad \text{and} \qquad q_a = \frac{p_i - p_a}{1-f} - \frac{p_a + p_m}{f}.$$

And when AMD is not active demand functions are:

$$q_m = q_i = 1 - p_m - p_i \qquad \text{and} \qquad q_a = 0.$$

3.3 Equilibrium (Non)Existence

We begin by showing that with zero costs there is no pure strategy equilibrium where AMD gets positive demand, regardless of how close AMD and Intel microprocessors may be.

Lemma 2 *With zero costs, there is no pure strategy equilibrium in which AMD obtains positive demand.*

Proof. If AMD has positive demand, Microsoft, Intel and AMD's best responses are

$$p_m = \frac{f - p_a}{2}, \tag{1}$$

$$p_i = \frac{1 - f + p_a}{2}, \tag{2}$$

and

$$p_a = \frac{f p_i - (1-f)\, p_m}{2}. \tag{3}$$

Substituting in (1) and (2) into (3) we get

$$p_a = \frac{1}{4} p_a.$$

Therefore, $p_a = 0$. Moreover, demand for AMD is

$$q_a = \frac{f \frac{1-f}{2} - (1-f)\frac{f}{2}}{f\,(1-f)} = 0.$$

■

We conclude that in any equilibrium in which MS, Intel and AMD are all active and choosing prices that are best-responses to each other, AMD must get zero demand. The intuition behind this result is that Intel has a strong incentive to cut price so long as AMD has positive share.

Given $p_a = 0$, the reaction functions above tell us that the candidate equilibrium prices for Microsoft and Intel are

$$p_m = \frac{f}{2} \qquad \text{and} \qquad p_i = \frac{1-f}{2}.$$

We now show that this candidate cannot be an equilibrium.

Proposition 2 *With zero costs, there is no pure-strategy equilibrium in which AMD is active.*

Proof. Lemma 2 has established that $p_a = 0; p_m = \frac{f}{2}$; $p_i = \frac{1-f}{2}$ is the only candidate equilibrium in which AMD is active. Although $q_a = 0$, AMD is active because it is on the margin of obtaining positive demand. However, $p_m = \frac{f}{2}$ is not a best response for Microsoft. If $p_i = \frac{1-f}{2}$ and $p_a = 0$, Microsoft would do better to charge $p_m = \frac{1+f}{4}$.

Microsoft raises it profits by picking its best response to Intel, ignoring the presence of AMD. Because AMD has zero demand when Microsoft charges $\frac{f}{2}$, AMD will also have zero demand for any higher p_m. Microsoft's new price is higher as $\frac{1+f}{4} > \frac{f}{2}$ for $f < 1$.

Microsoft's profits when $p_m = \frac{f}{2}$, $p_i = \frac{1-f}{2}$, and $p_a = 0$ are $\pi_m = p_m \left(1 - \frac{p_m}{f}\right) = \frac{f}{4}$. Microsoft's profits when $p_m = \frac{1+f}{4}$, $p_i = \frac{1-f}{2}$, and $p_a = 0$ are higher at $\pi_m = p_m \left(1 - (p_i + p_m)\right) = \frac{(1+f)^2}{16}$:

$$\frac{(1+f)^2}{16} - \frac{f}{4} = \frac{(1-f)^2}{16} > 0$$

for f in $[0, 1)$, establishing that $p_a = 0; p_m = \frac{f}{2}$; $p_i = \frac{1-f}{2}$ is not an equilibrium. ■

The intuition for the non-existence is as follows. With AMD in the game, Intel's marginal profits are discontinuous in a way that supports equilibrium. If Intel raises price, AMD takes share away quickly, at rate $\frac{1}{1-f}$. But if it lowers price, and AMD is squeezed out of the market, then Intel only expands the market with Microsoft at rate 1. Thus raising price reduces demand quickly while lowering price increases it slowly.

For Microsoft, the case is reversed. Lowering price expands demand quickly (rate 1/f) when AMD is active (but on the margin); in contrast, raising price only slowly reduces its joint demand with Intel (rate 1). Thus if Microsoft is indifferent about lowering price, then it will want to raise it, and if it is indifferent about raising price, then it would want to lower it. That explains why there is no equilibrium with AMD active in the market.

We now investigate if there is an equilibrium where AMD is not active. In this case, Microsoft and Intel face demand $1 - p_m - p_i$ and the unique candidate equilibrium is $p_m = p_i = \frac{1}{3}$. For this to be an equilibrium, it must be that AMD cannot attract any demand even when $p_a = 0$. For AMD to get positive demand requires $\frac{p_i - p_a}{1-f} > \frac{p_a + p_m}{f}$. With $p_i = p_m$ and $p_a = 0$, AMD's demand will be zero only if $f \leq 1/2$. This is a necessary condition for an equilibrium where AMD is not active.

At this point, we need to be careful with regard to how AMD acts in an equilibrium in which it is not active. Since AMD's demand is zero, there are a range of prices it could charge, all of which lead to zero demand and zero profits. If AMD charges a high price, then it will be less relevant when Intel and Microsoft consider alternative prices. Thus if p_a is large enough, there are pure strategy equilibria whenever $f \leq \frac{1}{2}$. However, we think these equilibria are artificial. A firm that is unable to attract customers should stand willing to take on customers at a price equal to marginal cost.

Assumption 1: If AMD is not active, it charges a price equal to marginal cost. Here, this implies $p_a = 0$.

Under Assumption 1, it turns out that a stronger condition, $f \leq \frac{4}{9}$ is both necessary and sufficient for there to be a pure-strategy equilibrium where AMD is not active. We first establish that this is sufficient.

Proposition 3 *Under Assumption 1, if $f \leq \frac{4}{9}$, then there exists a unique pure-strategy equilibrium in which Microsoft and Intel each charge $\frac{1}{3}$ and earn $\frac{1}{9}$. AMD has no demand and no effect on the market.*

Proof. We know that there is no pure-strategy equilibrium in which AMD is active. If AMD can be ignored. then the unique candidate equilibrium is the Cournot solution with $p_m = p_i = \frac{1}{3}$. In this case, provided $f \leq \frac{1}{2}$, AMD cannot achieve positive demand at $p_a = 0$. Thus we only need check that neither Microsoft nor Intel want to deviate. The only relevant deviation would be to price where AMD becomes active.

Consider Intel first. Lowering price will never make AMD a factor but will decrease Intel's profit. Raising price sufficiently may make AMD active but that would only be worse for Intel than its profits against Microsoft alone. Thus Intel maximizes profits at $p_i = \frac{1}{3}$.

Consider next Microsoft. Were Microsoft to raise price, AMD will not become active. However, were Microsoft to lower price sufficiently, AMD will have positive demand. Thus, we have to test that Microsoft doesn't want to price low and bring AMD into the game. With AMD active, we know from equation (1) that Microsoft's optimal price is $p_m = \frac{f}{2}$ and profits are $\pi_m = p_m\left(1 - \frac{p_m}{f}\right) = \frac{f}{4}$.[6]

Microsoft will not want to deviate from the proposed equilibrium provided that $\frac{1}{9} \geq \frac{f}{4}$ or $f \leq \frac{4}{9}$. ■

The proof also demonstrates that the conditions are necessary. The requirements are $f \leq \frac{1}{2}$ to ensure that AMD will not have positive demand and $f \leq \frac{4}{9}$ to ensure that Microsoft will not deviate to price at $\frac{f}{2}$. Both constraints are satisfied when $f \leq \frac{4}{9}$.

Intuitively, if f is small, then AMD is not a factor as its product is not a good substitute for Intel. Thus Microsoft will not want to pursue the undercut strategy to bring AMD in.

A corollary of our results is that when AMD microprocessors are sufficiently close in quality to Intel's ($f > \frac{4}{9}$), then there is no pure-strategy equilibrium.

Corollary 2 *Under Assumption 1, if $\frac{4}{9} < f < 1$, there is no pure-strategy equilibrium.*

[6] This assumes that $q_a \geq 0$ at $p_a = 0$, $p_m = \frac{f}{2}$, and $p_i = \frac{1}{3}$. This will be true provided $f \geq \frac{1}{3}$. Thus, for $f > \frac{4}{9}$, Microsoft will make AMD active when it prices at $\frac{f}{2}$.

To see this, recall that Proposition 2 demonstrates that there cannot be a pure-strategy equilibrium with AMD active (for any value of f). When AMD is not active, the only potential equilibrium is $p_m = p_i = \frac{1}{3}$. As demonstrated in the proof of Proposition 3, this is not an equilibrium when $f > \frac{4}{9}$. as Microsoft would deviate to $\frac{f}{2}$ from $\frac{1}{3}$

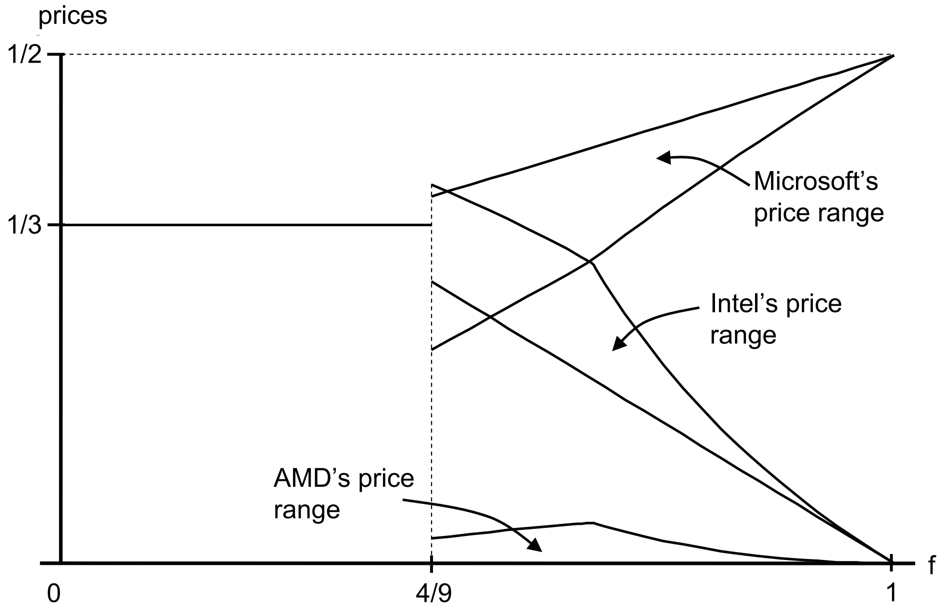
The analysis suggests that the complementors game is intrinsically unstable. Loosely speaking, the following 'dynamic' is at play. If we begin from a situation with $p_m = p_i = \frac{1}{3}$ and AMD inactive, Microsoft has an incentive to lower price and bring AMD in. But once AMD is active in the market, Intel wants to decrease its price to recover demand lost to AMD. At the point where AMD's demand is zero, Microsoft will choose to raise price, as the reduction in demand is low. But then AMD is no longer a threat even on the margin, which gives Intel an incentive to raise price. When Intel raises price, Microsoft once again prefers to lower price to make AMD active, and the cycle begins again.

In summary, either there is no equilibrium in pure strategies ($f > \frac{4}{9}$) or there is an equilibrium where AMD is irrelevant ($f \leq \frac{4}{9}$). While many games do not have pure strategy solutions, it would have been hard to anticipate that our simple extension of Cournot complements would have this dynamic. From a practical point of view, the result says that whenever it is relevant, the presence of a rival complementor causes instability in the game.

Even when there is no pure-strategy equilibrium, we can characterize the range where prices will fall in competitive interactions between Microsoft, Intel and AMD. In Appendix C we use the elimination of dominated strategies to derive bounds for p_m, p_i and p_a as functions of f). The range of undominated strategies are shown in the figure below.

One thing is apparent from this range of best responses: when f is large, Microsoft will do well, Intel will make a small amount and AMD will get close to zero. For example, when $f = 0.9$, the range of undominated prices for Microsoft is from 0.4487 to 0.4525, Intel is from 0.05 to 0.055 and AMD is from 0 to 0.0026. This is to be expected as in this case AMD is almost a perfect substitute for Intel.[7]

[7] In the boundary case where $f = 1$, there is a pure-strategy equilibrium. Here, Intel and AMD's microprocessors are perfect substitutes so that $p_i = p_a = 0$ regardless of Microsoft's price. Microsoft will respond by charging $p_m = \frac{1}{2}$ to obtain $\pi_m = \frac{1}{4}$.



4 Adding Marginal Costs (Positive and Negative)

Now we consider marginal costs different from zero and show that a pure-strategy equilibrium may exist when costs are negative. Let c_j be firm j's constant marginal cost. For simplicity, we will assume that $c_i = c_a = c$. This is beneficial to AMD in that we suppose that in practice $0 \leq c_i < c_a$ and so this is the best case for AMD.[8] Define $z = c_m + c$. We conduct our analysis below for all values of z, positive and negative.

Why consider *negative* z? We have written Microsoft's margin as $p_m - c_m$ with an implied understanding that $c_m \geq 0$. But we think that it is appropriate to consider the case where $c_m < 0$. The motivation is that while Microsoft's incremental cost from manufacturing additional copies of Windows is zero, it will eventually make a stream of revenue from the installed base. In the 1980s and early 90s, Microsoft made the stream from selling upgrades to the operating system. More recently this revenue has come from selling upgrades to Office,

[8] The manufacturing process for semiconductors is highly complex. Because of this complexity, production yields –the percentage of manufactured items that meet the necessary performance standards– are the most significant driver of marginal costs. Yields increase significantly with experience as firms identify and resolve trouble spots. Because Intel's cumulative volume is larger than that of AMD, it is likely that $c_i < c_a$ even if Intel's product is of higher quality.

mice, financial software and more.[9] Proposition 1 shows that in the two-player benchmark, Intel makes claims to half of those negative costs. Intel gets all of those profits *today* while Microsoft will get them *tomorrow* from the installed base of pcs.[10] We will see that when Microsoft's marginal cost is sufficiently negative, there is an equilibrium with AMD active.[11]

We begin with a generalization of Proposition 2.

Proposition 4 *With $z \geq 0$, there is no pure-strategy equilibrium with AMD active.*

Proof. The proofs of all remaining propositions are in Appendix A. ■

When marginal costs are positive, there is no pure-strategy equilibrium where AMD makes money. In fact, the result is slightly stronger: even if Microsoft's costs are negative, as long as $c_m \geq -c$, there is no pure-strategy equilibrium with AMD active.

Next we ask whether there is an equilibrium where AMD is not active. We have already seen (Proposition 3) that when $z = 0$, there *is* an equilibrium where AMD is not active as long as $f \leq \frac{4}{9}$. The following proposition generalizes that result.

Proposition 5 *Under Assumption 1 and $0 < f < 1$, there is an equilibrium with AMD inactive if and only if $z \geq z^*(f)$, where*

$$z^*(f) = \begin{cases} \frac{1}{2}\left(-7+3\sqrt{5}\right) & \text{if } \ 0 \leq f \leq \frac{1}{2}\left(7-3\sqrt{5}\right) \\ \frac{5f-6(1-f)\sqrt{f}}{9-4f} & \text{if } \ \frac{1}{2}\left(7-3\sqrt{5}\right) < f \leq 1 \end{cases}$$

In this equilibrium

$$p_m = \frac{1+2c_m-c}{3}, \tag{4}$$

$$p_i = \frac{1+2c-c_m}{3}. \tag{5}$$

As before, for $z \geq 0$, these equilibria arise only when $f \leq \frac{4}{9}$. Higher costs z, extends the range equilibrium where AMD is inactive. When Intel and AMD have large marginal costs, AMD will set a relatively high price even when inactive. As a consequence, the potential expansion in demand from having AMD in the market is modest. This makes it less attractive for Microsoft to price low and thereby break the candidate equilibrium.

[9] Note that in imagining that Microsoft will earn this future revenue stream, customers are myopic in that they do not take account of the fact that they will be paying it in the future. At the same time, they are not taking account of the benefits that will come with those upgrades, either.

[10] See Casadesus-Masanell and Yoffie [3] for an dynamic model where Intel makes money from the flow of pc sales and Microsoft makes money from the flow *and* the installed base.

[11] In this development, every customer contributes equally to Microsoft's future income (regardless of the customer's willingness to pay for a PC). We have also considered the case where the expected future income depends on the incremental value of the marginal consumer and shown through examples that the main results in sections 4 and 5 also hold. We have not characterized existence of equilibria for all values of cost and f in this case.

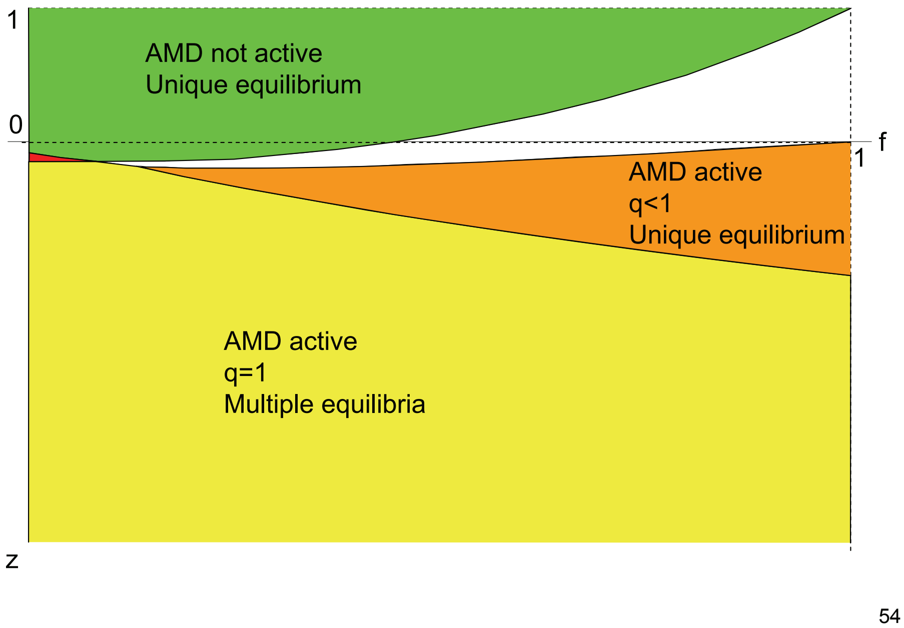
Finally, we ask whether there are combinations (z, f) such that there is a pure-strategy equilibrium with AMD active. We know that a necessary condition is $z < 0$. The proposition below shows that in this case we can always find f such that a pure-strategy equilibrium exists with AMD active and furthermore where AMD earns positive profits.

Proposition 6 *For every $z < 0$ there is an f such that AMD is active and earns positive profits in equilibrium.*

A necessary condition for $z < 0$ is $c_m < 0$. Therefore, when $z < 0$, Microsoft makes money from the installed base by selling applications and upgrades. Microsoft is willing to give up profits today to build the installed base of PCs by setting low prices. The low prices of Microsoft make it possible for AMD to be a viable competitor.

The surprising result is that even for z just slightly below 0, there is an f which leads to an equilibrium where AMD is active. The reason is that at $z = 0$ and $f = 1$, we have an equilibrium where AMD and Intel price at marginal cost. For small negative values of z, the AMD active equilibrium exists only when f is close to 1.

The figure below shows the combinations of f and z such that AMD inactive is an equilibrium and the combinations such that AMD active is an equilibrium. The small red area corresponds to pairs (f, z) where there are equilibria with both AMD active and AMD inactive. All equilibria on the yellow area have AMD active (getting positive demand and profit). On that area, Microsoft's marginal cost is so negative that it wants to grow the installed base as much as possible: p_m is set so low that $q_m = 1$, hitting the boundary. The white area are pairs (f, z) where no equilibrium (in pure strategies) exists.



5 Comparing Equilibria

We now study how Microsoft, Intel and AMD's profits are affected by changing z and f. Aside from competing in setting prices, Microsoft, Intel and AMD can influence the values of z and f. Microsoft, for example, can affect the value of z by developing new applications and managing the pace at which the installed base becomes obsolete. Microsoft can 'help' AMD raise f by contributing to AMD's R&D efforts or by forcing Intel to license new technologies to AMD. Likewise, Intel and AMD can affect f by investing in R&D, and z by developing new manufacturing processes.

It is a corollary to our previous results that the equilibria are entirely characterized by z and f.

Corollary 3 *When AMD is inactive, Microsoft's and Intel's profits are only a function of z. When AMD is active, the profits of all three firms are a function of z and f.*

Thus changing z and f are sufficient statistics for making comparisons across equilibria.

5.1 Choosing z (given f)

Microsoft might be better off with z close to zero so that AMD is not active even if this means earning less from the installed base.

Proposition 7 *Given f, Microsoft may prefer the equilibrium with a higher z so that AMD is inactive in equilibrium.*

The proposition is based on a comparison across equilibria. At a low value of z, the equilibrium will include AMD, while at a higher value of z, the equilibrium will only involve Microsoft and Intel. Microsoft may prefer to earn less future profits and interact with Intel as a monopolist, rather than with competition.

Intuitively, when f is small, AMD's products are perceived as far inferior. Since the willingness to pay for AMD is very low in this case, the only possibility for AMD to be active is if Microsoft sets very low prices so that the bundle is cheap and there are individuals willing to buy it. When f is low, however, Microsoft may prefer *larger* z so that AMD is not active. In this case, p_m is higher and Microsoft makes more of its profits today. The proposition shows, counterintuitively, that Microsoft may find it preferable to put itself at a disadvantage (by making z closer to zero and earning less from the installed base) to make sure that AMD is out. In other words, the absence of complement competition may result in higher profits to Microsoft.

What if MS just charged the same price as it did when there is only Intel? Why doesn't that lead to higher profits? The reason is that Intel anticipates that MS will be charging a low price and this allows Intel to charge a high price and capture more of the pie.

We now show that Intel may be better off with competition than without it.

Proposition 8 *Given f, Intel always prefers z such that AMD is active in equilibrium.*

There are two reasons why Intel prefers a world with competition from AMD to a situation where it is a monopolist supplier of microprocessors. First, given f, to have an equilibrium where AMD is active, z must be lower than what is required for an equilibrium without AMD. Intel captures part of the additional profit generated with the lower z. Second, for AMD to be active Microsoft must set lower prices and this benefits Intel.

5.2 Choosing f (given z)

Suppose now that z is fixed and that Microsoft can choose the degree of vertical differentiation between Intel and AMD by, for example, doing R&D on behalf of AMD. What will it want to choose?

Proposition 9 *Given z, Microsoft always prefers larger f.*

The closer the quality of Intel and AMD microprocessors, the stronger the intensity of price competition between them. With intense price competition in microprocessors, Microsoft is able to raise price and earn more.

Perhaps the clearest example of Microsoft affecting f is the MMX episode in the mid-1990s. MMX was a set of extensions in Intel's core processors that allowed the CPU to better handle multimedia, especially audio and video. PCs had not been designed to run graphic-intensive games or to play music or video clips. By adding 57 new instructions to the microprocessor, Intel wanted to increase the speed and quality of multimedia applications. To make MMX a success, Intel spent tens of millions of dollars in R&D resources and testing to develop the CPU extensions. In addition, it planned to spend another $250 million to make it successful in the marketplace.[12] Ultimately, however, these resources would be wasted if Microsoft did not support Intel. If the OS was not optimized to take advantage of MMX (Microsoft had to add one switch), then most games or other applications would see few performance enhancements. Clearly, MMX would result in lower f unless AMD also had access to that technology. After protracted discussions, Microsoft demanded, and Intel acceded, to license AMD *for free* in exchange for Microsoft adding support for MMX.

The following proposition shows that Intel has the exact opposite preferences.

Proposition 10 *Given z, Intel always prefers the lowest possible value of f.*

When z is so low that the only possibility is an equilibrium with AMD active, Intel's pricing power is larger when f is low. In addition, the more the vertical differentiation, the lower the price that Microsoft must set for AMD to be active and this also benefits Intel. Moreover, when Intel and AMD's products are highly differentiated, Intel sells larger volume.

When z is sufficiently close to zero so that for some values of f there are two-player equilibria and for some other values of f there are three-player equilibria, Intel still prefers small f and AMD not active. The reason is that the fs compatible with three-player equilibria are relatively high ($f \geq 0.568$). And when f is large, price competition between AMD and Intel is intense, making it difficult for Intel to capture value.

Consistent with the proposition, Intel has relentlessly pursued Moore's Law throughout its history, doubling the number of transistors on its CPU every 18 months. By increasing wafer sizes, shrinking transistor sizes, and decreasing the time and cost of production, Intel has managed to stay ahead of the competition. A significant percentage of Intel's capital and R&D spending has been pushing Intel 12-18 months ahead of AMD in process technology.

[12] Roughly $100 million would be dedicated to underwrite the development of new software, which could take advantage of the new instruction set; and another $150 million to market MMX as a brand new microprocessor that would drive consumers and business to buy new computers.

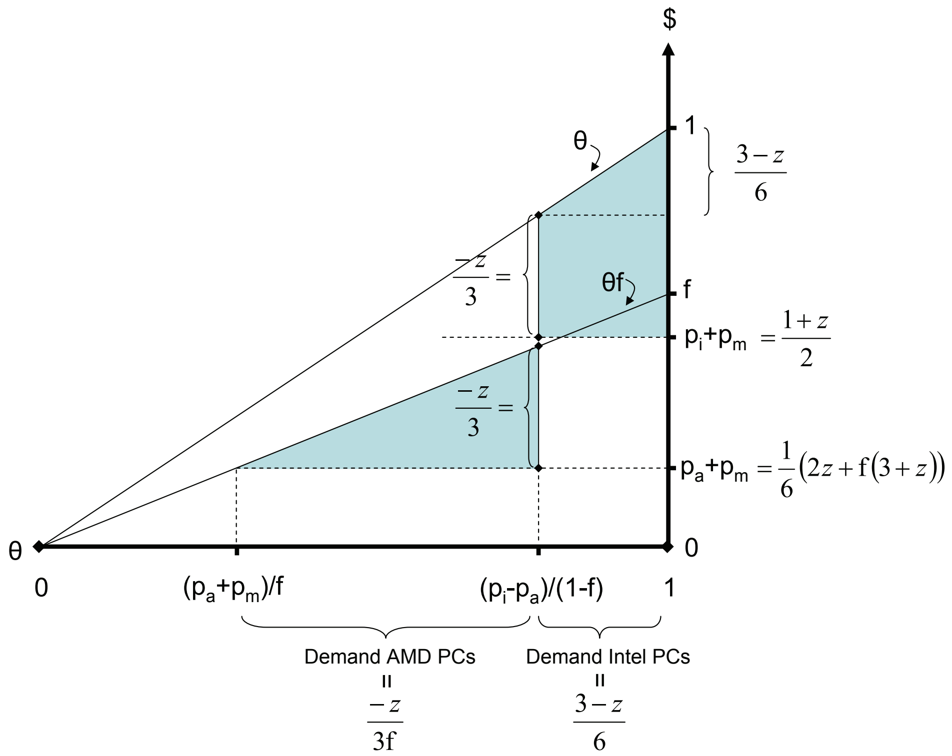
6 Welfare

In the final section, we compare total welfare in the two- and three-player worlds. In the equilibrium where AMD is ignored, total welfare $TS^{\text{AMD not active}}$ is given by:

$$TS^{\text{AMD not active}} = \pi_m^{\text{AMD not active}} + \pi_i^{\text{AMD not active}} + \frac{1}{2}\left(\frac{1-z}{3}\right)^2 = \frac{5\left(1-z\right)^2}{18}.$$

This is a decreasing function of z.

When AMD is active, the computation is a little bit more involved. The shaded area in the figure below is consumer surplus in this case:



Therefore,

$$\begin{aligned} TS^{\text{AMD active}} &= \pi_m^{\text{AMD active}} + \pi_i^{\text{AMD active}} + \pi_a^{\text{AMD active}} \\ &\quad \underbrace{+\frac{1}{2}\left(\frac{3-z}{6}\right)^2 + \left(\frac{3-z}{6}\right)\left(\frac{-z}{6}\right)}_{\text{Surplus of consumers buying Intel pcs}} + \underbrace{\frac{1}{2}\left(\frac{-z}{3f}\right)\left(\frac{-z}{3}\right)}_{\text{Surplus of consumers buying AMD pcs}} \\ &= \frac{1}{72}\left(27 - 48z + \frac{5\left(4+f\right)z^2}{f}\right). \end{aligned}$$

The first point to notice is that $TS^{\text{AMD active}}$ decreases with f (as $\frac{dTS^{\text{AMD active}}}{df} = \frac{-5z^2}{18f^2} < 0$). There are two effects at play. First, as Intel and AMD become closer substitutes, their pricing power decreases and this is good for total surplus. On the other hand, the more substitutable Intel and AMD are, the larger is Microsoft's pricing power. The exercise of such power is detrimental to total welfare. Notice that in the extreme case of $f = 1$, we have $p_i = p_a = c$ and Microsoft acts as a monopolist. This suggests that efforts by Microsoft to 'help' AMD become more competitive such as, for example, financing AMD's R&D investments, wind up hurting total welfare.

The second result is that, from a welfare point of view, a three-player world is always better than a world with two players. To see this, notice that given z, the worst case scenario for $TS^{\text{AMD active}}$ is $f = 1$. Assume that $f = 1$ and compute $TS^{\text{AMD active}} - TS^{\text{AMD not active}}$. It is immediate that this difference is positive for all z.

7 Concluding Remarks

We conclude with three observations:

- *Managers.* Competition between monopolist suppliers of complementary products results in equal profit sharing regardless of marginal cost differences. Attempts to increase value capture by reducing own costs or by investing in a better product are only 50% effective as profits are split 50:50 regardless of who bears the burden. We have shown that one way for A to increase value capture is by encouraging competition in B. The analysis has revealed that 'a little bit' of competition is not enough. In fact, mild competition between suppliers of B may be detrimental to A. The tactic works best when competition within complements is intense.

- *Welfare.* Competing complements raise total surplus compared to a situation with monopoly complements (abstracting from fixed/sunk cost considerations). However, from a public policy viewpoint, mild competition within complements is preferable to intense competition. When competition is intense in one side of the complements game, the other side becomes more powerful, hurting total welfare generation. Specifically, actions by A to help B_L become closer in quality to B_H appear to lower welfare.

- *Literature.* The paper constitutes a first step towards a general theory of competition between and within complements. We have shown that even the simplest departure from the standard model of monopolist complementors leads to surprisingly complicated interactions and nonexistence of equilibria. There are many possible ways in which this work can be extended but perhaps the most obvious directions are consideration of horizontal differentiation between suppliers of B and having competition on both sides of the complements game.

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Appendix

A Proofs

Corollary 1. *To the extent that a monopoly complementor can act as if it has higher variable costs, this will lead to a first-order increaser in its profits.*

Proof of Corollary 1. When the firm acts as if its fixed costs are variable, it doesn't actually pay the higher variable costs. It only acts as if it has higher variable costs for accounting and for pricing.

Firm i chooses p_i to maximize $\pi_i = (p_i - c_i - f)\, D\,(p_i + p_m)$, though its actual profits are $\pi_i^* = (p_i - c_i)\, D\,(p_i + p_m)$. As before, firm m maximizes $\pi_m = (p_m - c_m)\, D\,(p_i + p_m)$. We have

$$\left.\frac{d\pi_i^*}{df}\right|_{f=0} = \frac{\partial \pi_i^*}{\partial p_i}\frac{dp_i}{df} + \frac{\partial \pi_i^*}{\partial p_m}\frac{dp_m}{df} + \frac{\partial \pi_i^*}{\partial f}.$$

The first term is zero as p_i was optimally chosen at $f = 0$. As the two goods are complements, $\partial \pi_i^*/\partial p_m < 0$: Intel always prefers that Microsoft charge a lower price. Finally, $\partial \pi_i^*/\partial f = 0$ as f does not enter into π_i^*: Intel doesn't really pay higher variable costs, it just acts as if it does. Thus the effect on profits comes down to the sign of dp_m/df.

Now, by an argument similar to that in the proof of Proposition 1, $p_i - c_i - f = p_m - c_m$. Thus $dp_i/df - 1 = dp_m/df$. So dp_m/df is negative provided $dp_i/df < 1$. By differentiating firm i's first-order condition wrt f, we find that

$$[(p_i - c_i - f)\, D'' + 2D']\,\frac{dp_i}{df} - D' = 0. \tag{6}$$

The first-order condition implies

$$p_i - c_i - f = -\frac{D}{D'}.$$

Substituting in (6), we obtain

$$\left[-\frac{D}{D'}D'' + 2D'\right]\frac{dp_i}{df} - D' = 0$$

or

$$\left[-DD'' + 2D'^2\right] \frac{dp_i}{df} = D'^2.$$

Thus, $dp_i/df < 1$ provided

$$-DD'' + 2D'^2 > D'^2$$

or

$$-DD'' + D'^2 > 0$$

which is equivalent to log-concavity of D. Thus, we have shown that $\left.\frac{d\pi_i^*}{df}\right|_{f=0} > 0$ or that a small increase in f leads to higher profits. ■

Proof of Proposition 4. If AMD is active, then Microsoft's best response is

$$p_m = \frac{f + c_m - p_a}{2},$$

Intel's best response is

$$p_i = \frac{1 - f + c + p_a}{2},$$

and AMD's best response is

$$p_a = \frac{c + f p_i - (1-f)\, p_m}{2}.$$

Solving the system of equations for the Nash equilibrium we obtain

$$\begin{aligned} p_m &= \frac{3f - c\,(2+f) + (4-\mathrm{f})\, c_m}{6}, \\ p_i &= \frac{(3-c_m)\,(1-f) + c\,(5+f)}{6}, \end{aligned}$$

and

$$p_a = \frac{c\,(2+f) - (1-f)\, c_m}{3}. \tag{7}$$

Subtracting c from both sides of (7) and rearranging we obtain

$$p_a - c = \frac{-(1-f)\,(c + c_m)}{3}$$

Therefore, for AMD to have positive profit margin we need $0 > c_m + c \equiv z$. Therefore, with $z > 0$ AMD loses money in the candidate Nash equilibrium. Moreover, when AMD is a player, it gets demand

$$\frac{p_i - p_a}{1-f} - \frac{p_a + p_m}{f}. \tag{8}$$

Substituting the candidate equilibrium prices and simplifying, (8) becomes

$$\frac{-z}{3f}.$$

Therefore, when $z > 0$ AMD gets negative demand at the candidate equilibrium. ■

Proof of Proposition 5. We begin with a situation where Microsoft and Intel price against each other ignoring AMD. For $z \geq -2$, the candidate equilibrium prices in a two-player game are

$$p_m = \frac{1 + 2c_m - c}{3},$$

and

$$p_i = \frac{1 + 2c - c_m}{3}.$$

Profit margins are both equal to $\frac{1-z}{3}$. The quantity sold by both Microsoft and Intel is $\frac{1-z}{3} \leq 1$ and profits are each $\frac{(1-z)^2}{9}$.

For $z < -2$, there are multiple equilibria. In these equilibria, $p_i + p_m = 0$, so that all customers are buying the product. For such a pair of prices to be an equilibrium, it must be the case that neither player wants to deviate. Deviations downwards are never desirable as volume is at its maximum. For Microsoft and Intel not to deviate upward, it must be that raising price would lower profits, $d\pi_m/dp_m \leq 0$ and $d\pi_i/dp_i \leq 0$ at $p_i = -p_m$. These conditions deliver the following inequalities:

$$1 + c - p_m - 2p_i \leq 0 \qquad => \qquad 1 + c \leq p_i;$$

and

$$1 + c_m - p_i - 2p_m \leq 0 \qquad => \qquad 1 + c_m \leq -p_i.$$

Thus the range of candidate equilibria is

$$1 + c \leq p_i \leq -1 - c_m \qquad \text{and} \qquad p_m = -p_i.$$

For the two-player outcome to be an equilibrium, we need to ensure that AMD does not find it profitable to enter. In the case with $z < -2$, note that $p_i \geq 1 + c$ so that at $p_a = c$, all customers prefer AMD to Intel. The extra dollar price is not worth the extra value, even for a customer with $\theta = 1$ and $f = 0$. Since AMD can capture positive demand when pricing at cost, we can conclude that there are no equilibria with AMD inactive for $z < -2$.

For $z \geq -2$, Intel and Microsoft's prices are given by (4), (5). AMD's demand when pricing at cost is

$$q_a = \frac{1 + 2z - f(2 + z)}{3(-1 + f)f}.$$

For this to be negative, we require

$$-\tfrac{1}{2} < z < 1 \qquad \text{and} \qquad 0 < f < \tfrac{1+2z}{2+z}$$

or, equivalently,

$$0 < f < 1 \qquad \text{and} \qquad \tfrac{2f-1}{2-f} < z. \tag{9}$$

This says that given f, z cannot be too negative and that given z, f cannot be too large for AMD' inactive to be an equilibrium. We later show that this condition is satisfied for the proposed set of $z \geq z^*(f)$.

We now consider whether Intel or Microsoft will have an incentive to deviate. As in the model with zero costs, the presence of AMD only reduces Intel's incentive to deviate. When Intel considers lower prices, this will not make AMD active. Therefore demand (and profit functions) remain unchanged and so there is no gain from lowering price. Raising price may lead to AMD becoming active, but this would only be worse for Intel than if AMD is not active. Without consideration of AMD, Intel did not want to raise price. Thus raising price with AMD present can only be worse.

For Microsoft, raising price will not make AMD active. Because raising price does not change its demand or profit function, Microsoft will not want to deviate to a higher price. Lowering price sufficiently, however, may allow AMD to become active and this changes Microsoft's profits. Thus we consider potential deviations downwards by Microsoft.

By assumption, when AMD is inactive, $p_a = c$. Assuming that the deviation leads to AMD becoming active, Microsoft's optimal price is

$$p_m = \max\left[-c, \frac{1}{2}(f + c_m - c)\right].$$

We consider each of these cases in turn. Microsoft will price at $-c$ when z is sufficiently negative that Microsoft will want to capture the entire market with AMD. This case arises when $z \leq -f$. Here demand is 1 and profits are $-z$. This deviation will not be attractive provided

$$-z \leq \frac{(1-z)^2}{9}.$$

This implies

$$z \geq \frac{1}{2}\left(-7 + 3\sqrt{5}\right) \approx -0.146.$$

When $z \leq -f$, this a necessary condition for AMD inactive to be an equilibrium. It is also sufficient provided that AMD is truly inactive at the proposed solution. This follows as $f \leq -z < \frac{1+2z}{2+z}$ for $z \geq \frac{-7+3\sqrt{5}}{2}$ so that (9) is satisfied.

We turn now to the case where $z > -f$ and Microsoft's proposed deviation is to $p_m = \frac{f+c_m-c}{2}$. Microsoft's profit margin is $\frac{f-z}{2}$, $q_m = \frac{f-z}{2f}$, which leads to profits of

$$\pi_m = \frac{(f-z)^2}{4f}.$$

We need to confirm that at the proposed deviation AMD gets positive demand. When Microsoft lowers its price, the demand for AMD is

$$q_a = \frac{3z - f(-1 + 3f + z)}{6(-1 + f)f}.$$

Thus $q_a > 0$ implies

$$z < \frac{-f\,(1 - 3f)}{3 - f}. \tag{10}$$

We assume this is satisfied and show below that this is indeed the case.

For Microsoft not to deviate to this low price requires that it lead to lower profits:

$$\frac{(f - z)^2}{4f} \leq \frac{(1 - z)^2}{9}. \tag{11}$$

With a little bit of algebra, this condition reduces to

$$\frac{5f - 6\,(1 - f)\,\sqrt{f}}{9 - 4f} \leq z \leq \frac{5f + 6\,(1 - f)\,\sqrt{f}}{9 - 4f}$$

To show that this is a necessary condition, note that for $z < \frac{5f - 6(1-f)\sqrt{f}}{9-4f}$ it also follows (with some algebra) that $z < \frac{-f(1-3f)}{3-f}$ so that AMD obtains positive demand and Microsoft higher profits at the proposed deviation.

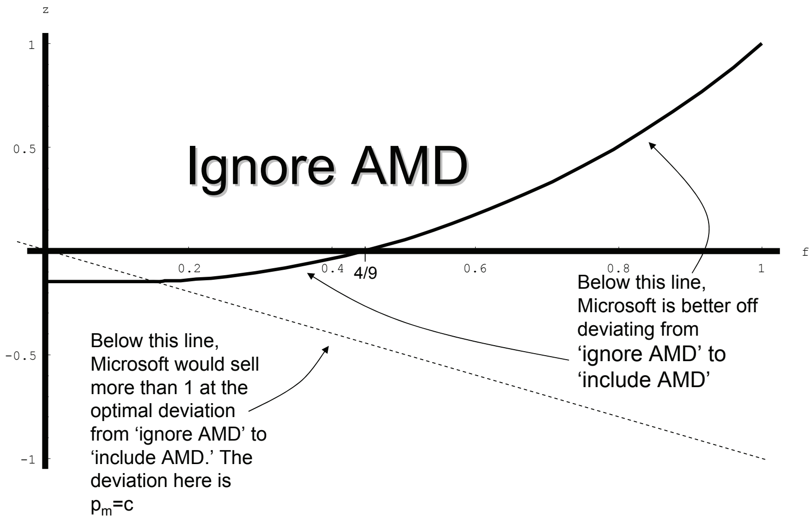
To show that this is a sufficient condition, we know that there is no profitable deviation for Microsoft. Thus we only need confirm that AMD does not obtain any demand in the proposed equilibrium. Again, with some algebra, it follows that $z \geq \frac{5f-6(1-f)\sqrt{f}}{9-4f}$ implies $z > \frac{2f-1}{2-f}$.

It only remains to link together the two cases, $z \leq -f$ and $z > -f$. Obviously, $z = \frac{1}{2}\left(-7 + 3\sqrt{5}\right)$ and $z = -f$ intersect at $f = \frac{1}{2}\left(7 - 3\sqrt{5}\right)$. This also happens to be the f where $z = \frac{5f-6(1-f)\sqrt{f}}{9-4f}$ has its minimum, which is $\frac{1}{2}\left(-7 + 3\sqrt{5}\right)$. Thus the two conditions fit together to become:

$$z \geq z^*(f) = \begin{cases} \frac{1}{2}\left(-7 + 3\sqrt{5}\right) & \text{if } \ 0 \leq f \leq \frac{1}{2}\left(7 - 3\sqrt{5}\right) \\ \frac{5f-6(1-f)\sqrt{f}}{9-4f} & \text{if } \ \frac{1}{2}\left(7 - 3\sqrt{5}\right) < f \leq 1 \end{cases}$$

■

The region where Microsoft does not want to deviate is shown in the following figure:



Proof of Proposition 6. When AMD is active profit functions are:

$$\pi_m = (p_m - c_m)\left(1 - \frac{p_a + p_m}{f}\right),$$

$$\pi_i = (p_i - c)\left(1 - \frac{p_i - p_a}{1-f}\right),$$

and

$$\pi_a = (p_a - c)\left(\frac{p_i - p_a}{1-f} - \frac{p_a + p_m}{f}\right).$$

Solving the system of FOCs we obtain a unique candidate to Nash equilibrium:

$$p_m = \frac{3f - c\,(2+f) + (4-f)\,c_m}{6}, \tag{12}$$

$$p_i = \frac{(3-c_m)\,(1-f) + c\,(5+f)}{6}, \tag{13}$$

and

$$p_a = \frac{c\,(2+f) - (1-f)\,c_m}{3}. \tag{14}$$

We know from Proposition 5 that for there to be a pure-strategy equilibrium with AMD active, it must be that $z < 0$. When $z < 0$, quantities at the candidate equilibrium prices are all positive:

$$q_m = \frac{1}{6}\left(3 - \frac{(2+f)\,z}{f}\right),$$

$$q_i = \frac{3-z}{6},$$

and

$$q_a = \frac{-z}{3f}.$$

In computing the candidate to equilibrium we have assumed that $q_m < 1$. When z is very negative, however, $q_m = \frac{1}{6}\left(3 - \frac{(2+f)z}{f}\right) > 1$ and thus the above solution does not apply. The condition on z that guarantees that $q_m < 1$ is:

$$z > -\frac{3f}{2+f}. \tag{15}$$

When (15) is satisfied, (12), (13), and (14) is the unique candidate to Nash equilibrium. Below we derive additional conditions on z and f that guarantee that that candidate is indeed an equilibrium.

In the case where $z < -\frac{3f}{2+f}$ there are multiple candidate equilibria. In all of these solutions the large negative costs lead to $p_m + p_a = 0$ and so the entire market will be served. Because AMD will never set p_a to be less than zero, the candidates must satisfy $p_m = -p_a \leq 0$. Moreover, for p_m to be part of an equilibrium, we need the derivative of π_m with respect to p_m evaluated at $p_m = -p_a$ to be less than zero (weakly) so that Microsoft has no incentive to raise prices. Note that Microsoft will never want to deviate down because volume cannot be increased. Furthermore, AMD is already a player. Therefore,

$$\left.\frac{d\pi_m}{dp_m}\right|_{p_m=-p_a} \leq 0 \Rightarrow \left.\frac{f + c_m - p_a - 2p_m}{f}\right|_{p_m=-p_a} \leq 0 \Rightarrow p_a \leq -f - c_m$$

For p_a to be part of an equilibrium we need to ensure that AMD does not want to raise price from $p_a = -p_m$. Thus,

$$\left.\frac{d\pi_a}{dp_a}\right|_{p_m=-p_a} \leq 0 \Rightarrow -\left.\frac{fp_i - (1-f)\,p_m + c - 2p_a}{f\,(1-f)}\right|_{p_m=-p_a} \leq 0 \Rightarrow p_a \geq \frac{fp_i + c}{1+f}. \tag{16}$$

We also must make sure that AMD does not want to deviate down. While lowering price will not expand total demand, it will result in larger market share for AMD, as the lower prices will persuade some Intel customers to switch to AMD. The first other condition that we must consider for price decreases is derived as follows:

When the market is covered, $q_a = \frac{p_i - p_a}{1-f}$ and $\pi_a = (p_a - c)\,\frac{p_i - p_a}{1-f}$. Therefore, AMD will not want to lower price if the following is satisfied:

$$-\left.\frac{d\pi_a}{dp_a}\right|_{p_m=-p_a} \leq 0 \Rightarrow -\left.\frac{p_i + c - 2p_a}{1-f}\right|_{p_m=-p_a} \leq 0 \Rightarrow p_a \leq \frac{p_i + c}{2}. \tag{17}$$

Intel is not at the boundary and thus its first-order condition will be satisfied with equality: $p_i = \frac{1-f+p_a+c}{2}$. Substituting this into (16) leads to

$$p_a - c \geq \frac{f\,(1-f)}{2+f}.$$

And substituting into (17) leads to

$$p_a - c \le \frac{1-f}{3}.$$

Let $d \equiv p_a - c$ and recall that $z \equiv c_m + c$. Our range of potential equilibria with AMD active is

$$\frac{f\,(1-f)}{2+f} \le d \le -f - z \tag{18}$$

and

$$d \le \frac{1-f}{3}. \tag{19}$$

Finally, we must worry about 'non marginal' deviations: Microsoft significantly raising p_m and moving to a solution where AMD is inactive. Absent a deviation, Microsoft's profits are

$$\pi_m = (p_m - c_m) * 1 = -p_a - c_m. \tag{20}$$

If Microsoft were to deviate, it would be to $p_m = \frac{1+c_m-p_i}{2}$. In that event,

$$p_m - c_m = \frac{1 - 2c_m - c + f - p_a}{4}.$$

Adding and subtracting c from the numerator, gross margin can be expressed as

$$p_m - c_m = \frac{1 - 2c_m - 2c + f - (p_a - c)}{4} = \frac{1 - 2z + f - d}{4},$$

and Microsoft's profits are

$$(p_m - c_m)^2 = \frac{(1 - 2z + f - d)^2}{16}. \tag{21}$$

For AMD active to be an equilibrium, we need to demonstrate that (20) is larger than (21) over the range given by (18) and (19). That is, we need

$$-d - z \ge \frac{(1 - 2z + f - d)^2}{16} \tag{22}$$

over

$$\frac{f\,(1-f)}{2+f} \le d \le -f - z \qquad \text{and} \qquad d \le \frac{1-f}{3}.$$

It is easy to see that (22) simplifies to

$$0 \le d \le -7 - 2z + f + 4\sqrt{3 + z - f}. \tag{23}$$

There is one more issue that we must take into account in characterizing the combinations f and z that are consistent with equilibria. This is that the only non-marginal deviations allowed are increases in price. A non-marginal deviation by Microsoft that involves a reduction in price will not result in volume increases as q is already at its maximum. Therefore, a reduction in price can only lead to lower profits.

When Microsoft deviates to ignore AMD it sets price $\hat{p}_m = \frac{1+c_m-p_i}{2}$. Intel is at its reaction function (as if AMD was active) $p_i = \frac{1-f+p_a+c}{2}$. Substituting in, we obtain

$$\hat{p}_m = \frac{1+f-2c+2c_m-d}{4}.$$

Now, because the only non-marginal deviations allowed are increases in price, whenever z, f, and d satisfying (15), (18), and (19) are such that $\hat{p}_m \leq p_m$, we must be at an equilibrium (even if (23) is violated)[13]. Therefore, the condition is

$$\begin{aligned} \frac{1+f-2c+2c_m-d}{4} &\leq -p_a \\ \frac{1+f-2c+2c_m-d}{4}+c &\leq -p_a+c \end{aligned}$$

or

$$-\frac{1+f+2z}{3} \geq d. \tag{24}$$

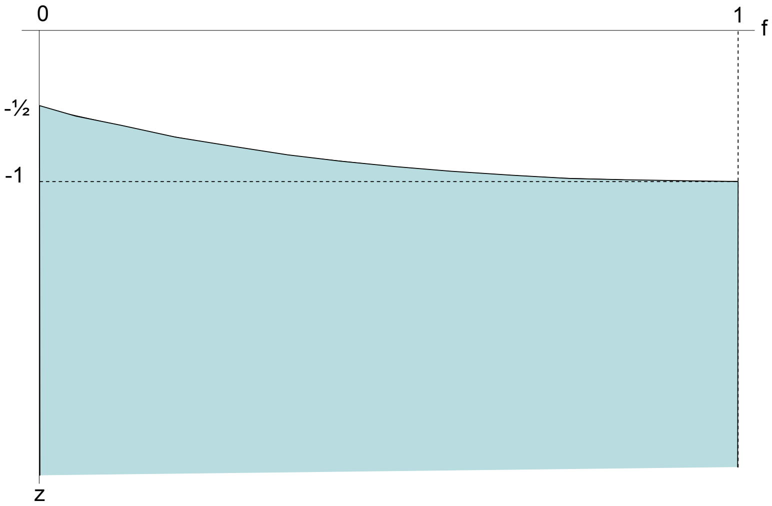
With all this preamble, we now present the combinations z and f such that equilibria exist. We proceed in two steps. First we show the pairs $(z,\ f)$ such that (18), (19), and (24) are satisfied. The only condition that we are not imposing in the first step is the profit condition (23). All pairs satisfying (18), (19), and (24) are part of an equilibrium even if the profit condition is violated because larger profits (violation of (23)) must come from a disallowed price move, a move *down.* The second step is to impose the profit condition. Pairs that violate (24) will be part of equilibria if (23) is satisfied.

Step 1: A little algebra reveals that z, f, and d satisfying (18), (19), and (24) are the following.

- $z \leq -1$, $0 \leq f < 1$, and $\frac{f(1-f)}{2+f} \leq d \leq \frac{1-f}{3}$.
- $-1 < z \leq \frac{-1}{2}$, $0 \leq f < \frac{3+z}{2} - \frac{1}{2}\sqrt{13+14z+z^2}$, and $\frac{f(1-f)}{2+f} \leq d \leq \frac{-1-f-2z}{3}$.
- $-1 < z \leq \frac{-1}{2}$, $f = \frac{3+z}{2} - \frac{1}{2}\sqrt{13+14z+z^2}$, and $d = \frac{-1-f-2z}{3}$.

The following figure shows the area (on $(f,\ z)$ space).

[13] The equilibrium has $p_m = -d-c$, $p_a = -p_m$, and $p_i = \frac{1-f+p_a+c}{2}$.



Step 2: Using Mathematica, we obtain z, f, and d satisfying (18), (19), and (23). Let $w_1(z)$ be the second root of the following polynomial:

$$1 + 12z + 4z^2 + \left(10 + 10z + 4z^2\right) x + \left(-1 - 2z + z^2\right) x^2 + -2\left(1 + z\right) x^3 + x^4.$$

Let $w_2 \approx -.899$ be the second root of the following polynomial:

$$857 + 1260x + 366x^2 + 28x^3 + x^4.$$

Let $w_3 \approx -.506$ be the third root of the following polynomial:

$$13 + 31x + 11x^2 + x^3.$$

Let $w_4 \approx -.184$ be the first root of the following polynomial:

$$9 + 51x + 11x^2 + x^3.$$

The answer is:

- When $\frac{1}{2}\left(-3 - 2\sqrt{2}\right) < z \leq \frac{1}{3}\left(-5 - 2\sqrt{3}\right)$, we have
  - $f = 0$ and $0 < d < -7 - 2z + 4\sqrt{3 + z}$.
  - $0 < f < w_1(z)$ and $\frac{f - f^2}{2 + f} \leq d < -7 + f - 2z + 4\sqrt{3 - f + z}$.
- When $\frac{1}{3}\left(-5 - 2\sqrt{3}\right) < z < -1$, we have

- $f = 0$ and $0 < d \le \frac{1}{3}$.
- $0 < f < \frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z)$ and $\frac{f-f^2}{2+f} \le d \le \frac{1-f}{3}$.
- $f = \frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z)$ and $\frac{f-f^2}{2+f} \le d < \frac{1-f}{3}$.
- $\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z) < f < w_1(z)$ and $\frac{f-f^2}{2+f} \le d < -7+f-2z+4\sqrt{3-f+z}$.

- When $z = -1$, we have
  - $f = 0$ and $0 < d \le \frac{1}{3}$.
  - $0 < f < 1$ and $\frac{f-f^2}{2+f} \le d \le \frac{1-f}{3}$.

- When $-1 < z < w_2$, we have
  - $f = 0$ and $0 < d \le \frac{1}{3}$.
  - $0 < f \le \frac{1}{2}(-1-3z)$ and $\frac{f-f^2}{2+f} \le d \le \frac{1-f}{3}$.
  - $\frac{1}{2}(-1-3z) < f < \frac{-2z}{3+z}$ and $\frac{f-f^2}{2+f} \le d \le -f-z$.

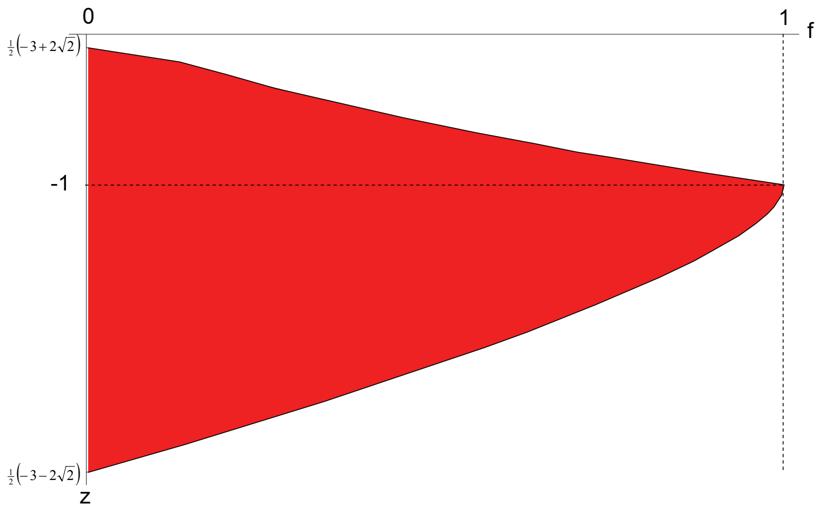
- When $w_2 < z < \frac{1}{3}\left(-5+2\sqrt{3}\right)$, we have
  - $f = 0$ and $0 < d \le \frac{1}{3}$.
  - $0 < f < \frac{1}{2}(-1-3z)$ and $\frac{f-f^2}{2+f} \le d \le \frac{1-f}{3}$.
  - $\frac{1}{2}(-1-3z) \le f < \frac{-2z}{3+z}$ and $\frac{f-f^2}{2+f} \le d \le -f-z$.

- When $z = \frac{1}{3}\left(-5+2\sqrt{3}\right)$, we have
  - $f = 0$ and $0 < d < \frac{1}{3}$.
  - $0 < f \le \frac{1}{2}\left(4-2\sqrt{3}\right)$ and $\frac{f-f^2}{2+f} \le d \le \frac{1-f}{3}$.
  - $\frac{1}{2}\left(4-2\sqrt{3}\right) < f < -\frac{2\left(\left(-5+2\sqrt{3}\right)\right)}{3\left(3+\frac{1}{3}\left(-5+2\sqrt{3}\right)\right)}$ and $\frac{f-f^2}{2+f} \le d \le \frac{1}{3}\left(5-2\sqrt{3}\right) - f$.

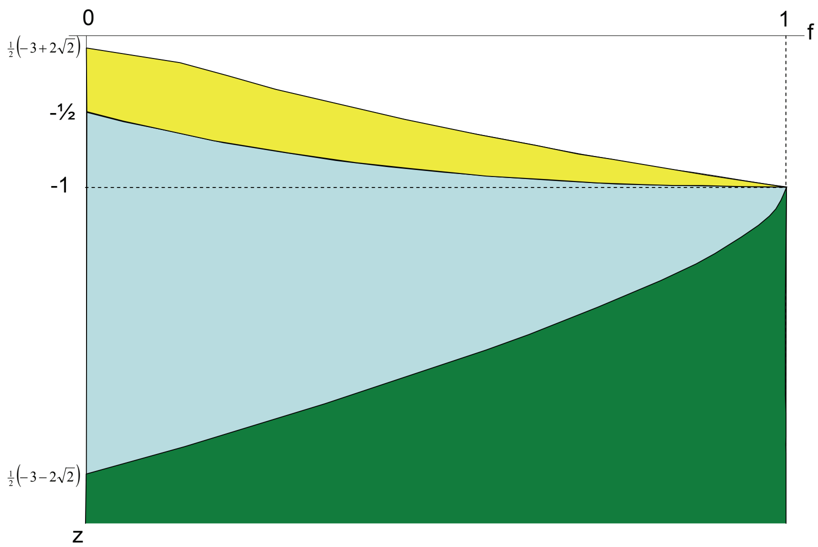
- When $\frac{1}{3}\left(-5+2\sqrt{3}\right) < z < w_3$, we have
  - $f = 0$ and $0 < d < -7-2z+4\sqrt{3+z}$.
  - $0 < f < -\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z)$ and $\frac{f-f^2}{2+f} \le d < -7+f-2z+4\sqrt{3-f+z}$.
  - $f = -\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z)$ and $\frac{f-f^2}{2+f} \le d < \frac{1-f}{3}$.
  - $-\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z) < f < \frac{1}{2}(-1-3z)$ and $\frac{f-f^2}{2+f} \le d \le \frac{1-f}{3}$.
  - $\frac{1}{2}(-1-3z) \le f < \frac{-2z}{3+z}$ and $\frac{f-f^2}{2+f} \le d \le -f-z$.

- When $f = w_3$, we have
  - $f = 0$ and $0 < d < -7 - 2z + 4\sqrt{3+z}$.
  - $0 < f < -\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z)$ and $\frac{f-f^2}{2+f} \leq d < -7 + f - 2z + 4\sqrt{3-f+z}$.
  - $f = -\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z)$ and $\frac{f-f^2}{2+f} \leq d < \frac{1-f}{3}$.
  - $-\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z) < f \leq \frac{1}{2}(-1-3z)$ and $\frac{f-f^2}{2+f} \leq d \leq \frac{1-f}{3}$.
  - $\frac{1}{2}(-1-3z) < f < \frac{-2z}{3+z}$ and $\frac{f-f^2}{2+f} \leq d \leq -f - z$.
- When $w_3 < z < \frac{-1}{2}$, we have
  - $f = 0$ and $0 < d < -7 - 2z + 4\sqrt{3+z}$.
  - $0 < f < -\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z)$ and $\frac{f-f^2}{2+f} \leq d < -7 + f - 2z + 4\sqrt{3-f+z}$.
  - $f = -\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z)$ and $\frac{f-f^2}{2+f} \leq d < \frac{1-f}{3}$.
  - $-\frac{3}{2}\sqrt{-1-2z} + \frac{1}{2}(2+3z) < f < \frac{1}{2}(-1-3z)$ and $\frac{f-f^2}{2+f} \leq d \leq \frac{1-f}{3}$.
  - $\frac{1}{2}(-1-3z) \leq f < \frac{-2z}{3+z}$ and $\frac{f-f^2}{2+f} \leq d \leq -f - z$.
- When $\frac{-1}{2} < z < w_4$, we have
  - $f = 0$ and $0 < d < -7 - 2z + 4\sqrt{3+z}$.
  - $0 < f < -\sqrt{2}\sqrt{1+z} + \frac{3+z}{2}$ and $\frac{f-f^2}{2+f} \leq d < -7 + f - 2z + 4\sqrt{3-f+z}$.
  - $f = -\sqrt{2}\sqrt{1+z} + \frac{3+z}{2}$ and $\frac{f-f^2}{2+f} \leq d \leq -f - z$.
  - $-\sqrt{2}\sqrt{1+z} + \frac{3+z}{2} < f < \frac{-2z}{3+z}$ and $\frac{f-f^2}{2+f} \leq d \leq -f - z$.
- When $z = w_4$, we have
  - $f = 0$ and $0 < d < -7 - 2z + 4\sqrt{3+z}$.
  - $0 < f < \frac{-2z}{3+z}$ and $\frac{f-f^2}{2+f} \leq d < -7 + f - 2z + 4\sqrt{3-f+z}$.
- When $w_4 < z < \frac{1}{2}\left(-3 + 2\sqrt{2}\right)$, we have
  - $f = 0$ and $0 < d < -7 - 2z + 4\sqrt{3+z}$.
  - $0 < f < w_1(z)$ and $\frac{f-f^2}{2+f} \leq d < -7 + f - 2z + 4\sqrt{3-f+z}$.

The following figure shows the area (on (f, z) space).



Finally, we put both areas together to find all the pairs (z, f) where an equilibrium exists.



The non-intersecting pairs are those on the yellow and green areas. On the yellow area, deviations by Microsoft to ignore AMD lead involve larger prices there. However, the profit condition (23) is satisfied. Therefore, Microsoft is better off not deviating and keeping AMD in. On the green area the profit condition does not hold but deviations are to lower prices. Since volume can never be larger that 1, Microsoft will prefer not to deviate. Therefore, the green area also corresponds to equilibria.

Let $w \approx 0.1304$ be the first root of $-1 + 7x + 5x^2 + x^3 = 0$. We conclude that there are equilibria with $q = 1$ and AMD active whenever z is below the following line (this is the upper bound of the yellow area):

$$\begin{cases} -3 + f + \frac{3}{2+f} + \frac{2\sqrt{(1-f)(2+f)^2}}{(2+f)^2} & \text{if } 0 \leq f \leq w \\ \frac{-3f}{2+f} & \text{if } w < f < 1 \end{cases}.$$

We now study the case where (15) is satisfied. As mentioned above, (12), (13), and (14) is the unique candidate to Nash equilibrium. Profit margins (at the candidate equilibrium prices) are all positive

$$p_m - c_m = \frac{1}{6}\left(3f - (2+f)\,z\right),$$
$$p_i - c = \frac{(-1+f)(-3+z)}{6},$$

and

$$p_a - c = \frac{(-1+f)\,z}{3}.$$

Finally, profits (at the candidate equilibrium prices) are

$$\pi_m = \frac{\left(f\,(z-3) + 2z\right)^2}{36f}, \tag{25}$$

$$\pi_i = \frac{(1-f)(3-z)^2}{36}, \tag{26}$$

and

$$\pi_a = \frac{(1-f)\,z^2}{9f}.$$

For (12), (13), and (14) to constitute an equilibrium, we need to check that there is no profitable deviation. In particular, we need to check that Microsoft does not want to price higher to move to a duopoly. (No need to check that Intel does not want to price lower to get rid of AMD. Lowering price when AMD is present is more effective than when AMD is not there.)

Assume that Intel and AMD stay put at the prices given by (13) and (14). Microsoft's best response function when pricing against Intel alone is $p_m = \frac{1}{2}(1 - p_i + c_m)$. Its demand is $q_m = 1 - (p_m + p_i)$. Substituting, we see that MS's profit from the deviation is

$$\pi_m = \frac{1}{144}\left(3 + 3f - (5+f)z\right)^2. \tag{27}$$

Microsoft will not deviate if (25) is larger than (27). Manipulating the inequality, the condition on z that guarantees that the three-player equilibrium is not broken by Microsoft (by pricing high) is

$$z \leq \frac{-12\sqrt{f} + 3f(3+f)}{16 + f(7+f)} = z^*(f). \tag{28}$$

Pairs (f, z) below $z^*(f)$ are such that Microsoft does not want to deviate upwards. To find the pairs (z, f) where the equilibrium is given by prices (12), (13), and (14), we must impose that $z > -\frac{3f}{2+f}$. When z is too negative the condition is violated. We conclude that the pairs (z, f) where (12), (13), and (14) are the equilibrium prices are those that satisfy:

$$-\frac{3f}{2+f} < z \leq \frac{-12\sqrt{f} + 3f(3+f)}{16 + f(7+f)} \qquad \text{and} \qquad w < f < 1,$$

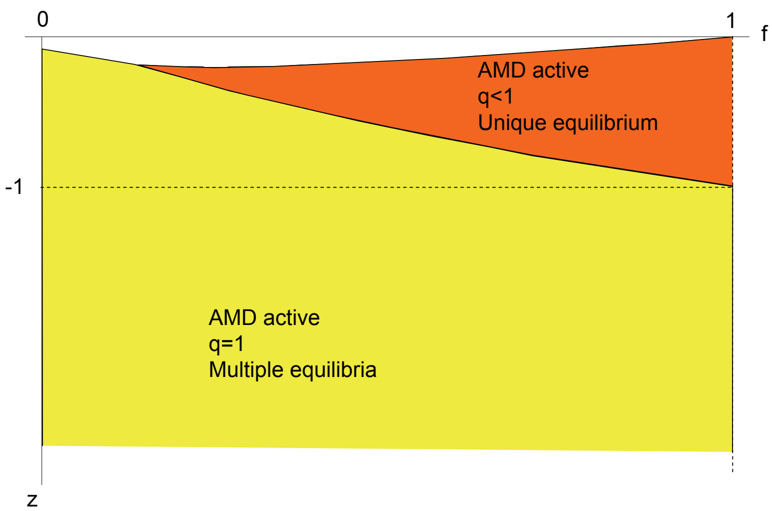
where w is the first root of $-1 + 7x + 5x^2 + x^3 = 0$ ($w \approx 0.1304$).

Finally, we need to check that when Microsoft desires to deviate, AMD gets negative demand, so that the relevant demand function faced by Microsoft when deviating is $1 - (p_m + p_i)$. AMD's demand at the deviation becomes

$$q_a = \frac{p_i - p_a}{1 - f} - \frac{p_a + p_m}{f} = -\frac{3 - 3f + (3+f)z}{12f}.$$

Notice that large z makes this expression negative. Substituting (28) we see that q_a is negative at the critical z. Therefore at any z such that Microsoft deviates (z larger than eq. 28), AMD gets negative demand.

Finally we put it all together in one single graph:



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Proof of Proposition 7. Microsoft's equilibrium profits when AMD is active are

$$\pi_m^{\text{AMD active}} = \frac{\left(f\left(z-3\right)+2z\right)^2}{36f}.$$

Differentiating $\pi_m^{\text{AMD active}}$ with respect to z and considering that z must be negative for AMD to be active, we see that Microsoft's profits increase as z becomes more negative. The best-case scenario in a three-player world is that z takes value $-\frac{3f}{2+f}$, the lower bound derived in the proof of Proposition 6. Substituting into $\pi_m^{\text{AMD active}}$ and simplifying we obtain $\pi_m^{\text{AMD active}} = f$. That is, given f, Microsoft's maximum profit in a world with AMD active is f (assuming that $-\frac{3f}{2+f} \leq z < \frac{-12\sqrt{f}+3f(3+f)}{16+\mathrm{f}(7+f)}$).

We turn now to Microsoft's profits in a two-player world. As shown above, we have

$$\pi_m^{\text{AMD not active}} = \frac{\left(1-z\right)^2}{9}.$$

Clearly, $\pi_m^{\text{AMD not active}}$ decreases with z. Therefore, if Microsoft was allowed to choose z within the set of zs compatible with a two-player equilibrium, it would choose the lowest possible z. As shown in the proof of Proposition 5, the lower bound is given by

$$z = \begin{cases} \frac{1}{2}\left(-7+3\sqrt{5}\right) & \text{if } 0 \leq f \leq \frac{1}{2}\left(7-3\sqrt{5}\right) \\ \frac{5f-6(1-f)\sqrt{f}}{9-4f} & \text{if } \frac{1}{2}\left(7-3\sqrt{5}\right) < f \leq 1 \end{cases}.$$

Substituting in $\pi_m^{\text{AMD not active}}$, we find Microsoft's maximum profits in a two player world.

Finally, we compare $\pi_m^{\text{AMD active}}$ and $\pi_m^{\text{AMD not active}}$.

The profits are equal at $f = \frac{1}{2}\left(7-3\sqrt{5}\right)$. Thus for any lower value of f, it is possible for Microsoft to earn more by increasing its costs and thereby moving to a duopoly equilibrium with Intel. We know from Proposition 6 that the three-firm equilibrium only exists if $f \geq w \approx 0.1304$.. ■[14]

Proof of Proposition 8. When AMD is active, Intel's equilibrium profits are

$$\pi_i^{\text{AMD active}} = \frac{\left(1-f\right)\left(z-3\right)^2}{36}.$$

Because $z < 0$, $\pi_i^{\text{AMD active}}$ decreases with z. The worst-case scenario for Intel in a three-player world occurs when z takes the maximum value compatible with a three-player world:

[14]Notice that although the range of f for which the proposition applies is small ($0.1304 \leq f < 0.1459$), we have set a very demanding benchmark against which to compare $\pi_m^{\text{AMD not active}}$. In assuming that Microsoft can choose any z, we are allowing for very high $\pi_m^{\text{AMD active}}$. In reality, there are constrains that prevent Microsoft from going all the way to $z = -\frac{3f}{2+f}$. For example, the prices of applications and other products sold to the installed base are bounded by competitors' offerings. This means that $\pi_m^{\text{AMD active}}$ is likely to be lower than what we have considered in the proof. Therefore, we expect the range of f for which $\pi_m^{\text{AMD not active}} > \pi_m^{\text{AMD active}}$ to be larger and the proposition to apply more generally.

$z = \frac{-12\sqrt{f}+3f(3+f)}{16+f(7+f)}$. Profits in this case are

$$\pi_i^{\text{AMD active}} = 4(1-f)\left(\frac{4+f+\sqrt{f}}{16+f(7+f)}\right)^2.$$

When AMD is not active, Intel's profits are

$$\pi_i^{\text{AMD not active}} = \frac{(1-z)^2}{9},$$

a decreasing function of z. If Intel was allowed to choose z within the set of zs compatible with a two-player equilibrium, it would choose the lowest possible z:

$$z = \begin{cases} \frac{1}{2}\left(-7+3\sqrt{5}\right) & \text{if } 0 \le f \le \frac{1}{2}\left(7-3\sqrt{5}\right) \\ \frac{5f-6(1-f)\sqrt{f}}{9-4f} & \text{if } \frac{1}{2}\left(7-3\sqrt{5}\right) < f \le 1 \end{cases}.$$

Substituting in $\pi_i^{\text{AMD not active}}$, we find Microsoft's maximum profits in a two player world.

Finally, we compare $\pi_i^{\text{AMD active}}$ and $\pi_i^{\text{AMD not active}}$. A little algebra reveals that $\pi_i^{\text{AMD not active}} < \pi_i^{\text{AMD active}}$ always. ■

Proof of Proposition 9. If z is positive (two-player world), f has no effect on Microsoft's profit and thus Microsoft is indifferent. If z is negative we may be in a two-player or a three-player world. Once again, Microsoft's profits ($\pi_m = \frac{(1-z)^2}{9}$) are independent of f in the two-player equilibrium. With three players, however, π_m depends on f: $\pi_m = \frac{(f(z-3)+2z)^2}{36f}$. Differentiating π_m with respect to f, we obtain $\frac{d\pi_m}{df} = \frac{1}{36}\left(-2z-3f+zf\right)\frac{2z-3f+zf}{f^2}$. Because $2z-3f+zf < 0$, if we show that $-2z-3f+zf < 0$ we will conclude that larger f is better for Microsoft. Notice that $-2z-3f+zf$ is largest when z is as negative as possible (because $2 > f$). The smallest possible z is $z = \frac{-3f}{2+f}$. Substituting into $-2z-3f+zf$ we obtain $-\frac{6f^2}{f+2}$ which is less than zero. Therefore, $\frac{d\pi_m}{df} > 0$.

Finally, we ask: Given $\frac{3\sqrt{5}-7}{2} \le z < 0$ so that both, two-player and three-player equilibria, are possible (depending on the value of f) what is f that maximizes Microsoft's profit? We compare $\pi_m = \frac{(1-z)^2}{9}$ and $\pi_m = \frac{(f(z-3)+2z)^2}{36f}$ evaluated at $f = 1$. Clearly the three-player world is better to Microsoft. ■

Proof of Proposition 10. We look at the case with $z < 0$ because Intel's profits are independent of f when $z > 0$. When $z < \frac{3\sqrt{5}-7}{2}$, the only possible equilibrium is a situation with AMD active. In this case, profits are $\pi_i^{\text{AMD active}} = \frac{(1-f)(3-z)^2}{36}$ a decreasing function of f. Therefore, the profit-maximizing f (in a three-player world) is the smallest f that is compatible with AMD active.

Finally, we need to check that when both two and three player equilibria are possible ($\frac{3\sqrt{5}-7}{2} \le z < 0$), Intel prefers the smaller f that makes AMD inactive. In the two-player equilibrium, Intel earns $\pi_i^{\text{AMD not active}} = \frac{(1-z)^2}{9}$. The smallest z such that both two and three player equilibria are possible is $z = \frac{3\sqrt{5}-7}{2}$. With this, we can easily compute the smallest f such that both two and three player equilibria are possible: $f = \frac{-61+36\sqrt{5}+3\sqrt{31\left(-17+8\sqrt{5}\right)}}{62} \approx 0.568$. Whenever $z > \frac{3\sqrt{5}-7}{2}$, the best f for Intel is given by $\frac{9+18z-7z^2+\sqrt{-3(z-3)^3(1+5z)}}{2(z-3)^2}$ (we computed this by solving (28) for f – considering those f larger than 0.568 only). We now substitute the "best f" in $\pi_i^{\text{AMD active}} = \frac{(1-f)(3-z)^2}{36}$ and compare the result to $\pi_i^{\text{AMD not active}} = \frac{(1-z)^2}{9}$ (under the constraint that $\frac{3\sqrt{5}-7}{2} \leq z < 0$). A little algebra shows that $\pi_i^{\text{AMD active}} < \pi_i^{\text{AMD not active}}$. Therefore Intel will want to choose f low enough to make sure that AMD is not active. ■

Proof of Proposition ??. AMD is active in equilibrium only when z is negative and $f \geq 0.1304$. In the three-player world, AMD's profits are:

$$\pi_a = \frac{(1-f)\,z^2}{f}.$$

This function is decreasing in f. ■

B Deriving the Best Response Functions

The first step it to derive best response functions of Microsoft, Intel, and AMD. This provides additional insight into the non-existence result in Corollary 2.

B.1 Microsoft's Best Response

Microsoft may ignore AMD or price assuming that AMD will be active. When Intel prices low, MS prices against Intel only. When Intel's price is high, MS prices against both Intel and AMD.

- If Microsoft ignores AMD and just prices against Intel, demand is $q_m = 1 - p_m - p_i$. The best response function is $p_m = \frac{1}{2}(1-p_i)$. Profits are $\pi_m = \frac{1}{4}(1-p_i)^2$.
- If Microsoft prices assuming that AMD will be a factor, demand is $q_m = 1 - \frac{p_a+p_m}{f}$. The best response function is $p_m = \frac{1}{2}(f-p_a)$. Profits are $\pi_m = \frac{(f-p_a)^2}{4f}$.

Two-player world is better than three-player world to Microsoft if

$$\frac{1}{4}(1-p_i)^2 \geq \frac{(f-p_a)^2}{4f}. \tag{29}$$

Taking into account that $0 \leq p_a < f$, condition (29) is equivalent to

$$p_i \leq 1 - \frac{f - p_a}{\sqrt{f}}.$$

Therefore, MS's best response function is:

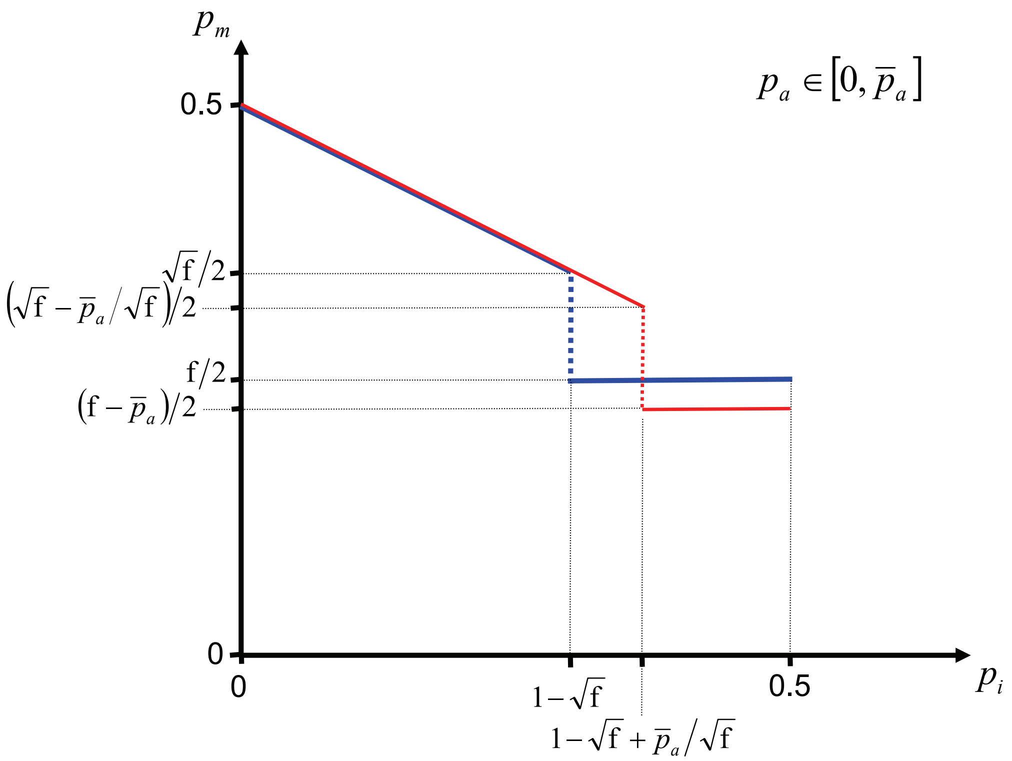
$$R_m(p_i, p_a; f) = \begin{cases} \frac{1}{2}(1 - p_i) & \text{if } 0 \leq p_i < 1 - \frac{f-p_a}{\sqrt{f}} \\ p_m \in \left\{ \frac{f-p_a}{2\sqrt{f}}, \frac{1}{2}(f - p_a) \right\} & \text{if } p_i = 1 - \frac{f-p_a}{\sqrt{f}} \\ \frac{1}{2}(f - p_a) & \text{if } 1 - \frac{f-p_a}{\sqrt{f}} < p_i \leq \frac{1}{2} \end{cases}.$$

Notice that $1 - \frac{f-p_a}{\sqrt{f}}$ is always greater than zero but may be larger than $\frac{1}{2}$ in which case MS prices against Intel (regardless of the price set by Intel). The condition for R_m to have a portion where MS prices against AMD is

$$\frac{1}{8}\left(1 + 8p_a + \sqrt{1 + 16p_a}\right) \leq f.$$

Clearly, when f is small AMD is ignored.

The following is a plot of $R_m(p_i, p_m; f)$. The blue reaction function is for $p_a = \underline{\mathrm{p}}_a = 0$. The red function is for $p_a = \bar{\mathrm{p}}_a > 0$. (We use the notation $\bar{\mathrm{p}}$ and $\underline{\mathrm{p}}$ for highest and lowest prices.) Of course, there is a continuum of reaction functions comprised between the blue and the red reaction functions in the figure below.



B.2 AMD's Best Response

We assume that AMD charges zero at the lowest. When MS charges high and Intel low, this is AMD's best response. It is important that AMD gets zero demand for AMD to wish to charge zero. We will need to check that AMD's demand is zero when Intel and Microsoft charge, respectively, $\underline{p}_i$ and $\bar{p}_m$. That is, we need

$$\frac{\bar{p}_m}{f} \geq \frac{\underline{p}_i}{1-f}.$$

We will show below that this is true.

When AMD gets positive demand $\left(\frac{p_i}{1-f} - \frac{p_m}{f} \geq 0\right)$, profits are $\pi_a = p_a\left(\frac{p_i - p_a}{1-f} - \frac{p_a + p_m}{f}\right)$. AMD's best response is $p_a = \frac{1}{2}\left(f p_i - (1-f)\, p_m\right)$. Therefore,

$$R_a\left(p_m, p_i; f\right) = \begin{cases} \frac{1}{2}\left(f p_i - (1-f)\, p_m\right) & \text{if } \frac{p_i}{1-f} - \frac{p_m}{f} \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

The largest price AMD will ever charge occurs when Intel charges the highest price and Microsoft the lowest:

$$\bar{p}_a = \frac{1}{2}\left(f\bar{p}_i - (1-f)\,\underline{p}_m\right).$$

We are assuming that in this case demand for AMD is positive.

B.3 Intel's Best Response

Intel can ignore AMD and just price against MS or assume that AMD will be active.

- Ignoring AMD, Intel faces demand $q_i = 1-(p_m+p_i)$. The best response is $p_i = \frac{1}{2}(1-p_m)$ and profits are $\pi_i = \frac{1}{4}(1-p_m)^2$.
- Assuming that AMD will be a factor, demand is $q_i = 1-\frac{p_i-p_a}{1-f}$. The best response is $p_i = \frac{1}{2}(1-f+p_a)$ and profits are $\pi_i = \frac{(1-f+p_a)^2}{4(1-f)}$.
- There is a corner solution also. If Intel ignores AMD it would choose to price high, so high that AMD becomes a factor. In that case, Intel would choose to price high, so high that AMD becomes a factor. In that case, Intel would like to lower its price (because it gains customers at rate $1/(1-f)$ rather than at rate 1).

  However, once Intel's price falls to the point that AMD gets zero demand, then Intel is back to pricing against Microsoft, which would lead it to raise price. But raising price makes AMD a factor again. Thus, Intel ends up at a corner solution where AMD is just excluded from the market.

  At the corner solution, p_i is such that $q_a = 0$. Thus, $p_i = \frac{p_a+(1-f)p_m}{f}$. Intel faces demand $q_i = 1-(p_m+p_i)$ or $q_i = 1-\frac{p_i-p_a}{1-f}$. Profits are $\pi_i = \frac{(f-p_m-p_a)(p_a+(1-f)p_m)}{f^2}$.

Let's now find the domains where each of these three best responses are relevant. In a two-player world Intel charges $p_i = \frac{1}{2}(1-p_m)$. AMD's demand is $q_a = \frac{p_i-p_a}{1-f} - \frac{p_a+p_m}{f}$. If we substitute p_i and solve $q_a = 0$, we obtain

$$p_m = \frac{f-2p_a}{2-f}. \tag{30}$$

This is Microsoft's price (as a function of p_a) such that Intel switches from pricing against MS to the corner solution.

In a three-player world Intel charges $p_i = \frac{1}{2}(1-f+p_a)$. Substituting in q_a and solving $q_a = 0$ for p_m, we obtain

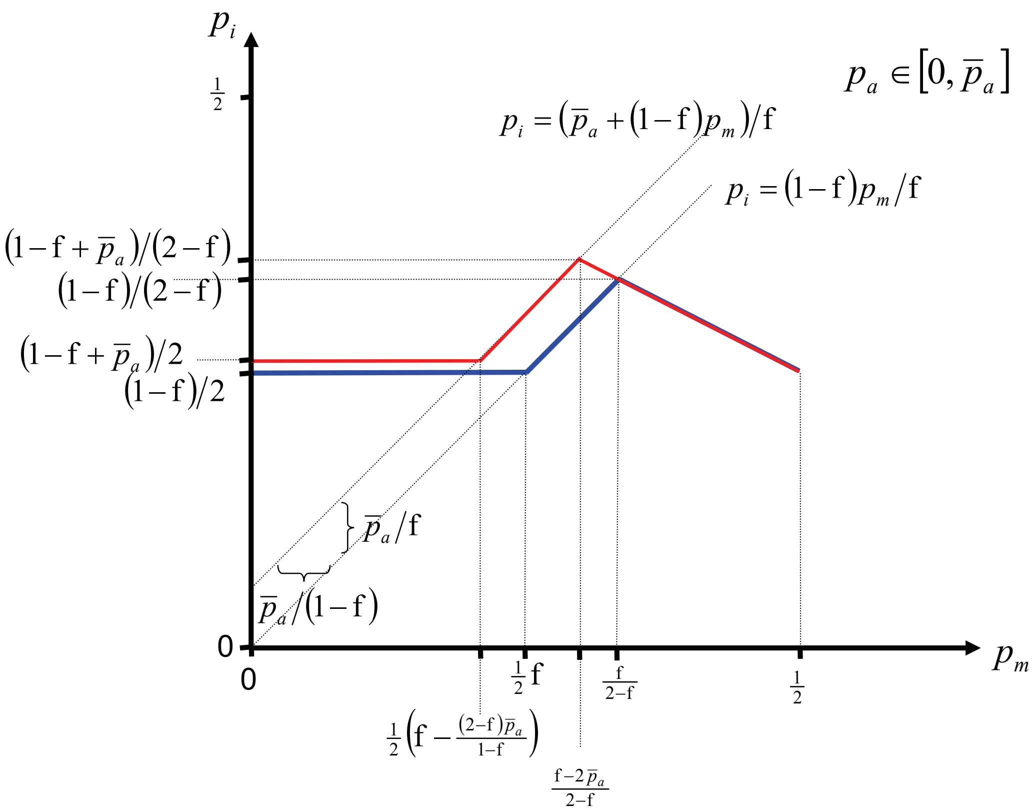
$$p_m = \frac{1}{2}\left(f - \frac{(2-f)\,p_a}{1-f}\right). \tag{31}$$

It is easy to see that (30) is always larger than (31).

Intel's reaction function is then:

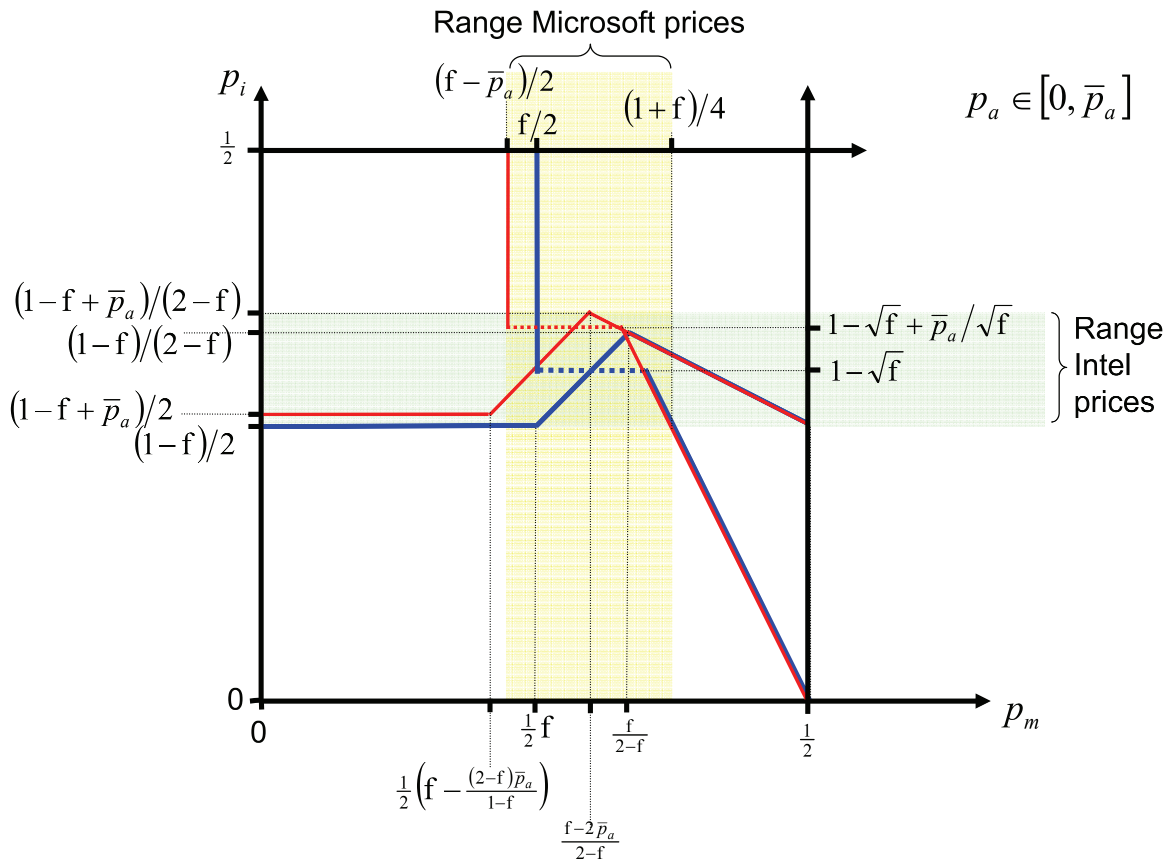
$$R_i\left(p_m, p_a; f\right) = \begin{cases} \frac{1}{2}\left(1 - f + p_a\right) & \text{if } p_m \leq \frac{1}{2}\left(f - \frac{(2-f)p_a}{1-f}\right) \\ \frac{p_a + (1-f)p_m}{f} & \text{if } \frac{1}{2}\left(f - \frac{(2-f)p_a}{1-f}\right) \leq p_m < \frac{f-2p_a}{2-f} \\ \frac{1}{2}\left(1 - p_m\right) & \text{if } \frac{f-2p_a}{2-f} \leq p_m \leq \frac{1}{2} \end{cases}.$$

This looks as follows (this figure is for the case where $\frac{1}{2}\left(f - \frac{(2-f)p_a}{1-f}\right) > 0$ and $\frac{f-2p_a}{2-f} < \frac{1}{2}$):



B.4 Equilibrium non-existence (revisited)

Now that we have derived Microsoft and Intel's reaction functions, we can put them in one single plot and see that when $f > \frac{4}{9}$, there is no equilibrium in pure strategies. The following plot is for $f = \frac{1}{2}$ and $p_a \in \left[0, \frac{1}{34}\right]$.



C Obtaining Price Bounds

Having obtained the best response functions, we now proceed to eliminating dominated strategies. We consider the case $f \geq \frac{4}{9}$.

C.1 $\underline{\mathrm{p}}_m$

We obtain $\underline{\mathrm{p}}_m$ by inspection of $R_m(p_i, p_a; f)$. Throughout the analysis, we will assume that $1-\sqrt{f}+\frac{\bar{\mathrm{p}}_a}{\sqrt{f}} \leq \bar{\mathrm{p}}_i$. We will later show that the condition is satisfied. With this, the lowest price for Microsoft occurs when p_a is maximum. That is

$$\underline{\mathrm{p}}_m = \frac{1}{2}\left(f-\bar{\mathrm{p}}_a\right).$$

C.2 $\underline{p}_i$ and $\bar{p}_m$

The figure above (Section B.4) reveals that $\underline{p}_i$ and $\bar{p}_m$ are jointly determined. Inspecting the figure, we see that

$$\underline{p}_i = \begin{cases} \frac{1-\bar{p}_m}{2} & \text{if } f < \bar{p}_m \text{ and } \frac{f}{2-f} < \bar{p}_m \\ \frac{1-f}{2} & \text{if } f \geq \bar{p}_m \text{ or } \frac{f}{2-f} \geq \bar{p}_m \end{cases}.$$

Now, because $\frac{f}{2-f} < f$, we have

$$\underline{p}_i = \begin{cases} \frac{1-\bar{p}_m}{2} & \text{if } f < \bar{p}_m \\ \frac{1-f}{2} & \text{if } f \geq \bar{p}_m \end{cases}.$$

Substituting this expression into $\bar{p}_m$ (assuming that $\underline{p}_i \leq 1-\sqrt{f}$) we obtain

$$\bar{p}_m = \begin{cases} \frac{1-\frac{1-\bar{p}_m}{2}}{2} & \text{if } f < \bar{p}_m \\ \frac{1+f}{4} & \text{if } f \geq \bar{p}_m \end{cases}.$$

Therefore, if $f < \bar{p}_m$, we have

$$\bar{p}_m = \frac{1-\frac{1-\bar{p}_m}{2}}{2} \Rightarrow \bar{p}_m = \frac{1}{3}.$$

Thus

$$\bar{p}_m = \begin{cases} \frac{1}{3} & \text{if } f < \frac{1}{3} \\ \frac{1+f}{4} & \text{if } f \geq \frac{1}{3} \end{cases}$$

and

$$\underline{p}_i = \begin{cases} \frac{1}{3} & \text{if } f < \frac{1}{3} \\ \frac{1-f}{2} & \text{if } f \geq \frac{1}{3} \end{cases}.$$

It is easy to see that $\underline{p}_i \leq 1-\sqrt{f}$ for all f and our assumption is correct. In addition to $\underline{p}_i \leq 1-\sqrt{f}$ we have also assumed that $\underline{p}_a = 0$ (for all f). In the next subsection we show that the guess is correct.

Since we are interested in the case $f > \frac{4}{9}$, in what follows we consider the simpler expressions $\bar{p}_m = \frac{1+f}{4}$ and $\underline{p}_i = \frac{1-f}{2}$.

C.3 $\underline{p}_a$

We have assumed that $\underline{p}_a = 0$ for all f. We now show that this is true. We plug $\underline{p}_i$ and $\bar{p}_m$ in

$$q_a = \frac{p_i - p_a}{1-f} - \frac{p_a + p_m}{f}$$

to obtain

$$q_a = -\frac{(1-f)^2 + 4p_a}{4(1-f)f} < 0.$$

With this, we confirm that $\underline{p}_a = 0$.

C.4 $\bar{\mathrm{p}}_i$

We have just shown that

$$\bar{\mathrm{p}}_m = \frac{1+f}{4}.$$

We also know that

$$\underline{\mathrm{p}}_m = \frac{1}{2}\left(f - \bar{\mathrm{p}}_a\right).$$

Notice that $\underline{\mathrm{p}}_m$ is largest when $\bar{\mathrm{p}}_a = 0$. (We use this fact below.) Now, we consider $\bar{\mathrm{p}}_m$ and $\underline{\mathrm{p}}_m$ in deriving $\bar{\mathrm{p}}_i$.

When f is large, the peak of the mountain on Intel's reaction function is at p_m to the right of $\bar{\mathrm{p}}_m$. The critical f such that below it the peak is to the left of $\bar{\mathrm{p}}_m$ is found by solving

$$\frac{f - 2\bar{\mathrm{p}}_a}{2-f} = \frac{1+f}{4}.$$

The solution is

$$f^* = \tfrac{1}{2}\left(-3 + \sqrt{17 + 32\bar{\mathrm{p}}_a}\right).$$

Therefore, when $f \geq f^*$, $\bar{\mathrm{p}}_i$ is on the upward sloping portion of Intel's reaction function and

$$\bar{\mathrm{p}}_i = \frac{\bar{\mathrm{p}}_a + (1-f)\frac{1+f}{4}}{f}.$$

As long as $f \geq \frac{1}{4}$, the peak is attained at p_m strictly between $\bar{\mathrm{p}}_m$ and $\underline{\mathrm{p}}_m$. In this case

$$\bar{\mathrm{p}}_i = \frac{1 - f + \bar{\mathrm{p}}_a}{2-f}.$$

To summarize,

$$\bar{\mathrm{p}}_i = \begin{cases} \frac{1-f+\bar{\mathrm{p}}_a}{2-f} & \text{if } f \leq \frac{1}{2}\left(-3+\sqrt{17+32\bar{\mathrm{p}}_a}\right) \\ \frac{\bar{\mathrm{p}}_a + (1-f)\frac{1+f}{4}}{\mathrm{f}} & \text{if } \frac{1}{2}\left(-3+\sqrt{17+32\bar{\mathrm{p}}_a}\right) < f \end{cases}.$$

C.5 $\bar{\mathrm{p}}_a$

We now look for $\bar{\mathrm{p}}_a$ as a function of f only. To do this, we put the highest price for Intel and the lowest for Microsoft in AMD's reaction function and then we solve for $\bar{\mathrm{p}}_a$ (Intel's highest price and Microsoft's lowest price depend on $\bar{\mathrm{p}}_a$)

$$\bar{\mathrm{p}}_a = \frac{1}{2}\left(f\bar{\mathrm{p}}_i - (1-f)\,\underline{\mathrm{p}}_m\right).$$

We obtain the following expression. Let w_2 solve $-6 + 9x + x^3 = 0$ ($w_2 \approx 0.63783$). Then

$$\bar{\mathrm{p}}_a = \begin{cases} \frac{f^2 - f^3}{6 - 3f - f^2} & f < w_2 \\ \frac{1 - 2f + f^2}{2(1+f)} & \text{if } w_2 \leq f < 1 \end{cases}.$$

C.6 $\underline{\mathbf{p}}_m$ revisited

We have shown above that

$$\underline{\mathrm{p}}_m = \frac{1}{2}\left(f - \bar{\mathrm{p}}_a\right).$$

Substituting $\bar{\mathrm{p}}_a$ we obtain $\underline{\mathrm{p}}_m$ as a function of f.

$$\underline{\mathrm{p}}_m = \begin{cases} \frac{f(3-2f)}{6-f(3+f)} & \text{if } f < w_2 \\ \frac{f(4+f)-1}{4(1+f)} & \text{if } w_2 \leq f < 1 \end{cases}.$$

C.7 $\bar{\mathbf{p}}_i$ revisited

We have shown that

$$\bar{\mathrm{p}}_i = \begin{cases} \frac{1}{2}\left(1 - \frac{f-2\bar{\mathrm{p}}_a}{2-f}\right) & \text{if } f \leq \frac{1}{2}\left(-3 + \sqrt{17 + 32\bar{\mathrm{p}}_a}\right) \\ \frac{\bar{\mathrm{p}}_a + (1-f)\frac{1+f}{4}}{\mathrm{f}} & \text{if } \frac{1}{2}\left(-3 + \sqrt{17 + 32\bar{\mathrm{p}}_a}\right) < f \end{cases}.$$

Substituting $\bar{\mathrm{p}}_a$ we obtain $\bar{\mathrm{p}}_i$ as a function of f:

$$\bar{\mathrm{p}}_i = \begin{cases} \frac{3(1-f)}{6-f(3+f)} & \text{if } f < w_2 \\ \frac{(1-f)\left(3+f^2\right)}{4f(1+f)} & \text{if } w_2 \leq f < 1 \end{cases}.$$

C.8 Checking that $1 - \sqrt{f} + \frac{\bar{\mathbf{p}}_a}{\sqrt{f}} \leq \bar{\mathbf{p}}_i$

Plotting both sides of the inequality it is easy to see that the condition is satisfied.