THE RATIONALLY SHRINKING UNION

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We analyze a voting model in which a democratic monopoly labor union rationally shrinks towards zero over time. In our model, preferences are not single-peaked so that this shrinkage may occur in spite of objections by the median voter. We characterize the wage-employment path as a function of the time horizon and the discount rate. We show that the employment path is independent both of the labor demand curve and of the workers’ utility functions.

1. INTRODUCTION

The presence of a seniority rule creates a well-known tension between older and newer union members when deciding on a wage demand. Blair and Crawford (1984) show that when workers are dismissed on the basis of seniority and labor demand is downward-sloping, a simple majority voting rule will result in the wage-employment combination preferred by the median worker.

One implication of their model is that if workers behave myopically, the median voter will become unemployed in the subsequent period when the new median voter raises the wage even further. Farber (1987) and Blair and Crawford (1984) suggest that this “shrinking union problem” will be mitigated when members consider the long-run effects of their current choices. Burda (1990) characterizes equilibrium voting behavior in a multiperiod model. He shows that if the uncertainty of future labor demand is sufficiently high, then voters’ preferences are single-peaked, permitting a stable equilibrium to exist.

We consider the shrinking union problem with nonstochastic labor demand. In this case, preferences are not single-peaked. As a result, changes in the wage level may be adopted despite opposition by the median voter. The lack of single-peakedness suggests a stable voting equilibrium may not exist and agenda restrictions may be needed for the union to reach a decision. Thus we ask: what is the largest wage increase that can obtain majority approval against the alternative of the status quo? Evaluation of this equilibrium wage path shows that even with rational expectations, union membership will shrink to zero over time.

*We have benefited from discussions with Giovanna Prennushi and Paul Johnson and the comments of Brendan O’Flaherty and an anonymous referee. This paper was motivated by Prennushi (1991).

1 Johnson (1990) provides an example where unraveling is possible even with perfect foresight.
Our results are consistent with the empirical findings of Blanchflower, Millward, and Oswald (1991), who find that 27% of British firms with closed shop labor unions had large (over 20%) decreases in employment between 1980 and 1984. In contrast, only 14% of non-union firms and 15% of firms with open shop labor unions suffered large employment decreases during the same time.

2. THE MODEL

The union votes for a wage level $w$ against the alternative of the status quo. At the status quo wage, all current members remain employed. The labor demand is known, so each member of the union rationally anticipates the employment level associated with each wage. The wage that leads to employment level $p$ is denoted by $w(p)$. Employment is determined by seniority; at wage $w(p)$, it is the $p$ most senior union members that remain employed.

We represent seniority as a continuum on the interval [0, 1]. The seniority variable may be thought of like a hiring date, so that smaller is better. Seniority of 0 is the oldest worker; seniority of 1 is the newest worker and the first to get fired. We define seniority level $p$ so that fraction $p$ of the workers have seniority of at least $p$.

A change in wage level is approved by the union if it receives at least fraction $\delta$ of the votes, $\frac{1}{2} \leq \delta \leq 1$, against the status quo. The case of simple-majority rule corresponds to $\delta = \frac{1}{2}$. Members are assumed to vote so as to maximize the undiscounted sum of their wages over the next $n$ periods. The wage of an unemployed member is normalized to zero. Once a member is unemployed, he is no longer a member of the union and cannot vote in subsequent periods.2

3. ANALYSIS

We begin analyzing the voting equilibrium first in a one-period and then in a two-period setting where decisions are made according to simple majority rule (i.e., $\delta = \frac{1}{2}$). This will provide the basis for our inductive argument.

We illustrate our main result for the case of a two-period model where decisions are made using simple majority rule.

At the status quo wage, $w(1)$, all current members will remain employed. What is the highest wage (lowest employment) that a union with rational expectations would approve under majority rule?

In a one-period model, each member seeks to obtain the highest wage available to him while remaining employed. Compared to the status quo, raising the wage maintains the support of a majority until the median seniority member

2If unemployed workers could vote, we would have to consider how they would choose between two wages, neither of which would lead to their employment. One argument suggests that they would prefer the higher wage. It is not just that misery loves company, but from the viewpoint of a signaling model, it is better to get laid off along with others rather than by yourself. Thus the unemployed members actually support further wage increases. However, we do not allow unemployed workers to vote in our model.

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would lose his job. At any higher wage, over half the workers would be unemployed — they would defeat the proposal to raise the wage above the status quo. In a one-period model, a majority of workers are in favor of raising the wage up to \( w(\frac{1}{2}) \). The median member becomes the most junior person employed.

In a two-period model, only those members who were employed in the first period are eligible to vote in the second period. If the first period wage is set at \( w(p) \), we use the one-period model to predict that the second-period wage will be \( w(\frac{1}{2}) \). Thus any wage increase in period 1 leads to the median seniority member being unemployed in period 2. The median voter would vote against setting the wage at \( w(p) > w(1) \) in the first period because he correctly anticipates that doing so would lead to a second-period wage of \( w(\frac{1}{2}) > w(\frac{1}{2}) \) and he would be unemployed. He prefers receiving \( w(1) \) in the first period and \( w(\frac{1}{2}) \) in the second period to \( w(p) \) in the first period and nothing in the second.\(^3\) Note that \( w(1) + w(\frac{1}{2}) \) dominates \( w(p) \) and so this argument does not depend on the utility function.

Here we use the assumption that workers do not discount future wages. However, in spite of the median voter's objection, there exists a majority voting coalition that supports a wage increase in the first period. Consider a proposal to increase the wage from \( w(1) \) to \( w(\frac{1}{2}) \) in the first period. All voters recognize that this would lead to a wage of \( w(\frac{3}{2}) \) in period two.

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This proposal will be supported by voters in the interval \([0, \frac{2}{5}]\); these very senior workers will be employed in both periods and will earn \( w(\frac{3}{2}) > w(1) \) and \( w(\frac{3}{2}) > w(\frac{1}{2}) \). The proposal will be opposed by voters in the interval \([\frac{2}{5}, 1]\), because they will become unemployed in both periods. Voters in the interval \([\frac{2}{5}, \frac{1}{2}]\) will become unemployed in period two whereas they would have worked both periods under the status quo. They also vote no, because \( w(\frac{3}{2}) < w(1) + w(\frac{1}{2}) \). However, members in the interval \([\frac{1}{2}, \frac{4}{5}]\) will vote yes, as they will be unemployed in period two in either case and prefer \( w(\frac{3}{2}) \) to \( w(1) \). The proposal will pass with 70% in favor, even though the median voter is opposed.

What is the largest wage increase that will prevail in the first period? Consider a wage increase that results in fraction \( x \) of the union being employed in period one. This will be supported by members in the interval \([0, \frac{x}{2}]\) and by members in the interval \([\frac{x}{2}, x]\), who will not be employed in period two regardless of whether

\(^3\)This assumes that \( p > \frac{1}{2} \). Since \( \delta > \frac{1}{2} \), we need not consider proposals with \( p < \frac{1}{2} \) in the first period since at least 50% of the workers will become immediately unemployed and thus defeat the proposal.
the measure passes. Thus the vote in favor of $w(x)$ over $w(1)$ is $\frac{x}{2} + x - \frac{1}{2}$. The smallest $x$ that still commands a majority vote is $x = \frac{2}{3}$: two-thirds of the original union are employed in period one and one-third of the original union are employed in period two.

Now suppose an arbitrary supermajority $\delta$ is needed to increase wages in the current period. In a one-period model, the wage $w(\delta)$ is the largest wage that will defeat the status quo. We henceforth assume that in each period, the wage that is chosen is the largest wage that will defeat the status quo. In a two-period model, an increase that results in fraction $x$ of the union being employed will be supported by those in the interval $[0, \delta x]$ and those in the interval $[\delta, x]$. The largest $x$ that solves $\delta x + x - \delta \geq \delta$ is $x = \frac{2\delta}{1+\delta}$. The generalization of these results to the $n$-period case with $\delta$-majority rule is presented in the following theorem.

**Theorem.** With $n$ periods remaining and super-majority rule $\delta \geq \frac{1}{2}$, the union will pick a wage $w$ so that the fraction of the membership that remains employed is

$$\frac{n}{n-1+\frac{1}{\delta}}.$$ 

**Proof.** See Appendix

It is worth emphasizing that the union shrinkage does not depend on the worker utility functions or even the labor demand function. The equilibrium wage depends on the labor demand function, but not employment levels. The reason, as seen in the proof, is that when a union member compares two initial wages and the anticipated wage–employment paths, one profile will always dominate the other. Members may disagree as to which profile dominates, but preferences are based one path dominating the other.

The results are simplest for the case of simple majority rule.

**Corollary 1.** With $n$ periods remaining and simple majority rule, the union will pick a wage so that the fraction of the membership that remains employed is

$$\frac{n}{n+1}.$$ 

In this case, the employment falls by a constant amount each period. Starting with an $N$ period time horizon, employment falls to $\frac{1}{N+1}$ of its initial level at the end of $N$ periods.

The rate at which the union membership shrinks is decreasing in both $n$ and $\delta$. Members can mitigate the effect of the shrinking union problem by extending the relevant time horizon and by adoption of super-majority voting mechanisms. Of course, preferences over the voting rules will not be single-peaked for the same reasons preferences over wages are not single-peaked.

We use the theorem to characterize the speed at which union membership shrinks. The fraction of the union that remains employed after $J$ periods is

$^4$This result depends on the no-discounting assumption.
\[\prod_{n=1}^{N+1-J} \frac{n}{n-1+\delta} = \frac{\Gamma[N+1]/\Gamma[N+1-J]}{\Gamma[N+\frac{1}{2}]/\Gamma[N-J+\frac{1}{2}]}
\]

\[\approx \left[\frac{N-J+\frac{1}{2}}{N+\frac{1}{2}}\right]^{\frac{1+\delta}{\delta}} \left[\frac{N+1}{N+\frac{1}{2}}\right]^N \left[\frac{N-J+1}{N-J+\frac{1}{2}}\right]^{N-J+\frac{1}{2}}\]

where we use the approximation \(\Gamma(x) = \sqrt{2\pi}e^{-x}x^{x-\frac{1}{2}}\) [see Feller (1968), p. 66].

The result in Corollary 2 substitutes \(J = N\) and the fact that \([1 - \frac{\delta}{N}]^N \to e^{-\delta}\).

When \(J < N\), the final two terms cancel and we have the result in Corollary 3.\(^6\)

**Corollary 2.** With an initial time horizon of \(N\) periods, final union membership falls to a fraction \(Q\) of its initial size.

\[Q \approx \left[\frac{1}{1 + \delta N}\right]^{\frac{1+\delta}{\delta}} e^{\frac{\delta}{2N}}\]

For any value of \(\delta\) other than 1, the final union membership shrinks to zero as the time horizon, \(N \to \infty\).

**Corollary 3.** With an initial time horizon of \(N\) periods, after \(J < N\) periods, union membership falls to a fraction \(Q\) of its initial size,

\[Q \approx \left[\frac{N-J+\frac{1}{2}}{N+\frac{1}{2}}\right]^{\frac{1+\delta}{\delta}}\]

**Corollary 4.** The median seniority member is employed for fraction \(\lambda\) of the time horizon,

\[\lambda = 1 - 2^\frac{\delta}{N}\]

Corollary 4 follows from solving for \(\lambda = \frac{\delta}{N}\) so that the unions shrinks by \(\frac{1}{2}\).

Employment falls slowly at first and then quickly near the end. Our first graph shows how long a worker is employed before the union shrinks sufficiently so that he loses his job. We use the result in Corollary 3 to illustrate the shrinkage with \(\delta = \frac{2}{3}\). Note, for example, that the median number is employed for \(\frac{1}{4}\) of the periods.

To give a sense of speed at which the union membership shrinks for various majority rules, we use the result in Corollary 4 to provide a graph of when the median number loses his job as a function of \(\delta\).

\(^5\)The result in Corollary 2 is good approximation even for small \(N\). The worst case occurs when \(\delta = \frac{1}{2}\); once \(N \geq 5\) the error is less than 10\%, once \(N \geq 10\), the error is less than 5\%, and once \(N \geq 20\), the error is less than 1\%. The absolute error is even smaller; for \(N = 20\), the absolute error is less than 0.0005.

\(^6\)The result in Corollary 3 is a good approximation once \(N-J\) is above 5. For example, with \(\delta = \frac{1}{2}\) and \(N = 20\), the accuracy is within 10\% for \(J \leq 14\) and within 5\% for \(J \leq 10\).
The shrinking union problem may be mitigated by alternative institutional arrangements. Our model has only considered the case in which all workers receive the same wage. Consider an alternative in which the $\delta$ most senior workers receive a higher wage than the less senior workers. For a sufficiently large wage disparity, the more senior workers will oppose shrinking the union, because they receive most of the rents generated by the less senior workers. Young workers may in turn be willing to join a union with a wage profile skewed in favor of senior workers if they believe this arrangement will last long enough.
for them to become part of the senior group as the older workers retire. Therefore, we believe the shrinking union problem will be most pronounced in declining or unstable industries in which a skewed wage scale will be difficult to sustain.

4. CONCLUSIONS

A democratic monopoly labor union in which all voters fully understand the long-term consequences of their actions may shrink over time, despite the objections of the median voter. The lack of single-peaked preferences suggests agenda restrictions are needed for the union to reach a decision. We characterize the largest wage increase that can prevail over the alternative of the status quo for an arbitrary super-majority voting rule and arbitrary time horizon. The union's shrinkage is decreasing in both the length of the time horizon and the size of the majority needed to defeat the status quo. Most surprising is that the equilibrium employment levels are independent of worker preferences and of labor demand.

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REFERENCES

APPENDIX

Theorem. With n periods remaining and super-majority rule δ ⩾ 1/2, the union will pick a wage w so that the fraction of the membership that remains employed is

\[ \frac{n}{n - 1 + \frac{1}{\delta}}. \]

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Proof. By induction. We compare maintaining wages at the status quo to immediately raising wages. Let the status quo workforce be size $s$ so that the status quo wage is $w(s)$.

For $n = 1$, all workers who remain employed prefer a higher wage while those who would lose their job prefer the status quo. Since higher wages lead to less employment, the highest wage that still employs fraction $\delta$ of the workers is $w(\delta s)$. Any higher wage would not command the support of a $\delta$-majority. Thus the workforce shrinks to a fraction $\delta$ of its original size. For $n = 1$, $\frac{n}{n-1+\frac{1}{2}} = \delta$, as claimed.

We now assume that the result is true for all $i \leq n - 1$ and show this implies the result for $i = n$. Again we compare maintaining wages at the status quo to immediately raising wages. If we maintain wages at the status quo, then, by induction, we will move to the proposed solution starting in the second period. Call this plan A. The alternative, plan B, is to move directly to the proposed solution.

All members who are employed for one extra period in plan A over B prefer the status quo. Consider a member who is employed for the first $t + 1$ periods under A and only $t$ periods under B. Under plan A he earns

$$w_1^\downarrow + w_2^\downarrow + \ldots + w_n^\downarrow + w_{t+1}^\downarrow.$$ 

Under plan B he earns

$$w_1^\uparrow + w_2^\uparrow + \ldots + w_n^\uparrow.$$ 

By construction, $w_2^\downarrow$ is greater than $w_1^\downarrow$; employment at $w_2^\downarrow = \frac{n-1}{n-2+\frac{1}{2}}$, while employment at $w_1^\downarrow = \frac{n}{n-1+\frac{1}{2}}$. This is true down the line. Thus, the sum under plan A is higher as

$$w_2^\downarrow > w_1^\downarrow, \quad w_3^\downarrow > w_2^\downarrow, \ldots, \quad w_{t+1}^\downarrow > w_t^\downarrow.$$ 

All the members who work the same number of periods in both cases prefer plan B, since it offers higher wages in each period. How many workers are there in each camp? It suffices to measure the size of camp B. For simplicity, we normalize the status quo union membership, $s$, to equal 1.

Those members with seniority between $\frac{n}{n-1+\frac{1}{2}}$ and $\frac{n-1}{n-2+\frac{1}{2}}$ will be employed only 1 period in either case. Therefore they prefer to have the higher wage (plan B) in the one period for which they are employed.

Those from $\frac{n}{n-1+\frac{1}{2}} \times \frac{n-1}{n-2+\frac{1}{2}}$ to $\frac{n}{n-2+\frac{1}{2}} \times \frac{n-2}{n-3+\frac{1}{2}}$ will be employed in the first two periods in either case. They too prefer the higher wages associated with plan B than plan A.

Summing up over all the members who are employed the exact same number of times in both plans, we find that the support for plan B over plan A is

$$\sum_{i=1}^{n} \left( \prod_{j=1}^{i} \frac{n+1-j}{n-j+\frac{1}{2}} - \prod_{j=1}^{i} \frac{n-j}{n-1-j+\frac{1}{2}} \right).$$
By Lemma 1 below, this sum equals

\[ V(n) - V(n - 1) = n\delta - (n - 1)\delta = \delta. \]

Any higher first-period wage would reduce the support for plan B (since more members would lose a period of employment) and therefore fail to get a \(\delta\)-majority vote against the status quo. QED

**Lemma 1.**

\[ V(n) = \sum_{i=1}^{n} \left( \prod_{j=1}^{i} \frac{n+1-j}{n-j+\frac{1}{\delta}} \right) = n\delta \]

**Proof.** By induction. For \(n = 1\), the sum is \(\frac{1}{\delta} = \delta\). We assume the result is true for \(n\) and show this implies it is true for \(n+1\).

\[
\begin{align*}
V(n+1) &= \frac{n+1}{n+\frac{1}{\delta}} + \frac{n+1}{n+\frac{1}{\delta}} \times V(n) \\
&= \frac{n+1}{n+\frac{1}{\delta}} + \frac{n+1}{n+\frac{1}{\delta}} \times n\delta \\
&= \frac{n+1}{n+\frac{1}{\delta}} \times (1 + n\delta) \\
&= (n+1)\delta.
\end{align*}
\]

QED