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Puzzles
Penny Stocks, Discount Brokers, Better Bidding, and More

Barry Nalebuff

In presenting economic puzzles, I have three goals in mind: some puzzles are chosen to stimulate research; others offer examples that will help undergraduate and graduate teaching; all should provide quality distractions during seminars. This feature begins with reader's comments on the first "Puzzles." Next are several speed puzzles, with answers provided at the end. Here we have two Fischer Black problems in finance and a dot game from Richard Zeckhauser. Following are several longer puzzles, for which readers are invited—nay, challenged—to submit their own answers. This issue's longer problems give you a chance to think about strategies for staying ahead (in sailboat racing to R&D racing) and not losing your head at an auction. The responses will be discussed in a future issue.

Please send your answers and favorite puzzles to: Barry Nalebuff, "Puzzles," c/o *Journal of Economic Perspectives*, Woodrow Wilson School of Public and International Affairs, Princeton University, Princeton NJ 08544. Good luck.

Mail

It was inevitable that my first letter would point out a mistake. The proposed answer to Puzzle 2 ("In Fact, it's a Gas") in the first issue, concluded that "self-service gasoline is outlawed in New Jersey (and Alaska)." Professor Gunnar Knapp at the University of Alaska, Anchorage writes that New Jersey has that distinction all to

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itself. This raises a new puzzle: why is self-service gas outlawed anywhere? Are there any special interest groups that benefit from this legislation? What is special about New Jersey? On the positive side, Professor J. Wilson Mixon from Berry College, Georgia, writes that his econometric tests (Mixon and Uri, 1987) support the hypothesis that the price premium for unleaded gasoline is due to differential price elasticities between consumers of leaded and unleaded gas.

Puzzle 1: A Penny Saved is a Penny Burned?

Imagine that the price of copper rises to the point where the copper value of a 1 cent coin is worth more than a penny. As a result, pennies disappear from circulation and no one misses them.

Your firm uses copper in its production process, and you can melt pennies down and retrieve their copper content at zero cost. At present, you have a six-month stockpile of copper reserves and you have managed to collect one million pennies. Should you melt the pennies down? If so, why, and if not, why not?

Puzzle 2: You Get What You Pay For

An investment adviser offers to manage your portfolio for a fee. In return for her investment expertise, she asks for 10 percent of the excess return (that is, the return above a safe asset, perhaps the short term treasury bill rate). If by chance she underperforms the safe asset, then she will also return to you 10 percent of the amount that she lost (relative to the safe asset).

As an alternative fee structure, the investment counselor offers to accept a flat percentage of the overall portfolio value. What yearly percentage of the portfolio would be equivalent, in your eyes, to her proposed fee of 10 percent of the excess return?

Puzzle 3: This Dot is For You

Any game in which players move in sequence and which always ends after a finite number of moves has a "best strategy." In theory, this best way of playing could be discovered by examining every possible sequence of moves. This approach is relatively easy for tic-tac-toe and impossible (at present) for chess. In the game below, the best strategy is unknown. Yet, even without knowing it, the fact that it exists is enough to show that it must lead to a win for the first player.

Richard Zeckhauser describes the following two player dot-game. The game starts with dots arranged in any rectangular shape. The object of the game is to force your opponent to take the last dot. For example, look at the 7 × 4 array below.

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Each turn a player removes a dot and with it *all* remaining dots to the northeast. In our example, if the first player chooses the fourth dot in the second row, this leaves the second player opponent with

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Each play, at least one dot must be removed. The person who is forced to take the last dot loses.

For any shaped rectangle with more than one dot, the first player has a winning strategy. Although this strategy is known for special cases (including the 7 × 4) a general winning strategy remains to be discovered. How can we know who has the winning strategy without knowing what it is?

Answers to Puzzles 1, 2, and 3 appear following Puzzle 5.

Puzzle 4: Keeping One Step Ahead

After the first four races in the 1983 America's Cup finals, Dennis Conner's *Liberty* led 3-1 in a best out of seven series. On the morning of the fifth race, "cases of champagne had been delivered to *Liberty's* dock. And on their spectator yacht, the wives of the crew were wearing red-white-and-blue shorts, in anticipation of having their picture taken after their husbands had prolonged the United States' winning streak to 132 years."[1] It was not to be.

At the start, *Liberty* got off to a 37-second lead when *Australia II* jumped the gun and had to recross the starting line. The Australian skipper John Bertrand tried to catch up by sailing far to the left of the course in the hopes of catching a wind shift. Dennis Conner choose to keep *Liberty* on the right hand side of the course. Bertrand's

[1]The *New York Times*, September 22, 1983, p. B19.

gamble paid off. The wind shifted five degrees in *Australia II* 's favor and they won the race by one minute and forty-seven seconds. Two races later, *Australia II* won the series.

Conner was criticized for his strategic failure in the race. In sailboat racing, the leading sailboat usually follows the route of the trailing boat, even when the follower is clearly pursuing a poor strategy. Why? Because in sailboat racing, overall speed doesn't count. Only winning matters. If you have the lead, the surest way to preserve the lead is to play monkey see, monkey do.

While you might never get the chance to skipper an America's Cup race, here you have a chance to apply the same principles in a research and development race.

Going into the home stretch of an R&D project, you are six months ahead of the competition. To bring the project to a successful completion requires finishing the development stage. There are two strategies you can pursue, RISKY and SAFE. SAFE takes two years but is guaranteed to work. RISKY takes only one year, but there is a fifty percent chance that it will get you nowhere, in which case you will have to return to the SAFE strategy and take an additional two years.

Six months from now, your competitor will be faced with a similar problem of how to bring their development stage to completion. They too have a SAFE and a RISKY strategy available. Their SAFE strategy takes them two years to reach completion. Their RISKY strategy takes only one year, but has only a fifty percent chance of success. The competitor's chance of success is *independent* of whether your RISKY strategy works. When their RISKY strategy fails, they must turn to their SAFE option and wait an additional two years.

Only the first to completion is awarded the patent. There is no time discounting. Due to limited resources, it is not possible to pursue both strategies simultaneously. Which strategy should you pursue to maximize your chance of winning? Do you want to keep your move hidden? What do you think your competitor will do? How likely are you to win?

This problem differs from the sailboat racing example in one important way. The two sailboats can observe each other. It is relatively straightforward for Dennis Conner to follow John Bertrand. Staying ahead is more complicated when the moves are unobserved; prediction rather than observation is needed. It's your move.

Puzzle 5: Be a Better Bidder

One of the more interesting developments in auction theory is the revenue equivalence theorem.[2] This result demonstrates that a wide range of auction formats all provide the seller with the same expected revenue. Equivalence arises when there is a single indivisible object is for sale, the bidders do not collude, each bidder knows his own valuation of the object but not the value to others, and these private valuations are independently and identically distributed.

[2]Several examples of equivalence were first noticed by Vickrey (1962); the general equivalence theorems are due to Myerson (1981) and Riley and Samuelson (1981).

One type of problem where the revenue equivalence theorem does not apply is when the bidders have a common valuation for the sale object. For example, the value of an offshore oil lease may be the same for all bidders, but they each could have different estimates of this common value. This auction problem is characterized by a "winner's curse" and it is sometimes thought that this curse is what destroys revenue equivalence.[3]

In the problem below, Jeremy Bulow describes an auction where the bidders' valuations are still affiliated (as in the oil example) but where there is no winner's curse. Although each bidder has a private valuation and these values are identically distributed, different auctions result in different levels of expected payments to the seller. Hence it is not just the winner's curse which destroys revenue equivalence.[4]

There are two people bidding. Person 1 values the object at A + B. Person 2 values the object at B + C. The values A, B, and C are all independently and uniformly distributed on $[0, 1]$. Finally, each bidder only knows his total valuation; person 1 only knows the sum of A + B while person 2 only knows the sum of B + C.

It is worth emphasizing that there is no "winner's curse" in this problem. In this auction, both bidders know their own valuation exactly and their estimate is unaffected by the other's valuation. Now consider three types of auctions.

First, a sealed bid auction: the highest bidder wins the object and pays his bid.

Second, a second price sealed bid auction: the highest bidder wins the object but pays only the second highest bid.

Third, an auction where both bidders pays their bid, regardless of who wins, and the highest bidder is awarded the object.

What are the seller's expected revenues from each of these three auction formats? Why do they differ? Can you predict in advance which will work best? How much of the buyer's surplus can the seller get?

Compare these outcomes to a common value auction problem. Let the value to both bidders be A + B + C, where bidder 1 observes only A + B and bidder 2 observes only B + C. Now which auction works best for the seller?

I will report the answers to Puzzles 4 and 5 in a subsequent issue.

Answers to Speed Problems

Answer to Puzzle 1

Melting down copper pennies is never a good idea as long as you have some other source of copper already available. For that matter, it's not important, except for transportation costs, that you have the copper. So long as anyone has copper, if

[3]The winner's curse arises when the high bidder discovers that everyone else had a lower estimation of the sale good's value. This then leads him to lower his expectation of the good's value. Of course, each bidder places his bid accounting for the curse, as his bid counts only if it is the highest. For a more detailed discussion, Richard Thaler describes the anomaly of the winner's curse in this issue.

[4]Milgrom and Weber (1981) provide the now standard treatment of symmetric buyers with affiliated values, and this problem is meant to help illustrate some of their results.

transportation costs are zero, it pays to hoard pennies and not melt them down. Why? If the price of copper stays high or rises further, it won't make any difference whether you melt the pennies or not. If the price of copper falls, however, the price of copper stockpile will decline but the price of the pennies will not fall to less than one cent each. Pennies can be turned into copper, but you can't mint the copper back into pennies. Because the value of pennies has a lower bound of one cent each, they are worth more than their copper value alone. Once you have melted the pennies, this surplus is lost. If the choice is between melting already-purchased pennies or buying more copper, and copper costs more than pennies, then pennies might be worth melting for immediate use. But as long as the copper stockpile last, it is always better to save or sell the pennies.

This problem is just a special case of Bob Merton's (1973) result that it never pays to exercise early a stock option that doesn't pay dividends. Pennies are like copper with a free option.

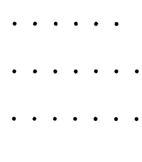
Answer to Puzzle 2

The broker who is willing to accept 10 percent of the *excess* return (positive or negative) provides her services for *free* in a world where one can borrow and lend at the risk-free rate. One way of understanding this result is to recognize that 10 percent of the excess return on the entire portfolio is equivalent to all of the excess return on 10 percent of the portfolio. So from the investor's point of view, the offer of the broker to take 10 percent of excess return on a $100 investment is equivalent to putting $90 into the market and $10 into risk-free Treasury bonds. If you would have preferred putting all $100 into the market, then borrow $11 and give the broker $111. After she takes the 10 percent of the excess return, she will give you the equivalent of the risk-free rate on $11 (which you use to pay back your loan) and the market return on the other $100. Of course, in this world, the broker doesn't need your money. She could just as easily borrow money from the bank and offer to keep 100 percent of the excess return, leaving them with just the risk-free rate that they require.

The broker's services are costly to the extent you cannot borrow at the risk-free rate. But as long as you are not up against that constraint, her services are free under the first scheme.

Answer to Puzzle 3

If the second player in the game has a winning strategy, that means that for *any* opening move of the first player, the second has a response that puts him in a winning position. Imagine that the first player takes just the upper right hand dot.



No matter how the second player responds, the board will be left in a position that the first player could have chosen as a first move. If this is truly a winning position, the first player should have and could have opened the game this way. There is nothing the second player can do to the first that the first player cannot do unto him beforehand.

■ *The first two speed puzzles are adapted from a course Fischer Black taught at MIT (15.423). Thanks to Pete Kyle for reminding me of two favorites. Problems 3 and 4 are adapted from my forthcoming book with Avinash Dixit,* Strategic Thinking and Action. *Jeremy Bulow deserves the credit for Problem 5.*

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