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Puzzles
Noisy Prisoners, Manhattan Locations, and More

Barry Nalebuff

The purpose of this feature is to create a dialogue between the journal and its readers. In presenting economic puzzles, I have three goals in mind: Some puzzles are chosen to stimulate research; others offer examples that will help undergraduate and graduate teaching; all should provide quality distractions during seminars. I welcome readers to share their favorite teaching problems and research puzzles (with answers, please). Readers are also invited, nay challenged, to submit their answers to the puzzles below; the responses will be discussed two issues later.

Each "Puzzles" will begin with a few speed problems. These puzzles have answers provided in the same issue. Puzzles 1 and 2 below will give you a chance to get up to speed. Then, we continue with longer puzzles taken from two very broadly defined categories: strategy puzzles and theory puzzles. (Empirical puzzles will be presented in Richard Thaler's feature "Anomalies.") Strategy puzzles will give the readers an opportunity to compete against each other in problems of coordination and competition. The third puzzle, a noisy prisoner's dilemma tournament, falls dead center in this category. Theory puzzles are meant to offer mathematical problems that have an economic interpretation. The fourth puzzle, an optimal location problem, is in this category.

Please send your answers and favorite puzzles to: Barry Nalebuff, Puzzles, c/o *Journal of Economic Perspectives*, Woodrow Wilson School of Public and International Affairs, Princeton University, Princeton, N.J. 08544.

Good luck.

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Puzzle 1: Turn Out the Lights

In the event of a temporary shortage of electricity, standard practice is to ration all customers equally, that is, to reduce each customer's available electricity by the same proportion. This practice results in the familiar summer brown-outs. Imagine that priorities for electricity could be bought and sold. A market could be set up so that all payments are made as transfers among the users; the money collected for obtaining a high priority would be paid out to those who are willing to take a lower priority.[1] Since the payments are made as transfers, the electric company generates no revenue from the sale of priorities. In the event of a shortage, the person with lowest priority would be cut off first. If this rationing is insufficient, then the next lowest priority would be cut off and so on up the priority scale. It seems intuitive that those who most value uninterrupted electrical supply and those who least value an uninterrupted supply are both made better off as a result of the marketing of priorities. Hospitals would be willing to pay a premium for an uninterrupted electrical supply while struggling artists would be willing to endure more frequent power outages in return for compensation. But what about the consumers in the middle? Show that under the competitive equilibrium prices for priority, everyone is at least as well off as they were before the market was created.

Puzzle 2: In Fact, it's a Gas

The production costs of leaded and unleaded gasoline are almost exactly equal.[2] At the wholesale level, the price differential is usually no more than 1 cent per gallon. Why does this translate into about 8 cents per gallon at the pumps? Similarly, the production cost of unleaded premium is only 2 cents per gallon more expensive than standard unleaded. At the pumps, the price differential is often over a dime. Why?

Answers to Puzzles 1 and 2 appear after Puzzle 4.

Puzzle 3: Prisoner in a Noisy Cell

The "prisoners' dilemma" should be familiar to many readers. Two prisoners are being questioned about a crime in which both participated. They are separated in the police station and faced with a choice: keep silent or confess. If one prisoner keeps

[1] Everyone is required to buy some level of priority. Once the market is established, the initial endowment point is no longer relevant.

[2] Until recently, leaded gasoline was about 2 cents/gallon cheaper to produce (and it sold for 15 cents/gallon more). The equalization of production costs is due to the tighter restriction on the lead content of leaded gas combined with the fact that leaded gas has an 89 octane while unleaded has only an 87 octane.

quiet and the other informs the police about the crime, the informer gets off with a suspended sentence while the quiet one gets the maximum term. If both prisoners keep silent, it will be hard for police to prove their case; the result is a short plea-bargained jail term. If both prisoners confess, they benefit from having cooperated with the police but are hurt by the corroborating testimony; the result is a medium term sentence for both. The result is that "confess" is a dominant strategy for both prisoners even though this makes them both worse off than if they both remain silent.

A standard response to the prisoners' dilemma is to observe that most real-life applications are not a one-shot problem. If the "dilemma" is repeated, each time the two recidivists will know the choice their partner made in previous plays and can use this information in deciding how to respond. Axelrod's (1984) pioneering study of the prisoners' dilemma demonstrated that in the repeated problem a simple Tit for Tat strategy can seemingly resolve the dilemma. Both sides begin by cooperating with each other (keeping quiet). Subsequently, each side copies the other's previous move. The threat of future punishments helps maintain the honor among thieves.

It is time to repeat Axelrod's experiment to see what has been learned, while adding a new twist.[3] Communication is not perfect. Although the prisoner hears what his partner in crime decided in the last round, that signal is correct only 95 percent of the time. Thus, the prisoners' decisions must take into account the possibility that the partner did the opposite of what he heard. Further, the prisoner must remember that the partner may be reacting on the basis of a misperception of the prisoner's previous decision.

When mistakes in communication are possible, Tit for Tat breaks down. Once a mistake in communication leads one side to believe that the other has defected (confessed), both sides will get into a mindless and almost slapstick routine of punishing each other: player A punishes B for defecting and then B punishes A for punishing him and so on. This outcome suggests that the desirability of Tit for Tat may not be robust to situations where players' actions may be misinterpreted. Yet, misperceptions are an unavoidable and an intrinsic element in many problems, including that of attaining nuclear deterrence (see Jervis, 1982). To apply the prisoners' dilemma model to a wide variety of political science applications, it is important to find strategies that are robust to misperceptions. This search is the topic of research by Downs and Rocke (1987) and the focus of the current tournament.

What is the best strategy when communication is imperfect? To help make the comparison with Axelrod's tournament, the payoffs in each period are as in Axelrod (p. 8): if both players cooperate with each other then each receives 3; if both defect to help the police then they each receive 1; if one defects and the other cooperates then the defector receives 5 and the cooperator receives 0. The aggregate payoff in the multiperiod problem is simply the average of the single period payoffs: there is no discounting.

[3]Ted Bergstrom has already begun this task. The results from his tournament (which is similar, but not identical, to the one below) show that complicated strategies that make use of conditional probabilities were able to take advantage of the simpler entries. Contestants are advised to read his column in *The Free Lunch*, vol. 1 (Department of Economics, University of Michigan), for more hints.

Instructions for submitting your strategy: Design a computer program to play your strategy in a repeated prisoners' dilemma. There is no fixed time deadline but only the first *fifty* entries will be considered. Please submit your program written in Basic on an IBM formatted disk (along with a hard copy). If your program is very short, you may send in a flow chart indicating your strategy and we will convert it to a Basic program.

A master program will run a loop for a random number of periods, with a 1 percent probability of quitting each time after the fiftieth round.[4] Each period, the master program will call up the two competing programs and read their current moves. Your move should be denoted by the variable DecisionX. (Your opponent's move will be called DecisionY.) The two possible values for DecisionX are Cooperate or Defect. Your strategy can be a function of all your previous moves and all of your opponent's previous moves as reported. The variable turn denotes the round; initially, turn = 1. Your opponent's previous moves will be reported to you in a vector called ChoiceY(turn). For example, ChoiceY(2) is your observation of your opponent's second move; this particular value will be available to you once turn equals three or more. It is important to emphasize that your signal, ChoiceY(turn), may not be a correct report of your opponent's move. After reading both players' moves, the master program updates ChoiceY(turn) for you and ChoiceX(turn) for your opponent. The current moves will be reported correctly to the other side with a 95 percent probability and reversed the other 5 percent of the time. This probability of a reversal is independent across periods and independent across the reports to ChoiceX and ChoiceY.[5] Finally, the initial value of ChoiceX(0) and Choice Y(0) is arbitrarily set equal to Cooperate.

The Tit for Tat strategy can be expressed in one line,

$$\text{DecisionX} = \text{ChoiceY}(\text{turn} - 1).$$

You begin by cooperating. Subsequently, your decision equals your opponent's previous move. After reading both moves, the master program updates your history of your opponent's previous move by setting ChoiceY(turn) = DecisionY (or with a 5 percent probability, ChoiceY(turn) = the opposite of DecisionY). Once turn is greater than 50, with a 99 percent probability, the master program will run an additional loop, setting turn = turn + 1 and then look for your next move.

Randomized strategies are certainly allowed. In Basic, the command RND(N) provides a random number uniformly distributed between 1 and N. In conjunction

[4] This condition is set to avoid problems with the logic of backwards induction from a known final period. After the first 50 rounds, the chance that there will be one more period of play will be constant at 0.99. Hence, the expected length of the tournament will be 150 plays.

[5] You may wish to keep track of all your moves in a vector called HistoryX(turn). Trying to compare this vector to the vector ChoiceX would be cheating: you will be disqualified. Also, the payoffs will be based on the true moves; when the signals get crossed, the payoffs are not affected in that period. Of course, future payoffs may change because future strategies may be different.

with IF and THEN statements, any (rational) probabilities may be used. For example, if the decision to cooperate on the Nth turn is chosen to be $1/N$, then this strategy can be represented by

IF RND(turn) = 1 then Decision = Cooperate ELSE Decision = Defect.

On the Nth turn, there are N possible values for RND(turn), only one of which will lead to cooperation.[6]

A final example of a simple conditional strategy is to Cooperate until the opponent is seen to defect twice. Upon observing two defections, you play Tit for Tat thereafter. This rule might be called "Fooled me once, shame on you; fool me twice, shame on me."[7]

I will report the results of this tournament in a subsequent issue.

Puzzle 4: The Best Location in Manhattan

Consider the most efficient pattern of firm locations when consumers are uniformly distributed over a two-dimensional plane. If transportation costs are measured by Euclidean distance (the distance as the crow flies) then Mills and Lav (1964) and Bollobas and Stern (1972) have shown that the most efficient location pattern results in hexagonal market areas. Any other location pattern leads to a greater average distance traveled by consumers for a given density of firms. An interesting alternative to the Euclidean distance metric is the "Manhattan" metric. Under the Manhattan metric, the distance between A and B equals the sum of the vertical and horizontal differences; if $A = (a_1, a_2)$ and $B = (b_1, b_2)$ then the distance between A and B, denoted by $\|A - B\|$, equals $|a_1 - b_1| + |a_2 - b_2|$.[8] With this metric, what is the most efficient location pattern and what is the resulting shape of market areas?

Again, I will report the answers in a subsequent issue.

[6]There is no danger that both players will get correlated values of RND(turn); each will have an independent draw.

[7]The program for this strategy is

If turn = 1 then Defections = 0

If ChoiceY(turn − 1) = Defect then Defections = Defections + 1

If Defections ≤ 2 then DecisionX = Cooperate ELSE DecisionX = ChoiceY(turn − 1).

The program starts out cooperating. Defections keeps a running total of how many times the opponent has cheated (or so it appears). Once defections reach two or more, the strategy is to defect forever. Note that the value of defections is initialized only for the first turn; although the IF statement is a somewhat inefficient way of doing this, it works fine.

[8]The Manhattan metric is so named because it measures the distance between two points in Manhattan as measured by a taxi-cab driving along streets and avenues.

Answers to Speed Puzzles

Answer to Puzzle 1

Since all payments are transfers, the average price for a priority level must be zero. What does the average price buy? It buys the average bundle. Given N priority levels, a consumer could buy $1/N$ of each priority level at a total cost of zero. The interpretation of owning $1/N$ of a priority level is that $1/N$ of a consumer's electricity is supplied at that priority. In the event of a shortage which requires, say, a 20 percent rationing, the lowest 20 percent of priorities would be cut off. The consumer with $1/N$ of each priority would then receive 80 percent of his demands. Note that this outcome is precisely what would have occurred before the market for priorities was established. Since each consumer has the option of costlessly maintaining the status quo, the market equilibrium can only improve everyone's welfare.

This result would have been obvious if the question had been posed slightly differently. Imagine that everyone is given equal endowment of $1/N$ of each of the N priority levels. A market is opened and trade is allowed. In the resulting competitive equilibrium, no one can be worse off than at their initial endowment; they don't have to trade! The trick in this question is to recognize the equivalence between equal endowments and the requirement that aggregate of the payments for priorities be zero.

Answer to Puzzle 2

Nick Nichols, at Harvard's John F. Kennedy School of Government, offers two convincing explanations. First, leaded gasoline is used as a loss-leader or fighting grade. Second, consumers of leaded gas appear much more price sensitive than consumers of unleaded gas, especially premium unleaded.

In most states, gasoline stations advertise on large posters the price of their cheapest gas, which historically has been leaded gas. By the time the driver has pulled into the station and then discovers that unleaded gas is significantly more expensive, the commitment to buy has been made. In addition, since leaded gas accounts for only a small proportion of sales (most customers cannot use it), it is the cheapest grade to use as a loss leader or fighting brand to get customers into the station. Of course, this strategy raises the question of why consumers of unleaded gasoline pay any attention to the advertisements of leaded gas prices. To test this hypothesis of the fighting grade, Nichols notes that several states require gasoline stations to post all their prices in equal size lettering. If the fighting grade argument is true, then those states should have smaller price differentials between leaded and unleaded gas. Do they?

Nichols has a second independent reason why unleaded gas should be priced more competitively. Gas stations have a small local monopoly. The stations are able to exploit their customers' cost of searching for another station.[9] But the value of searching varies systematically with the type of gas desired. The primary consumers of leaded gas are truckers and those who drive old gas-guzzling cars. Truckers are buying large quantities and therefore have a greater value of searching. Many of those who drive exprienced autos (like me and my '74 Celica) are providing a credible signal of their greater sensitivity to price. They don't want to pay a lot for cars or for their gasoline. Gasoline stations recognize the greater price elasticity of their leaded gasoline customers and respond with lower margins. In this explanation, leaded gasoline customers may be completely inelastic in their demand for leaded as opposed to unleaded gas—the lead is needed to lubricate their aging engines. The greater elasticity comes from their greater wilingness to search for another station selling cheaper leaded gas. The flip side of this argument helps explain why unleaded premium has such a high mark-up. Those who have invested over $25,000 for their premium automobiles may place a sufficiently high value on their time that they are unwilling to search in the hopes of finding a 5 cent per gallon discount. Further evidence is provided by the discrepancy in diesel gas prices between filling stations located in cities and those on major highways. Highway prices are significantly lower because the primary consumers are truckers. In contrast, the primary consumers of diesel fuel in city stations are people who are driving either Mercedes or Volvo diesel cars.

As a parting thought, what should advertising and search theory imply about the price differential between self-service and full-service gasoline? Although Princeton has many attractions, it is not an ideal place to do empirical work on this question—self-service gasoline is outlawed in New Jersey (and Alaska).

■ *The speed puzzles were just some of the many questions offered as an antipasto over dinner with Richard Zeckhauser. He attributes the electrical pricing problem to a seminar given by Bob Wilson. The location problem was offered as food for thought over dinner with Richard Arnott; he heard it from Ralph Braid. My thanks to all these puzzle masters for their generous contributions.*

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[9] At the entrance to the Holland Tunnel into New York, twelve stations are lined up in a row. At this point, the search costs approach zero and stations lose any local monopoly power. Accordingly, the price differential betwen leaded and unleaded ranges from 1 cent to 3 cents and unleaded premium costs only about a nickel more than the standard unleaded.