Price Discrimination and Welfare

A Comment on *Tying, Bundled Discounts, and the Death of the Single Monopoly Profit* by Einer Elhauge

Barry Nalebuff

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I. Introduction

Elhauge (2009)[1] provides a wide-ranging article that is impressive both in its clarity and its holistic attack on the practice of bundling and tying. In this commentary, I will focus my attention on one aspect of his presentation, namely the effect of price discrimination via metering and tying on consumer welfare and total welfare.

Elhauge makes the claim that we should not suppose that the total welfare effects of price discrimination are positive. Even if they are, he suggests that this perspective is too narrow; a price-discriminating monopolist will make more money and so may incur greater ex ante costs to secure its market position. And if total welfare still rises after taking these costs into account, Elhauge makes the further argument that antitrust is and should be focused on consumer welfare, not total welfare. In that domain, the presumption should be that price discrimination lowers consumer welfare.

The first claim that (what Elhuage calls ex post) total welfare goes down may be surprising since it runs counter to the intuition that comes from first-degree or perfect price discrimination. Perfect price discrimination is typically thought to achieve the efficient outcome and therefore it raises total welfare. As I discuss below, I think that perspective is too simplistic, as it ignores the real costs associated with implementing a price discrimination system. But, putting that issue aside, it is easy to see why there is a presumption that imperfect price discrimination moves total welfare in the same direction as perfect price discrimination. Elhauge argues that this intuition is unfounded.

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This comment provides some models and examples that challenge Elhauge's argument for the case of metering and tying. My primary focus is on the given market structure and thus I do not consider how price discrimination may change the ex ante competition.[2] Using his framework, I redo one of his models using continuous rather than discrete variables. This simplifies the mathematics and allows me to provide conditions under which price discrimination via tying will raise total welfare. I show that total welfare rises in a more general version of his model. I also show that a small amount of price discrimination generally increases total welfare in a model with linear demand. While it is certainly possible for price discrimination via tying to lower total welfare, the results here suggest why this might be the exception to the rule for the case of tying.

> WHILE IT IS CERTAINLY POSSIBLE FOR PRICE DISCRIMINATION VIA TYING TO LOWER TOTAL WELFARE, THE RESULTS HERE SUGGEST WHY THIS MIGHT BE THE EXCEPTION TO THE RULE FOR THE CASE OF TYING.

Turning to consumer welfare, here there is more support for Elhauge's presumption that price discrimination is harmful to consumers. Consumer welfare falls in the first set of models and the effect is ambiguous in the second set.

It helps to distinguish between the types of imperfect price discrimination, namely second- and third-degree price discrimination. Under third degree price discrimination, a monopolist can charge a different price to different groups based on some exogenous identifying feature. Thus a firm might offer a lower price to senior citizens (or a higher price to non-senior citizens) that reflects different price elasticities. An early example of such price discrimination (discussed by Pigou[3]) is that the English railroads charged a higher price for transporting copper compared to coal. The service provided by the railroad was the same, but copper had a higher value and thus the producers could be charged more. The cost of this type of price discrimination is that it may lead the monopolist to price some of the highest-value customers out of the market. In the case of a linear demand, only half the highest-value customers will be served. This inefficiency lowers consumer welfare and might lower total welfare.

My first result presents a model in which the total welfare effect of third-degree price discrimination is unambiguously positive. The model is a variation of the model presented by Elhauge, the primary difference being that consumer types are continuous rather than discrete and have no minimum demand. This ends up making an important difference as it implies that absent price discrimination, the monopolist will choose to exclude some consumers from the market. Once the monopolist charges a high enough price to exclude some customers, raising the price further becomes even more attractive as there are no more of these customers to be lost. It is this positive feedback that leads to lower total welfare under the single monopoly price.[4] One surprising result is that total welfare rises under price discrimination even though output (as measured by the tied or metered good) falls.

Under second-degree price discrimination, the firm can't target customers by their type, only by their actions. Thus a monopolist might offer a quantity discount, a Saturday-night stayover discount, or a tied-in sale (such as overpriced ink) to capture more of the surplus from high-value customers and expand the market to low-value customers.[5] While the monopolist charges more to high-value customers, the higher price isn't so much more that they end up being excluded from the market. Because high-value customers can always act like low-value customers, they have the option of taking the same price/quantity choice as the low-value customers. Were they to do so, they would get higher surplus than the low-value types and hence the high-value customers are more likely to participate in the market. While second-degree price discrimination will generally keep the highest-value customers in the market, their demand will be curtailed and so there will be some inefficiency.

> ONE SURPRISING RESULT IS THAT TOTAL WELFARE RISES UNDER PRICE DISCRIMINATION EVEN THOUGH OUTPUT (AS MEASURED BY THE TIED OR METERED GOOD) FALLS.

The main result I show is that a small amount of metering will lead to an increase in total welfare under a reasonably general set of conditions. The result is general in that it doesn't depend on the distribution of consumers in the market. It is limited in two important regards. First, the result depends on a linear demand specification. Second, the result is only for a small amount of metering (or tied-in sales). A monopolist would generally like to do more than a small amount of price discrimination and the result is silent as to the impact of the profit-maximizing amount of price discrimination.

I have two reasons for looking at this case. Firms may be limited in amount of metering, and thus the impact of small amounts of price discrimination is relevant. Second, the result provides some intuition for why imperfect price discrimination via metering may more generally raise total welfare. We know that price discrimination initially raises welfare and so any subsequent negative impact must be large enough to counter the initial gains.

Why might price discrimination be limited in size? Take the case of ink cartridges. HP can sell them at a price premium, but not without limit. At some point, users can and will figure out how to refill cartridges or buy them on the black market. The ability to enforce a tie is limited.

My starting point is a model in which the welfare effects are unambiguous: Price discrimination increases total welfare and hurts consumers. The weaknesses of this model are clear and it is designed to be a jumping off point for more realistic extensions.

II. Baseline Model

Consumers buy a base good for the purpose of using it to make some exogenous output amount n, where n varies by consumer. The value of each unit of output is v, where v is equal across all consumers. The base good and its output are both produced at zero cost.

Absent price discrimination, the firm has to charge a single price p for the base good. Consumers buy the good if

$$n \geq p/v.$$

Let $F()$ represent the cumulative distribution of n. In the case where F is uniform on $[0, 1]$,

$$\Pi = p * [1 - \frac{p}{v}].$$

The profit-maximizing price is $p=v/2$ and only half the consumers (those with $n \geq 1/2$) purchase the good. More generally, the first-order condition is always positive at $p=0$. Thus some consumers will be excluded from the market and the monopoly price is inefficient.

In contrast, if the monopolist is able to meter or engage in a tied-in sale, the monopolist will set a price of v per unit of output and provide the base unit for free to all customers. The net result is perfect price discrimination. Consumers end up with zero surplus, and the result maximizes total welfare.

This model is a special case in that price discrimination is usually imperfect and thus unable to achieve a fully efficient outcome. Even though it is a special case, this model provides some intuition for the claim that second-degree price discrimination (and metering in particular) will typically increase total welfare and decrease consumer welfare. Firms might not be able to achieve the perfect result, but as long as they get close, the results will be directionally the same.

Elhauge argues that this model is such a special case that any intuition drawn would be misleading. In particular, we should not use this model to draw the presumption that price discrimination increases total welfare. Clearly the model is a special case. A first question to consider is whether it does a good job representing *any* real-world model of preferences. The surprise is that this obviously stylized model does a good job describing the facts in *Independent Ink v. Trident*.[6]

THE SURPRISE IS THAT THIS OBVIOUSLY STYLIZED MODEL DOES A GOOD JOB DESCRIBING THE FACTS IN *INDEPENDENT INK V. TRIDENT*.

Trident manufactures a proprietary printer head that is typically used for high-speed printing. Their printer head might be used to date-stamp boxes along a production line—for example, the sell-by date on a carton of beer. The cost of this printing is truly insignificant when measured as part of the total production

cost. A beer manufacturer is unlikely to adjust its price of beer in response to the cost of ink used to print the sell-by date. Thus it is reasonable to assume that the number of boxes stamped (and hence the demand for ink) is exogenous, at least from the perspective of Trident.

How much is the manufacturer willing to pay for the use of Trident's printer head (and contractually provided ink)? The customer compares the unit cost of Trident's product to that of a rival technology. To the extent that the unit cost of Trident's product is v lower than that of a rival, the customer would be willing to pay v per box stamped.[7] Customers that are using similar production technologies would likely have similar cost savings per unit. Thus it seems like a reasonable first approximation to consider the case where the units demanded n varies exogenously across customers, while the value of each unit is equal to v. In such a world, it is not surprising that the manufacturer would seek to engage in price discrimination via metering or a tied-in sale.

There are two quite restrictive assumptions in the baseline model. The first is that the value of each output unit is constant across all consumers. While that may describe some applications, we should consider how the results change when the value of the output varies across the population. Thus in Model I we retain the assumption that the total demand by a customer of type n is exogenous and equal to n. But the value of each of those n units will vary across the population. For example, the production line for beer might move slower than one for soda and so the incremental value of the Trident printer head could be lower.

A second restrictive assumption in the baseline model is that all units have a constant incremental value up to the exogenous demand n. There is no declining marginal utility of consumption. While that may be appropriate to a commercial application like Trident, for many consumer applications we expect the incremental value of consumption to decline. The analysis of the case with declining marginal utility is presented in Model II.

III. Model I

As before, a consumer buys a base good for the purpose of using it to make some output. Thus a printer has no utility of its own other than through making copies, or a car has no utility other than through the miles it is driven. For clarity of exposition, we will continue to use the example of a printer as the base good and copies as the output.

Different customers have different levels of utility for copies. The value of each copy is distributed across the population uniformly over $[0, A]$. At a price of c per copy, customers with per-copy values over $[c, A]$ would want to make copies, but the surplus created may not justify the purchase of the printer. If the printer is

sold at a price of p, then the customer with value v who has demand for n copies will buy the printer provided

$$n(v - c) \geq p.$$

Finally, the number of copies demanded, n, is distributed uniformly over $[0, \bar{N}]$.

Absent metering or tied-in sales, the monopolist is forced to charge the competitive price (here 0) for the copies. We compare this outcome to the scenario where the monopolist is able to charge the consumer a per-copy fee of c. This may be done via direct metering or via the required purchase of a tied-in good such as ink or a cartridge at an above-market price.

This setup is almost identical to the model considered by Elhauge. The primary difference between our approaches is that he considers the case where the number of copies demanded is discrete ($n=1, 2, 3, \ldots$). Clearly it is the case that copies are not truly divisible. The assumption that the demand for copies is continuous is a convenience that greatly simplifies the mathematics. For the most part, this assumption has little impact on the results except for the case where the market is concentrated on small n. Even here one should think of copies as being measured in units of 2,000, where the copies are sold via proprietary toner cartridges.[8]

Unlike the analysis of Elhauge, in the continuous model we find that the comparison result does *not* depend on the range of copies demanded. Our results are unchanged if n is uniform over $[0,1]$ or $[0,2]$ or $[0,100]$, while his results vary based on whether there are just two groups (who want either 1 or 2 copies) or more groups, say four, who want either 1, 2, 3, or 4 copies.

Theorem 1: For all $\bar{N}$, Total Surplus is higher under price discrimination than under the single monopoly price; the gain is 4.9 percent.

The proof for this result and all following theorems and corollaries are presented in a mathematical appendix.

Corollary: Base-unit demand increases by forty percent under price discrimination, while the total demand for the complementary product decreases by two percent.

At first glance, it might seem peculiar that total welfare goes up even while demand falls, albeit by only 2 percent. Consumers only care about the base unit (printer) for the use of the complementary product (copies). Thus it would seem that the increase in total welfare combined with a decline in relevant output contradicts the results from Varian[9] and Schmalensee[10] that an increase in output is a necessary condition for welfare to rise under 3rd-degree price discrimination.

The explanation is not due to a difference in the type of price discrimination. While the present model is set up to be 2nd-degree price discrimination, this is a special case where the results of the two types of discrimination coincide. Instead

of charging group "n" a price of $A/2$ per copy, the group could be charged a fixed price of $nA/2$. The same set of customers would buy printers and the same number of copies would be made. The explanation is due to the fact that customers are buying different quantities, which takes us outside of the Schmalensee/Varian framework.

Price discrimination expands the set of high-value low-quantity customers. Consider the case where $\bar{N}=36$, which results in a monopoly price p just above $10A$. Absent price discrimination, customers with the highest value per copy (A) and a demand for only nine or ten copies are excluded from the market. It is better to include both these two customers and have them get 19 copies together (a welfare addition of $19A$) rather than sell thirty copies to the one customer who values each copy at $0.4A$ each. The latter customer was willing to buy the printer at a price up to $12A$ and so purchased the printer absent price discrimination. Under price discrimination, her value per copy is below $A/2$ and so she (and her 30 copies) are excluded from the market, for a welfare loss of $12A$. The net gain here is $7A$, although total copies purchased falls from 30 to 19.

The numbers work out easier with high values of $\bar{N}$, but that is not essential to the argument. Absent price discrimination, consumers are ranked by the product of their value per copy (v) times n, or $v*n$. Total welfare can't go up with the same number of customers buying fewer total copies, as the welfare-per-customer is maximized under one price. Once we allow the set of customers to expand in number, then it is welfare enhancing to include a large number of customers with high valuations per copy and a low demand for copies. Essentially, that is what happens under price discrimination. The customer base expands by 40% and the average value per copy rises. The total number of copies purchased falls slightly under price discrimination, but not enough to offset the gain in average value.

Theorem 2: For all $\bar{N}$, Consumer Surplus is lower under price discrimination than under the single monopoly price; the loss is 18.7 percent.

While the result on consumer surplus is in accord with the finding of Elhauge, the total surplus result is similar, but not identical. Elhauge finds that total surplus is lower under price discrimination for $\bar{N} = 2$ and 3 and higher for $\bar{N} \geq 4$.

There are two reasons for the differences in total welfare results. The first is that in the continuous distribution of types leads to somewhat different mathematics than the discrete case. The second difference is that in Elhauge's model, the lowest value of n is 1, not 0. Putting these elements together means that with a continuous distribution of n between $[0, \bar{N}]$, at any positive price charged by the monopolist there will be some group that is on the margin of being entirely excluded from the market (specifically, the group with $n=p/A$.) In contrast, with a discrete number of groups, the monopolist may not be on the margin of losing a group. Raising the price will reduce the fraction of each group that buys the

printer but may not change whether an entire group is excluded from the market or not.

The potential to exclude a group gives the single-price monopolist a greater incentive to raise price and this further reduces total surplus. The reason is that once the group has been excluded, an additional increase in price loses fewer consumers and this makes price increases more attractive on the margin.

THE POTENTIAL TO EXCLUDE A GROUP GIVES THE SINGLE-PRICE MONOPOLIST A GREATER INCENTIVE TO RAISE PRICE AND THIS FURTHER REDUCES TOTAL SURPLUS.

Consider the solution to Elhauge's model when $A=200$ and $\bar{N} = 4$. There are two candidates for an equilibrium. Under the assumption that all four groups are served in the market, the optimal price ($192) does indeed lead to positive demand from all four groups.[11] The maximum willingness to pay per unit is $200, and so even the customer group with demand for a single unit has some positive demand at a price of $192. Consumer welfare is $84,800, profits are $76,800, and total welfare is $161,600.

The issue is that the fixed-point argument is only a necessary and not sufficient condition for profit maximization. There is another fixed-point solution under a different assumption and one that leads to higher profits. Under the assumption that the customers with demand of 1 unit are all excluded, the profit-maximizing price becomes $277, and so the group with $n=1$ is indeed excluded.[12]

As the monopolist raises its price from $192 to $200, this reduces profits (as $192 is the profit-maximizing price when all four groups are being served). However, once the monopolist raises its price above $200, there are no more customers from group one to be lost. This makes further price increases more profitable and leads the monopolist to go all the way up to $277. At $p=\$277$, profits are higher at $83,077. The resulting consumer welfare is $55,392 and total welfare is $138,468. Total welfare under price discrimination is higher at $150,000, while consumer welfare is lower at $50,000.

Our results are the same once $\bar{N}$ is large. This is not because the continuous case is the same as large $\bar{N}$. Rather, the fact that Elhauge's model uses 1 as the lowest value rather than 0 becomes much less important when $\bar{N}$ is large. Elhauge finds that price discrimination lowers welfare for $\bar{N} = 2$ or 3.[13] (The case with just one type leads to the same result with or without price discrimination.) The reduction in total welfare is a result of the assumption that the minimum number of copies demanded is 1 unit (where a unit represents a cartridge or 2,000 copies). Because of the minimum, even with a continuous distribution the monopolist finds that no consumer groups are on the margin. For example, with n distributed uniformly on [1, 3], the monopolist would set a price of $A/\ln(3)=0.91A$, a price that leads to strictly positive demand from the $n=1$ customer group.

The primary reason why price discrimination raises welfare is that it allows half the consumers of each type to be served, including half the low-value types who might be excluded under a one-price monopoly. If there are no low-value types to be excluded, price discrimination will lead to lower welfare. A necessary condition for total welfare to increase is that price discrimination expand the base-unit sales. Thus price discrimination is put at a disadvantage when the minimum type is $n=1$ rather than $n=0$. Once $\bar{N}$ becomes large enough (3.51), some groups will be excluded and total welfare soon thereafter is higher under price discrimination.

Theorem 3: Assume that n is distributed uniformly over $[1, \bar{N}]$. For $\bar{N} > 4.58$, total surplus is higher under price discrimination.

Our results above all rely on a uniform distribution on n over $[0, \bar{N}]$ or $[1, \bar{N}]$. It is possible to make some progress with a more general distribution. For any continuous and positive distribution of n over $[0, \bar{N}]$, price discrimination will increase total sales of the base unit. Of course, this is only a necessary and not sufficient condition to improve total welfare.

Theorem 4: Under the assumption that n has continuous support over $[0, \bar{N}]$, the one-price monopolist restricts sales of the base unit relative to the price discrimination case.

From these results so far, I take away the presumption that price discrimination via metering raises total welfare and lowers consumer welfare. It is possible that price discrimination leads to lower total welfare. This typically arises when there is some minimum demand type and the single-price monopolist chooses to serve all consumer groups in the market.

> FROM THESE RESULTS SO FAR, I TAKE AWAY THE PRESUMPTION THAT PRICE DISCRIMINATION VIA METERING RAISES TOTAL WELFARE AND LOWERS CONSUMER WELFARE.

Even if total welfare usually rises, the fact that total welfare might fall under price discrimination is icing on the cake for Elhauge. A presumption or even a demonstration that total welfare rises would not create an antitrust defense. Elhauge is quite clear and explicit that antitrust law is based—and should be based—on consumer welfare, not total welfare. Thus his argument against price discrimination, tied sales, and metering does not depend on this leading to a reduction in total welfare. But others may disagree on how the law is and how it should be in terms of evaluating consumer welfare versus total welfare. I will return to this issue in the conclusion.

I turn now to a model in which customers experience a declining marginal value of output. The results from this model provide further support for the presumption that price discrimination via tying and metering raises total welfare. The effect on consumer welfare is ambiguous.

The main result is the following: Given preferences that lead to linear demands with different intercepts, a small amount of price discrimination always

raises total welfare. This is a general result in that it does not depend on the specific distribution of consumer preferences.

While the result does not tell us what happens when the amount of price discrimination is chosen to maximize monopoly profits, it helps us appreciate that price discrimination at least starts off on the right foot in terms of increasing total welfare. The effect of a small amount of price discrimination is more relevant than it might at first appear because firms may be limited in how much metering or tying they can practically accomplish.

The case of ink or toner cartridges is again instructive. HP might like to completely restrict others from selling toner cartridges that are compatible with their laser printers. They have designed cartridges with a patented shape that blocks entry into the new cartridge market. They have added a chip to some cartridges that detects when the ink is empty and prevents the printer from working even if the once empty cartridge has been refilled. However, if the price of the cartridge gets too high, entrants will find ways to collect and remanufacture the spent cartridges. At some point, the savings become too large and buyers find a way to avoid paying the large mark up.

The case of *Independent Ink v. Trident* also illustrates this issue. According to Independent Ink, Trident was selling refills for a price of \$325.[14] This was sufficiently high that buyers were willing to break their contractual agreement with Trident and buy refills from Independent Ink, who could profitably sell refills at \$125 to \$189. The cost of monitoring and enforcing the contracts with each buyer is substantial. From Trident's perspective it was cheaper to go after the rival manufacturer than each customer.

This cost of monitoring and enforcement may limit how much a monopolist can raise the price of the tied-in product above the competitive level. For that reason, we are interested in the welfare impact of a small amount of price discrimination. The fact that tied-in sales initially improve total welfare may present a defense to those who seek to defend tying on the grounds that it is efficient.

IV. Model II

A consumer buys a base good for the purpose of using it to make some output. Again, for clarity of exposition, we will use the example of a printer as the base good and copies as the output.

Different customers have different levels of utility for copies. Even a customer with a high value of copies will not value all copies equally. For simplicity, we assume that a customer of type a values the qth copy at $a - q$. Here the number of copies is assumed to be a continuous variable. Thus if copies are priced at c, the consumer of type a will demand

$$D(a, p) = a - c$$

copies, if that consumer has bought a copier. Without loss of generality, we assume that the marginal cost of copies is zero and so the price per copy should be interpreted as a markup over cost.[15]

The price per copy is best thought of as a metering device or tied-in sale. For example, if the printer monopolist forces the customer to use its special ink, then we can calculate c as the implied price of the ink per standardized page of printing. While toner cartridges are typically sold in packages of 2,000 copies, over the lifetime of the printer this integer problem should not be an important factor. It might also be possible for the monopolist to meter output directly, either through a counter on the printer or the odometer of a car (as is done with car leases).

The printer is sold for a fixed price, p. Given a base price of p and a per-unit cost of c, a consumer of type a will only buy the printer if her total surplus is weakly positive. The total surplus of consumer type a is $(1/2)(a - c)(a - c) - p$. Thus consumer of type a will make the purchase provided

$$a \geq \sqrt{2p} + c.$$

Total profits for the monopolist consist of the profits from the base sales along with the profits from the tied-in sales. Profits from the sale of each base good are $p - \mu$, where μ is the constant production cost of each base unit. Total profits are thus:

$$\Pi = \int_{a \geq \sqrt{2p} + c} [p - \mu + c(a - c)] f(a) \, da$$

where consumer preferences are distributed according to the atomless density function $f(a)$.

Total surplus is

$$TS = \int_{a \geq \sqrt{2p} + c} [\frac{a^2 - c^2}{2} - \mu] f(a) da.$$

Absent a tied sale, the monopolist is constrained to the competitive price (0) for the tied-in product. Thus the single-price monopolist maximizes profits subject to the constraint that $c=0$. We denote this profit maximizing price by $p(0)$. More generally, let $p(c)$ be the profit-maximizing base price when the monopolist charges c for the tied good and correspondingly $\Pi(p(c), c)$ are the profits, and $TS(p(c),c)$ is the total surplus.

Theorem 5:

$$p'(c) < -\sqrt{2p}$$

$$d\Pi(p(c), c)/dc > 0$$

$$dTS(p(c), c)/dc > 0$$

The formal proof is in the appendix. It is important to note that the results do not depend on the distribution $f(a)$ other than the requirement that the maximization problem is quasi-concave.

When the monopolist is allowed to engage in a small amount of price discrimination via a tied-in sale, it will strictly want to take advantage of this opportunity: profits go up. Because incremental sales are now more valuable, the monopolist will lower the price of the base good. The base price falls enough so that the net result is a lower price to the marginal consumer, thus expanding total demand and increasing total welfare. (It is also the case that each consumer buys slightly less of the tied-in good, however this has no welfare loss as the incremental unit lost has almost no value.) The total change in consumer surplus is ambiguous. The increased price for the tied-in product is harmful, but the reduction in base price might lead to an overall gain for consumers.

V. Conclusions

Although the models in this paper suggest that price discrimination via metering or tied-in sales will typically increase total welfare, these economic models miss many of the most important costs associated with price discrimination. Price discrimination is not free. Firms spend a large amount to implement the tying practices and consumers spend resources to avoid them. For example, HP spends resources to design a proprietary shape for its toner cartridge and further resources to ensure that spent cartridges cannot be refilled. Trident spends large amounts enforcing its customers' contractual obligations to buy its expensive ink. These costs are inefficiencies that are typically left out of the model.[16]

EVEN WHEN TYING LEADS TO HIGHER TOTAL WELFARE, WE SHOULD RECOGNIZE THAT THE PRIMARY SOURCE OF THE GAIN IS THAT THE MONOPOLIST IS BEING LESS INEFFICIENT.

Further, tying may impose collateral damage costs on the tied-good market. The forced tied sale may make the complementary market less competitive. This could make it more difficult for others to enter the monopolized base-good market.

Even when tying leads to higher total welfare, we should recognize that the primary source of the gain is that the monopolist is being less inefficient. While a gain is a gain is a gain, there is something different about a firm increasing total welfare by inventing a better mousetrap versus being a less inefficient monopolist. We want to reward firms for making better products, and not for becoming better monopolists.

Section II of the Sherman Act prohibits a firm from monopolizing any part of trade or commerce. To the extent that the firm engages in price discrimination, it becomes a more powerful monopoly. Thus even a firm that has earned its

monopoly by being the best product on the market starts to cross the line and monopolize the market when it engages in price discrimination.

A similar perspective is provided in the U.S. DOJ/FTC, Horizontal Merger Guidelines (1992, revised 1997): "The unifying theme of the Guidelines is that mergers should not be permitted to create or enhance market power or to facilitate its exercise." It seems clear that price discrimination is an enhancement of a firm's market power.

While we have been focused on total welfare and consumer welfare, there is no requirement that we pick either as the antitrust standard. These are two extremes. Total welfare equals profits + consumer welfare. More generally, we can look at a weighted sum of profits and consumer welfare. If we place equal weight on profits and consumer welfare, the result is a total welfare standard. If we place zero weight on profits, the result is a consumer welfare standard. In between, there is an infinite range of options. For example, the courts could weight consumer welfare twice as high as profits.

Before we condemn tying, we should reflect on the fact that the same results can be achieved via metering as with tying. Instead of requiring the consumer to use its overpriced ink, the monopolist could simply charge a price per copy. While metering is legal, for a firm with market power, tying is *per se* illegal. Since the effect on consumers and total welfare is the same, there is the question of why the law treats these two cases differently.[17]

One solution would be to harmonize the law to make tying legal or at least subject to a rule-of-reason test. Alternatively, one could make metering (by a firm with market power) subject to antitrust. It could be seen as a violation of the Sherman Act, Section II, as it allows a firm with market power to further monopolize the market.

Elhauge's response (section IV. A) is to emphasize that firms may find it more difficult or expensive to engage in direct metering than a forced tie. He observes that direct metering is uncommon. If it turns out that tying is the easier method of engaging in price discrimination, there is no reason to facilitate this practice.

Of course, there are efficiency reasons to employ metering besides price discrimination, the primary one being risk sharing. A buyer who is unsure if the service will work or the quantity it will require might prefer to pay on a per-unit basis rather than a single upfront price.

THERE ARE AT LEAST TWO ADVANTAGES OF DIRECT METERING OVER TYING.

There are at least two advantages of direct metering over tying. One is that the effect is transparent. A buyer may not appreciate what the true cost is per use when the base good comes with a tied-in sale. For example, many buyers may not know the implied per copy charge that comes with each printer due to the markup on ink

and proprietary toner cartridges. A second concern is that the forced tied sale may make the complementary market less competitive.[18] This could make it more difficult for others to enter the monopolized base-good market.

While I have argued that tying may typically lead to improved social welfare, I want to reiterate that this is not a legitimate justification according to Elhauge (section IV. C).[19] In that light, I hope that this comment will help focus the debate. The reasons to condemn tying are in spite of, not because of, its potential to improve total welfare.

VI. Appendix

Theorem 1: For all $\bar{N}$, Total Surplus is higher under price discrimination than under the single monopoly price; the gain is 4.9 percent.

Proof: Recall that a consumer of type n is interested in n units of output. The value of each unit is constant and distributed in the population uniformly between 0 and A. Thus the total demand from type n customers at price p is $(1/A)(A - p/n)$.

Costs are zero for both the base good and the tied good. Thus at price p, total profits are

$$\Pi = \int_{\frac{p}{A}}^{\bar{N}} \frac{p}{A}[A - \frac{p}{n}]f(n)\,dn.$$

Profits are maximized when

$$\frac{d\Pi}{dp} = \int_{\frac{p}{A}}^{\bar{N}} [1 - \frac{2p}{An}]f(n)\,dn = 0.$$

The uniform distribution of n allows us to replace $f(n)$ by f and then integrate to find a closed-form solution.

$$\frac{d\Pi}{dp} = \{[\bar{N} - \frac{p}{A}] - \frac{2p}{A}[\ln(\bar{N}) - \ln(\frac{p}{A})]\}f = 0.$$

Define $z = \bar{N}A/p$.

$$\frac{d\Pi}{dp} = \frac{p}{A}f\{z - 1 - 2\ln(z)\} = 0 \Rightarrow z \approx 3.51.$$

Thus we have the general solution for p:

$$p \approx \frac{\bar{N}A}{3.51}.$$

At this price, roughly 72 percent of the highest-value customers are served in the market. Overall, only 36 percent of customers are served.

Turning to Total Surplus:

$$TS = \int_{\frac{p}{A}}^{\bar{N}} \frac{n}{A} \frac{A + \frac{p}{n}}{2} (A - \frac{p}{n})f(n)\,dn$$

$$= \int_{\frac{p}{A}}^{\bar{N}} \frac{n}{2A} [A^2 - (\tfrac{p}{n})^2] f(n) dn$$

$$= \frac{1}{4} \int_{\frac{p}{A}}^{\bar{N}} [2nA - \frac{2p}{n} \frac{p}{A}] f(n) dn.$$

Using the first-order condition for p, we can substitute A for $2p/n$ in the integral.

$$TS = \frac{1}{4} \int_{\frac{p}{A}}^{\bar{N}} [2nA - p] f(n) dn$$

$$= \frac{1}{4\bar{N}} \{A[\bar{N}^2 - (\tfrac{p}{A})^2] - p(\bar{N} - \tfrac{p}{A})\}$$

$$= \frac{\bar{N}A - p}{4}$$

$$p = \frac{\bar{N}A}{3.51} \Rightarrow TS_{NoPD} = \frac{\bar{N}A * (1 - \frac{1}{3.51})}{4}$$

When the monopolist is allowed to engage in price discrimination, it charges a positive price per tied good (cartridge). Since all groups are identical, the monopolist would charge the same price per cartridge to each group. Profits are maximized when the monopolist sells to half the consumers at a price of $A/2$. In that case, the customers who buy have an average valuation of $3A/4$ and buy $\bar{N}/2$ units:

$$TS_{PD} = \frac{1}{2} \frac{3A}{4} \frac{\bar{N}}{2} = \frac{3A\bar{N}}{16} \qquad .$$

It then follows that total surplus is almost 5 percent higher under price discrimination:

$$\frac{TS_{PD}}{TS_{NoPD}} = \frac{3/16}{(1 - \frac{1}{3.51})/4} \Rightarrow \frac{3}{(4 - \frac{4}{3.51})} = 1.049.$$

Corollary: Base-unit demand increases by forty percent under price discrimination, while the total demand for the complementary product decreases by two percent.

Proof: Absent price discrimination, demand for base units is

$$D(p) = \frac{1}{2} \int_{\frac{p}{A}}^{\bar{N}} f(n) dn = \frac{\bar{N} - \frac{p}{A}}{2\bar{N}} = 0.358.$$

With price discrimination, demand for base units is 40 percent higher at ½.

Absent price discrimination, the number of copies sold is

$$\int_{\frac{p}{A}}^{\bar{N}} \frac{n}{A} [A - \frac{p}{n}] f(n) dn = \int_{\frac{p}{A}}^{\bar{N}} [n - \frac{p}{A}] f(n) dn$$

$$= \frac{1}{2\bar{N}} [\bar{N} - \frac{p}{A}]^2 = \frac{\bar{N}}{2} [1 - \frac{p}{\bar{N}A}]^2 = \frac{\bar{N}}{2} * 0.511.$$

With price discrimination, half the customers are served. Since the average customer demands $\bar{N}/2$ copies, the number of copies sold is 2.29% lower:

$$\frac{\bar{N}}{2} * \frac{1}{2} \,.$$

Theorem 2: For all $\bar{N}$, Consumer Surplus is lower under price discrimination than under the single monopoly price; the loss is 18.7 percent.

Proof: Under price discrimination, half the consumers in each group buy and get $3A/4 - A/2 = A/4$ of surplus on $\bar{N}/2$ units. Thus total consumer surplus is

$$CS_{PD} = \frac{1}{2} \frac{A}{4} \frac{\bar{N}}{2} = \frac{A\bar{N}}{16} \,.$$

Absent price discrimination, consumer surplus is

$$CS_{NoPD} = \int_{\frac{p}{A}}^{\bar{N}} \frac{n}{A} \frac{A - \frac{p}{n}}{2} (A - \frac{p}{n}) f(n) dn$$

$$= \int_{\frac{p}{A}}^{\bar{N}} \frac{n}{2A} (A - \frac{p}{n})^2] f(n) dn$$

$$= \frac{1}{2A} \int_{\frac{p}{A}}^{\bar{N}} (nA^2 - 2pA + \frac{p}{n} p) f(n) dn$$

$$= \frac{1}{2} \int_{\frac{p}{A}}^{\bar{N}} (nA - 2p + \frac{p}{n}\frac{p}{A}) f(n) dn \,.$$

Using the first-order condition for p, we have

$$CS_{NoPD} = \frac{1}{2} \int_{\frac{p}{A}}^{\bar{N}} (nA - \frac{3}{2}p) f(n) dn$$

$$= \frac{1}{4\bar{N}} [A(\bar{N}^2 - (\frac{p}{A})^2) - 3p(\bar{N} - \frac{p}{A})]$$

$$= \frac{1}{4} [A\bar{N} + 2\frac{p^2}{A\bar{N}} - 3p] \,.$$

Given that $p \approx \bar{N}A/3.51$, consumer surplus is cut by 18.7 percent under price discrimination:

$$CS_{NoPD} \approx \frac{A\bar{N}}{4} [1 + \frac{2}{3.51^2} - \frac{3}{3.51}] = \frac{A\bar{N}}{13}$$

$$\frac{CS_{PD}}{CS_{NoPD}} = \frac{\frac{A\bar{N}}{16}}{\frac{A\bar{N}}{13}} = \frac{13}{16} = 1 - 0.187 \,.$$

Theorem 3: Assume that n is distributed uniformly over $[1, \bar{N}]$. For $\bar{N} > 4.58$, total surplus is higher under price discrimination.

For $p \le A$, all types have positive demand, and the profit-maximizing price satisfies:

$$p = \frac{(\bar{N} - 1)A}{2\ln(\bar{N})} \,.$$

It then follows that $p \leq A$ only if $\bar{N} \leq 3.51$. (It can be checked that $p > A$ are not profit maximizing.) For $\bar{N}$ in this range, total surplus will be higher absent price discrimination. This is because half the market will be served in both cases, but the allocation of consumers is more efficient with a single price.

Once $\bar{N} > 3.51$, we are back to our earlier model in which some consumer types will be excluded from the market by the high price. Under one price, the calculation of consumer surplus is exactly as before except that the density is now $1/(\bar{N} - 1)$ rather than $1/\bar{N}$:

$$TS = \frac{\bar{N}}{\bar{N}-1} \frac{A\bar{N}-p}{4} = \frac{\bar{N}}{\bar{N}-1} A\bar{N} \frac{2.51}{14.04}.$$

Turning to the case of price discrimination, the only difference is that the demand for cartridges varies from 1 to $\bar{N}$, rather than 0 to $\bar{N}$. Hence the average demand is $(\bar{N} + 1)/2$.

$$TS_{PD} = \frac{3A(\bar{N}+1)}{16}$$

$$\frac{3A(\bar{N}+1)}{16} > \frac{\bar{N}}{\bar{N}-1} A\bar{N} \frac{2.51}{14.04} \text{ iff}$$

$$1.05(\bar{N}+1) > \frac{\bar{N}^2}{\bar{N}-1} \text{ iff}$$

$$0.05\bar{N}^2 > 1.05 \text{ or } \bar{N} > \sqrt{21} = 4.58.$$

Theorem 4: Under the assumption that n has continuous support over $[0, \bar{N}]$, the one-price monopolist restricts output relative to the price discrimination case.

Proof: Absent price discrimination, demand is

$$D = \int_{\frac{p}{A}}^{\bar{N}} \frac{1}{A}[A - \frac{p}{n}]f(n)dn = \int_{\frac{p}{A}}^{\bar{N}} f(n)dn + pD'(p)$$

$$= \int_{\frac{p}{A}}^{\bar{N}} f(n)dn - D(p) \text{ at the profit-maximizing } p$$

$$= \frac{1}{2}\int_{\frac{p}{A}}^{\bar{N}} f(n)dn < \frac{1}{2}$$

where the strict inequality follows from the fact that $p > 0$ and so some consumers will be excluded from the market. The result then follows as the price-discriminating monopolist sells to precisely half of each customer group.

Theorem 5:

$$p'(c) < -\sqrt{2p}$$

$$d\Pi(p(c), c)/dc > 0$$

$$dTS(p(c), c)/dc > 0$$

Proof: The first-order condition that determines the optimal base price is:

$$\frac{d\Pi}{dp} = 1 - F(\sqrt{2p} + c) - \frac{1}{\sqrt{2p}} f(\sqrt{2p} + c)(p - \mu + c\sqrt{2p}) = 0.$$

Note that at $c = 0$, $\frac{d\Pi}{dp} > 0$ at $p = \mu$, so $p(0) > \mu$. This first-order equation implicitly defines $p(c)$:

$$\frac{d^2\Pi}{dp^2} \frac{dp}{dc} + \frac{d^2\Pi}{dpdc} = 0.$$

The two parts of the equation are

$$\frac{d^2\Pi}{dpdc} = -2f(\sqrt{2p} + c) - f'(\sqrt{2p} + c)(\tfrac{p-\mu}{\sqrt{2p}} + c)$$

$$\left.\frac{d^2\Pi}{dpdc}\right|_{c=0} = -2f(\sqrt{2p}) - \tfrac{p-\mu}{\sqrt{2p}} f'(\sqrt{2p})$$

$$\frac{d^2\Pi}{dp^2} = -\frac{3}{2} \frac{1}{\sqrt{2p}} f(\sqrt{2p} + c) - \frac{\mu}{2\sqrt{2}p^{3/2}} f(\sqrt{2p} + c) - \frac{1}{\sqrt{2p}} f'(\sqrt{2p} + c)(\tfrac{p-\mu}{\sqrt{2p}} + c)$$

$$\left.\frac{d^2\Pi}{dp^2}\right|_{c=0} = -\frac{3}{2} \frac{1}{\sqrt{2p}} f(\sqrt{2p}) - \frac{\mu}{2\sqrt{2}p^{3/2}} f(\sqrt{2p}) - (\tfrac{p-\mu}{2p})f'(\sqrt{2p}).$$

Thus $p'(c)$ at $c=0$ is

$$\left.\frac{dp}{dc}\right|_{c=0} = \frac{2f(\sqrt{2p}) + \tfrac{p-\mu}{\sqrt{2p}} f'(\sqrt{2p})}{\tfrac{-3}{2\sqrt{2p}} f(\sqrt{2p}) - \tfrac{\mu}{2\sqrt{2}p^{3/2}} f(\sqrt{2p}) - \tfrac{p-\mu}{2p} f'(\sqrt{2p})}$$

$$= -\sqrt{2p} \left[\frac{4f(\sqrt{2p}) + \tfrac{\sqrt{2}(p-\mu)}{\sqrt{p}} f'(\sqrt{2p})}{(3 + \tfrac{\mu}{p}) f(\sqrt{2p}) + \tfrac{\sqrt{2}(p-\mu)}{\sqrt{p}} f'(\sqrt{2p})} \right]$$

$$= -\sqrt{2p} \left[1 + \frac{f(\sqrt{2p})(1 - \tfrac{\mu}{p})}{(3 + \tfrac{\mu}{p}) f(\sqrt{2p}) + \sqrt{2p} f'(\sqrt{2p})} \right] < -\sqrt{2p},$$

as $p(0) > \mu$ and the denominator is positive by the local concavity of the profit function. Turning to the effect on profits:

$$\frac{d\Pi}{dc} = \frac{\partial\Pi}{\partial p} \frac{dp}{dc} + \frac{\partial\Pi}{\partial c} = \frac{\partial\Pi}{\partial c}$$

$$\Pi = \int_{a \geq \sqrt{2p}+c} [p - \mu + c(a - c)] f(a) da$$

$$\left. \frac{d\Pi}{dc} \right|_{c=0} = \int_{a \geq \sqrt{2p}+c} a f(a) da - [p(0) - \mu] f(\sqrt{2p}).$$

Employing the first-order condition that determines the optimal p, we have

$$\left. \frac{d\Pi}{dc} \right|_{c=0} = \int_{a \geq \sqrt{2p}} a f(a) da - (\sqrt{2p})(1 - F(\sqrt{2p}))$$

$$= \int_{a \geq \sqrt{2p}} (a - \sqrt{2p}) f(a) da > 0.$$

Finally, the effect on Total Surplus is positive:

$$TS = \int_{a \geq \sqrt{2p}+c} [(a - c) \frac{a + c}{2} - \mu] f(a) da$$

$$= \int_{a \geq \sqrt{2p}+c} [\frac{a^2 - c^2}{2} - \mu] f(a) da$$

$$\frac{dTS}{dc} = \frac{\partial TS}{\partial p} \frac{dp}{dc} + \frac{\partial TS}{\partial c}$$

$$= -c[1 - F(\sqrt{2p} + c] - \frac{(\sqrt{2p} + c)^2 - c^2 - 2\mu}{2} f(\sqrt{2p} + c) \left[1 + \frac{1}{\sqrt{2p}} \frac{dp}{dc} \right]$$

$$\left. \frac{dTS}{dc} \right|_{c=0} = -(p - \mu) f(\sqrt{2p}) [1 + \frac{1}{\sqrt{2p}} \frac{dp}{dc}] > 0$$

as

$$\frac{dp}{dc} < -\sqrt{2p}. \ \blacktriangledown$$

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1   Einer Elhauge, *Tying, Bundled Discounts, and the Death of the Single Monopoly Profit Theory*, 123 Harv. L. Rev. (forthcoming Dec. 2009).

2   Going forward, for simplicity I will simply refer to social welfare, which is what Elhauge considers to be ex post social welfare.

3   A.C. Pigou, The Economics of Welfare (1920).

4   The model leads to an unusual use of third-degree price discrimination, namely one that comes out exactly the same as metering or second-degree price discrimination. In the model, the price-discriminating monopolist is able to charge a different price to each consumer group based on the group's exogenous demand for the tied good (copies). Normally, under third-degree price discrimination, a consumer type n is charged p_n. Here, all consumer groups are identical other than in their demand for copies and the optimal price-discrimination charge increases linearly with the type. Thus a consumer group who wants n copies is optimally charged $nA/2$ for a printer. This is 3rd-degree price discrimination as the price varies based on the group type where type n is taken to be exogenous and observable. In theory, group n could make any number of copies for the fixed price of $nA/2$, but they are only interested in n copies. Charging $A/2$ per copy leads to the exactly same result when the demand for copies is exogenous (as the customer of type n ends up buying n copies and paying $nA/2$), even if unobservable. In this case, the monopolist doesn't have to know which consumer is which in setting

the price. Since the demand for copies is exogenous and not sensitive to the price (assuming that the customer finds it worthwhile to buy a printer), there is no issue of self-selection. Thus the monopolist can achieve the third-degree price discrimination result using a linear two-part tariff or metering. This would not be true if the distribution of valuations were different according to the number of copies desired. It would also not be true of the customer type who wanted n copies had some declining marginal valuation and would consider buying fewer than n depending on the pricing.

5   Although it may be a dispute over semantics, Elhauge is incorrect when he seeks to classify metering ties as third-degree price discrimination (in footnote 82). As discussed in the previous note, his model is the exceptional case where the two approaches lead to the same result. Elhauge writes "The categorization is a bit ambiguous because while tying does present all buyers with the same price schedule (like second-degree price discrimination), it also effectively charges buyers higher prices for the tying product if they likely value it more (like third-degree price discrimination)." The fact that high-value buyers pay more has nothing to do with the classification type. Second-degree price discrimination arises whenever a monopolist can't tell one consumer type from another and is thus limited to a price schedule that induces self-selection. If all consumers are charged the same price per copy, this is 2nd-degree price discrimination. If consumers are charged different prices for the printer based on their observable and exogenous type (e.g., business or residential), this would be third-degree price discrimination.

6   Illinois Tool Works Inc. v. Independent Ink, Inc., 547 U.S. 28.

7   If, for example, Trident's printer is faster or more reliable, this would lead to a production costs savings of some amount v per box.

8   Customer demand can be continuous even if the units sold are discrete. Thus a customer might ideally want to purchase 2,200 copies or 1.1 toner cartridges. With discrete units,, the customer will be forced to choose between buying one or two cartridges, and therefore cutting back his printing to 2,000 or expanding it to 4,000. Neither I nor Elhauge consider such a model.

9   Hal R. Varian, *Price Discrimination and Social Welfare*, 75 Amer. Econ. Rev. 4, 870-75 (1985).

10 Richard Schmalensee, *Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination*, 71 Amer. Econ. Rev. 1, 242-247 (1981).

11 The optimal price under the assumption that consumers with demands 1, 2, 3, and 4 are served is $4A/[2*(1+1/2+1/3+1/4)] = 24A/25 < A$, so all groups are served. In the numerical example, $A=200$ and so $p=192$.

12 The optimal price under the assumption that consumers with demands 2, 3, and 4 are served is $3A/[2*(1/2+1/3+1/4)]=18A/13$. Here $A < 18A/13 < 2A$ which implies that groups 2, 3, and 4 are all partially served, while the first group is excluded. In the numerical example, $A=200$ and so $p=276.9$.

13 The Elhauge case with $\bar{N}=2$ or 3 is similar to earlier results from Robinson, *see* Joan Robinson, The Economics of Imperfect Competition (1933) and Richard Schmalensee, *Output and Welfare Implications of Monopolistic Third-Degree Price Discrimination*, Am. Econ. Rev. 71 (1981): 242-247. In these models, demand from each customer group is linear. If all consumer groups are served, the monopolist sells to half the customers in the market, the same fraction as under price discrimination. Since the allocation is better under a single price, price discrimination is inefficient.

14 *See* Barry Nalebuff, Supreme Court Amici Curiae in support of Independent Ink, Inc. in Illinois Tool Works, Inc. v. Independent Ink, Inc., 2005 (with Ian Ayres and Lawrence Sullivan).

15 If there is a marginal cost, then change a to a' where the a' is reduced by the marginal cost. Since the result does not depend on the distribution of a, replacing a by a' has no effect.

16 These costs are not limited to tying. Think of all the resources that airlines employ to price discrimination against business customers; these include advance booking discounts, Saturday-night stayover requirements, and frequent-flyer rewards. Then think of all the strategies that business consumers use to avoid being subject to price discrimination, including back-to-back ticketing, phantom returns, and staying over the weekend. For more on this subject, *see* Barry Nalebuff, *Exclusionary Bundling*. 50 ANTITRUST BULL. 3, 321-370 (2005).

17 In some cases with commercial customers, direct metering may lead to a violation of the Robinson Patman Act.

18 The two points are discussed further in Barry Nalebuff, *Unfit to Be Tied: An Analysis of Trident v. Independent Ink (2006)*, in THE ANTITRUST REVOLUTION, 5th edition, 365-88 (J. Kwoka & L.White eds., 2009).