Individuals and Institutions

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What happens when political and economic forces interact in society? Recent developments in politics and economics focus attention on how outcomes are determined by the interaction between institutional structure and individual preferences. Specifying the "rules of the game" is often seen as a way of producing institution-specific answers to how social decisions are made (see e.g., David P. Baron and John Ferejohn, 1987; David Austen-Smith and Jeffrey Banks, 1988; Baron, 1991). In this tradition, membership of the institution (the set of players of the game) is typically taken as given. This paper proposes to endogenize the institutional structure by considering the case in which multiple institutions compete for members. We present general definitions of equilibrium with multiple institutions and explore conditions for existence.

Charles M. Tiebout's (1956) approach to the free-rider problem demonstrates the application to local public goods. There are several communities, each of which has a political process for deciding the provision of its local public goods based on the preferences of its population. In turn, individuals reveal their preference for the provision of local public goods through their choice of a community. The existence of an equilibrium in this context was first demonstrated by Frank Westhoff (1977). If communities use majority rule to determine their public goods and if preferences are single-peaked, then there exists an equilibrium in which each community follows the preference of its median member and each person belongs to his most-preferred community. In this paper, we consider multidimensional choice problems, political processes other than majority rule, and other applications (such as platform choice of political parties).

I. A Model

There are m exogenously given institutions.[1] These institutions are meant to include political parties, communities, private clubs, and other decision-making bodies. Each institution k has available a compact set of possible policy positions $X_k \subset \mathbb{R}^w$. The vector of positions is represented by $\bar{\mathbf{x}} \in X$, where X is the product of the sets X_k.

Individual preferences vary across the population as summarized by a vector $\boldsymbol{\alpha} \in \mathbb{R}^n$. The utility of an α-type joining institution k given choices $\bar{\mathbf{x}}$ is $U(\boldsymbol{\alpha}, k, \bar{\mathbf{x}})$. The distribution of types across society is then represented by a hyperdiffuse probability measure f on utility parameters $\boldsymbol{\alpha}$ with compact support $A \subset \mathbb{R}^n$.

We assume that each individual joins the institution k that maximizes $U(\boldsymbol{\alpha}, k, \bar{\mathbf{x}})$. Since an individual is an infinitesimal part of an institution, no one person believes that he can influence the party's position by his action. The membership of institution k is then denoted by $S_k(\bar{\mathbf{x}})$.

Each institution responds to its membership through a political process. We allow the feasible choices of an institution to be constrained by its membership. For example, an institution may be constrained to

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[1]The potential entry and exit of institutions has been addressed in related models by Joseph Greenberg and Shlomo Weber (1986) and Alessandra Casella and Jonathan Feinstein (1990).

have a balanced budget. Given any distribution of individuals into institutions, we denote the kth institution's set of feasible choices by $\mathrm{X}_k(\mathrm{S}_k) \subset \mathrm{X}_k$.

It remains to define the political decision-making process. Here we consider two alternative approaches. The first is a membership-based definition, and the second is a position-based definition. In the first, the political decision-making process of party k is defined by a policy function, $\mathbf{P}_k$, which maps the memberships of all parties into its set of feasible positions, $\mathrm{X}_k(\mathrm{S}_k)$:[2]

$$\mathbf{x}_k = \mathbf{P}_k(\mathrm{S}_1,\dots,\mathrm{S}_m).$$

In the second approach, the political decision-making process of party k is defined by a policy function, $\mathbf{Q}_k$, which maps the membership of the kth institution and the positions of all rival institutions into its set of feasible positions, where the feasible set for the kth institution depends on the rationally anticipated membership given the fixed position of rival institutions and its proposed position:

$$\mathbf{x}_k = \mathbf{Q}_k(\mathrm{S}_1,\bar{\mathbf{x}}_{-k}).$$

The real difference is that in the first case the current members of an institution regard the memberships as fixed when selecting their position. In the second case, the current members anticipate changes in their membership as they adjust their positions, holding rival institutions fixed. The first is appropriate when the decisions take place after commitments have been made to the institutions, as in choice of neighborhood. The second is appropriate when the institution must commit to a policy before individuals commit to the institutions, as in many political processes.

Corresponding to these two definitions of an institution are distinct definitions of institutional equilibrium. In the membership-based case, each individual rationally anticipates the membership choices of others and the resulting institutional policies and selects the most preferred institution.

Definition 1 (membership-based institutional equilibrium): Given the set of political processes, $\{P_1,\dots,P_m\}$, $\bar{\mathbf{x}}^*$ is a political-economic equilibrium if and only if

$$\bar{\mathbf{x}}_k^* = \mathbf{P}_k(\mathrm{S}_1(\bar{\mathbf{x}}^*),\dots,\mathrm{S}_m(\bar{\mathbf{x}}^*)) \qquad k=[1,\dots,m].$$

In the position-based case, each individual rationally anticipates the positions that each institution will take and chooses among institutions based on these anticipated positions. Given these tentative memberships, the institutions pick the predicted positions, and no individual wishes to switch to another institution.[3]

Definition 2 (position-based institutional equilibrium): Given the set of political processes, $\{Q_1,\dots,Q_m\},(\mathrm{S}_1,\dots,\mathrm{S}_m,\bar{\mathbf{x}}^*)$ is a position-based institutional equilibrium if and only if

$$\mathbf{x}_k^* = \mathbf{Q}_k(\mathrm{S}_k,\bar{\mathbf{x}}_{-k}^*) \qquad k=[1,\dots,m].$$

In the next section we explore the existence of equilibrium. We work throughout with the membership-based definition: essentially similar results are available for the position-based definition (Caplin and Nalebuff, 1992).

II. Existence

There are three distinct mathematical approaches to the existence issue, based on maximization, continuity, and algebraic topology, respectively. The first covers only the most trivial of cases. The second is quite general but relies crucially on the smoothing effect of idiosyncrasies. The third dispenses with idiosyncratic factors and is the most challenging approach. It most clearly

[2]This can be generalized to cover the case of policy correspondences.

[3]This definition is similar to that proposed by Dennis Epple et al. (1984) in the context of majority voting over local public goods.

indicates the special mathematical character of allowing for political-economic interactions.

Utilitarian Parties.—Consider first a society in which each individual cares only about the position adopted by his own institution. Assume that each institution has as its objective the maximization of the sum of its members' utilities and that the choice sets are independent of membership.[4] In this case it is immediate that the method of dividing the population into m groups that maximizes the grand sum of utilities defines an institutional equilibrium: each institution will play its part and maximize the internal sum, while the individual plays his part by going to the institution that makes his individual utility the highest.

Continuity Approaches.—The continuity approaches make the assumption that an individual not only has opinions about the policy that an institution adopts, but also an idiosyncratic view of the institution. For example, there are those who prefer the Democratic party to the Republican party, even if the two parties take the same position on all observable issues. An individual now has a type vector $(\boldsymbol{\alpha}, \boldsymbol{\varepsilon})$ where the $\boldsymbol{\alpha} \in \mathbb{R}^n$ component refers to the explicit policies adopted, and the $\boldsymbol{\varepsilon} \in \mathbb{R}^m$ gives the m-vector of institution-specific idiosyncratic factors,

$$U(\boldsymbol{\alpha}, \boldsymbol{\varepsilon}, i, \bar{\mathbf{x}}) = U(\boldsymbol{\alpha}, i, \bar{\mathbf{x}}) + \varepsilon_i.$$

We make two continuity assumptions. We assume that the utility functions $U(\boldsymbol{\alpha}, i, \bar{\mathbf{x}})$ are continuous in $\bar{\mathbf{x}}$ and that the joint density $f(\boldsymbol{\alpha}, \boldsymbol{\varepsilon})$ is diffuse and has an unbounded support. Thus, each party's constituency will be a continuous function of the platforms. We assume that a party's platform is a continuous function of its constituency. The political process is continuous in the weak topology: a slight change in membership leads to only a slight change in the party's position. With these assumptions, we use Brouwer's theorem to prove the existence of equilibrium (Caplin and Nalebuff, 1992).

[4]For example, the choice of a political position is not constrained by the size of the group.

While this is a general positive result, it is somewhat unsatisfying in that it relies on the addition of idiosyncratic factors that are unconnected to the original question and may not be present in many cases. This makes it of interest to explore the model in which the $\boldsymbol{\varepsilon}$ vectors are set to zero. It is the case without idiosyncracies that highlights the conceptual difficulties involved in establishing existence of political-economic equilibria. We present two different problems in which there is no political-economic equilibrium (other than a trivial one in which all institutions are identical) for the case with no idiosyncracies.

Our first example is an adaptation of the redistribution model of Epple and Thomas Romer (1991). Consider individuals who differ only in income level y and who are considering which of two towns to live in. The towns must set a proportionate tax rate, $t \in [0,1]$, and also a level of government transfers, g. Each individual cares only about his income net of taxes and transfers,

$$U(y, g, t) = y(1-t) + g.$$

The (t, g) combination in each town is chosen by some political process subject to a balanced-budget constraint. Epple and Romer specify majority rule, while we allow for general political processes.

We look for an equilibrium in which the two towns differ. The town that sets the higher tax rate must also have the higher level of government transfers; otherwise it will not attract any inhabitants. Without loss of generality, suppose that $(t_1, g_1) > (t_2, g_2)$. Now note that if a type y is attracted to town 1, so will be all those with lower income levels: the high-tax high-transfer town attracts the poorest inhabitants. Anyone in town 1 who has above that town's average income level will prefer to move to town 2: such individuals are net losers in town 1 as its richest inhabitants, and would at worst break even in town 2 as its poorest inhabitants. This implies that the poorest type of individuals must form a community by themselves, with everyone else in the second community. Even this will not be an equilibrium unless the second community chooses to set a zero tax rate.

If the political process is majority rule (as in Epple and Romer [1991]), then whenever the median income is lower than the mean income, the tax rate will be positive, and hence there will be no equilibrium in which the towns differ.[5] More generally, the existence problem arises for any continuous political process that results in a positive tax rate, given membership of all but the poorest types.

A second example of a political process for which no equilibrium exists is the "rotation" process. Each party chooses the position most favored by its most counterclockwise member. As a result, the division between the two parties continues to rotate counterclockwise, and there is never any stable point. For example, on the left in Figure 1, the division of the population into parties S_1 and S_2 lead to positions x_1 and x_2, which in turn lead to the new divisions S_1 and S_2 on the right in the figure.[6]

To see the issues involved in establishing existence, we consider cases with two parties and restricted voter preferences. Each individual evaluates a party by a weighted sum of its perceived benefits. The population differs only in the weights used to evaluate benefits. The utility benefits are determined by a continuous function t which maps the w dimensions of the platform into an $(n+1)$-dimensional vector of utility benefits,

$$U(\boldsymbol{\alpha},k,\bar{\mathbf{x}}) = \sum_{j=1}^{n} \alpha_j t_j(\mathbf{x}_k) + t_{n+1}(\mathbf{x}_k)$$

[5]There remains the trivial equilibrium in which both towns pick identical tax and transfer schemes and each type in the population divides evenly between the towns.

[6]The mapping described above is not continuous in the dividing line. The problem occurs when one party contains the entire population. In that case, there is no most counterclockwise point. This problem can be fixed by taking the most counterclockwise point until a party has 90 percent of the total mass. Thereafter, we take a weighted average of the center of gravity of the set and the most counterclockwise point, where the weight on the center of gravity goes to 1 as the party mass approaches 1.

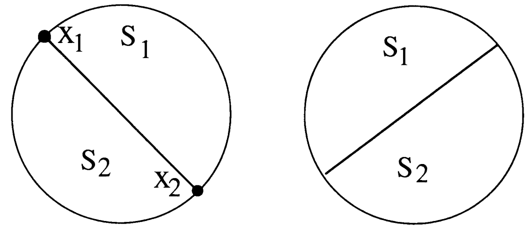

FIGURE 1. THE ROTATION PROCESS

where U: $\mathbb{R}^n \times \mathrm{X} \mapsto \mathbb{R}$, and the functions $t_j(x_k)$ continuously map the institution's position into a utility valuation. The linear preference model covers many of the standard utility functions used in economics, such as Euclidean preferences and constant-elasticity-of-substitution preferences.

The most critical feature is that it results in a very simple description of the sets that prefer the different parties. When the two political parties have picked distinct positions, the division is always by a hyperplane. This hyperplane has some gradient vector $\boldsymbol{\pi}$, which we normalize to unit length, and intercept b. The orientation of the hyperplane tells us which individuals go to which institution. In our notation, the gradient vector points into the constituency of party 1. Thus we can characterize S_1 and S_2 by $(\boldsymbol{\pi}, b)$. We continue to assume that the probability density of consumers' utility parameters is hyperdiffuse over its compact support and that the political process is continuous.

One natural way to look for an equilibrium is to use the division of the population between the two parties as defined by the hyperplane $(\boldsymbol{\pi}, b)$ to produce a new division $(\boldsymbol{\pi}', b')$. The mapping must be continuous and must have the property that a fixed point of the map is a political-economic equilibrium. In pursuing this approach, in Caplin and Nalebuff (1992), we first make assumptions that ensure that it is possible to construct such a continuous map and then ensure that the map has a fixed point.

In broad terms, the idea of the mapping is a multidimensional analog of Westhoff (1977). It begins with an arbitrary division of the population defined by a $(\boldsymbol{\pi}, b)$ pair such that both institutions have strictly positive population. We then look at the choices they will make and redivide the population (with some continuous adjustments made in the actual mapping). For this map to be well defined it is necessary that parties whose constituents lie on opposite sides of a hyperplane will choose distinct political positions. Another assumption required to ensure that a fixed point of our map is an equilibrium is that starting with parties of equal size, someone in each party must prefer his own party's position to that of the rival party. This is a minimal form of positive responsiveness of institutions to their members that rules out complete flips of the population.

To ensure that all institutions have positive membership in equilibrium, we assume, following Westhoff (1977), that small institutions lose members: there exists a size $\mu > 0$ such that if one institution has population less than or equal to μ, then some individual will prefer to switch to the other institution. This is reasonable in settings where size is an advantage, as when there is a fixed cost of supplying a public good or power is at stake in a political process. It is this assumption that the tax-subsidy example contradicts: a small rich town attracts members.[7]

Using these assumptions, in Caplin and Nalebuff (1992) we define a continuous map $(\boldsymbol{\pi}, b)$ into itself whose fixed points are equilibria. This is not quite enough: recall the rotation example. Remarkably, this counterexample is a problem only in even-numbered dimensions.[8] The dimensionality of the voter preferences is determined by the dimensionality of the $\boldsymbol{\alpha}$ vector. The theorem is that there exists a two-party political-economic equilibrium for social choices in $\mathbb{R}^{2n+1}$. In this equilibrium, both institutions will have strictly positive membership. The proof of this result relies on the Lefschetz fixed-point theorem (see James Munkres, 1984).[9]

III. Conclusions

This paper introduces a general framework for studying the interaction between institutional structure and individual preferences in the presence of mobility. We provide two definitions of institutional equilibrium designed to capture this interaction. We believe this question has broad applicability (from local public goods to political parties to corporate control) and leads to a distinctive theory.

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[7]A second route is to assume that parties which are sufficiently small tend to expand. This would be true if small institutions could design a platform that was tailor-fitted to their constituency. The tax example also fails: small poor towns lose members.

[8]One cannot simply "fix up" the problem for the even-numbered dimensions by adding another "fake" dimension. The continuity assumption will fail: a discontinuity arises when the two parties differ only in the "fake" dimension.

[9]This positive result for odd dimensions is connected to the "hairy-ball theorem" which states that any continuous vector field on the surface of a sphere in $\mathbb{R}^{2n+1}$ must have a zero. For example, since the winds on the surface of the (three-dimensional) earth change direction continuously, somewhere on the earth's surface there is no wind. In contrast, along the surface of a circle (such as the equator), there can be a clockwise (or counterclockwise) wind everywhere.

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