A Model of Two-Sided Costly Communication for Building New Product Category Demand

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Abstract

When a firm introduces a radical innovation, consumers are unaware of the product’s uses and benefits. Moreover, consumers are unsure whether they even need the product. In this context, we model the marketing communication process as a two-sided process that involves both firms’ and consumers’ costly efforts to transmit and assimilate the novel product concept. Its success endogenously generates consumers’ need recognition and thereby market demand for a novel product. We study a firm’s different information disclosure strategies for a radical innovation. We find that sharing an innovative idea, instead of extracting a higher rent by keeping the idea secret, can be optimal. A firm may benefit from the presence of competitors and their communication efforts. The innovator can share an innovation so that competitors can also profit and thus have incentives to create and expand the market.

Keywords: Communication, information disclosure, endogenous demand, strategic complementarity, strategic substitutability, free-riding.

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1 Introduction

In 2014, within four years of its IPO, Tesla Motors had already achieved a market value half that of Ford’s.¹ That same year, the company decided to share all of its patents with other car manufacturing companies, opening its vault to expand the long-range electronic car market category.² This controversial move wasn’t entirely unprecedented. In the semiconductor industry, for instance, it is common practice for an innovative firm – one that has developed a new generation of processor – to license its innovation to one or more competing firms. This strategy creates multiple sources of supply (Semiconductor Licensing Trends 2013³) and can spur broader product demand for the next generation semiconductor (Shepard 1987).

In fact, publicly releasing innovative technology has become a routine practice in high-tech markets, with many companies following similar practices. For some products, firms pre-announce an innovation by revealing the details and marketing plans for the product well in advance of its release, practically inviting competitors to participate in the category.

Consider Google: In April 2012, Google announced its marketing plan for the new product, “Google Glass” – a radical wearable technology. Interestingly, the product was at the conceptual stage; Google had not even developed a prototype. Rather, the Google Glass Team posted a video about an ideal day using their (then) imagined product. The design team shared prototype ideas online. Google marketed “Glass” so early that Samsung, Microsoft, Intel, and many other technology companies quickly followed and announced they would commercialize a similar product in the mass market. Although the project was declared a failure due to consumer concerns over privacy and social awkwardness⁴ – in January 2015, Google officially announced its plan to withdraw the product from the market – many industry experts acknowledge that Google Glass fostered a market for new wearable technologies.⁵ Nowadays, technology experts widely recognize wearable technology like the Apple Watch and Samsung Gear as the next big thing for consumers⁶: in 2014, wearable technology has thus far generated $3 billion in revenue; and it is estimated to grow to $6 billion by 2016⁷ and $19 billion by 2018.⁸

³“Trends and Opportunities in Semiconductor Licensing”:
⁴http://www.cnn.com/2015/01/20/opinion/pease-google-glass-what-went-wrong/.
⁶⁴ “Get Ready: Wearable Tech is About to Explode;” http://www.entrepreneur.com/article/2293475#.
In the examples of Tesla and Google Glass, firms released key information on a new technology or product into the public domain. This behavior is curious since this type of information is usually considered a trade secret that serves as a key competitive advantage against potential competitors. When companies market a half-baked idea or share a technological breakthrough, they risk prematurely disclosing a potentially disruptive innovation. This may lead to greater competition and weak appropriability in the product market competition. This is why firms sometimes opt for great secrecy around new products: to maximize their competitive surprise (i.e. Apple’s iPhone).

In this research, we focus on the introduction of revolutionary new products and firms’ marketing strategies for such products. Specifically, we try to explain the puzzling differences in information disclosure strategies among innovating firms: how much do these firms choose to keep secret, and how much of their innovation do they reveal? We propose an explanation for why a firm sometimes shares an innovative idea with competitors, which might potentially undermine its dominant position in the market, and when this choice (as opposed to keeping the innovation secret) is optimal. Our explanation is based on the persuasion motive of firms in communication and the strategic role of consumers.

When a firm introduces a novel product for which the market does not exist, consumers do not understand the product’s uses and benefits. They are even unclear as to whether they need the product. In this context, the role of marketing communication is to educate consumers and encourage them to deliberate their potential needs, and thus increase product acceptance. In contrast to incremental innovations, where consumers generally know the benefits from the category, and can passively learn about the value of new features from advertising, consumers bear relatively high costs to learn about the potential benefits of revolutionary new products. Therefore, a key challenge to marketing radical innovation is persuading consumers to expend more effort in paying attention to the content, assimilating the new idea, and deliberating their needs by examining an unproven product category. For example, consumers can be aware of new wearable technology products like Google Glass from advertising alone. However, awareness of a radical product does not necessarily translate into consumers’ need recognition or product acceptance. Consumers might be unwilling to incur further learning costs for a product that seems to aim at tech geeks and not them. The potential promise of wearable technology can fail to emerge and expand as a category due to consumers’ aversion to costly effort.9 Marketers therefore need to not only advertise the product and its features, but also create the right marketing environment in which customers are willing to invest in costly learning. Without consumers on board, the firm’s effort alone is insufficient to convey the new idea and convince consumers that they need the product.

We model marketing communication for a radical product as two-sided costly communication, a process that involves both firms’ and consumers’ efforts to transmit and assimilate information.

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9“Glass, Darkly for its wearable computer to be accepted, Google must convince people that the device isn't creepy,” MIT Technology Review, http://www.technologyreview.com/review/524576/glass-darkly/.
(Dewatripont and Tirole 2005, Wernerfelt 1996). This two-sided perspective of communication is widely recognized in sociology and economics, but has not yet been embodied into formal modeling in marketing literature. Most marketing literature considers marketing communication as one-sided costly communication; that is, only firms exert communication effort (such as advertising) and consumers effortlessly become informed. Such one-sided communication is reasonable in many situations where consumers know about the product and their needs (for example, existing product categories and incremental innovations). However, one-sided communication is not applicable to radical innovation, in which consumers are clear neither about the uses and the benefits of a product nor their needs for it.\textsuperscript{10} When a firm communicates the values of radical innovations to consumers, consumers have to expend effort in learning about the product and considering whether it suits their needs. In this paper, we recognize that the acts of both formulating and absorbing the content of a communication are privately costly, and it takes both parties (i.e., firms and consumers) for communication to succeed.\textsuperscript{11} Through this two-way communication process, consumers can come to understand product’s uses and benefits, and become potential buyers. Therefore, successful marketing communication is a critical process whereby a firm creates a market for a novel product.

In the situation where two-sided costly communication plays an important role in endogenously creating and expanding the demand for the new product, the firm’s incentive to disclose its innovation to competitors or to keep it secret is determined by the trade-off between the two effects. On the one hand, by disclosing its innovation to competitors, firms can invite the competitors to share costly communication efforts and thus expand the total market size through cooperation in communication stage. Nevertheless, this demand-enhancing effect does not necessarily arise because of the free-riding incentive of firms – that is, firms may not expend costly efforts to increase the total demand, and may instead simply free ride on the other firm’s efforts.\textsuperscript{12} However, we show that when both firms’ and consumers’ communication efforts endogenously determine the market demand for the new product, strategic consumers alleviate the potential free-riding effects of rival firms. By inviting the competitor, the innovator can credibly commit to providing consumers with greater surplus, which increases the incentives for consumers to expend more costly effort in deliberating about the new innovation. In turn, this increased consumer effort encourages firms further to put

\textsuperscript{10}We follow the notion that a radical innovation is a new product that represents substantial improvements in technology or consumer benefits (Chandy and Tellis 1998) and thus consumer preferences for such products are not yet fully formed (Christensen 2003). On the other hand, incremental innovation concerns an existing product or service whose performance has been enhanced or upgraded. Consumers already have a stable preference for such products.

\textsuperscript{11}In reality, we can observe the necessity of two-sided efforts in a wide variety of scenarios; for example, when a teacher tries to convey a new idea to students, if the students are not interested or do not spend time to pay attention or think, they will not absorb and internalize the new knowledge no matter how hard or clearly the teacher tries to elaborate. On the other hand, no matter how hard students try to learn, if the teacher does not explain things clearly then, the idea is hardly understood and accepted by the students. Both efforts are necessary for the communication to succeed.

\textsuperscript{12}The effects of free-riding on competitor’s service efforts on retail competition (also, known as showroaming effect) is analyzed in Shin (2007).
more effort into the communication stage. This complementarity in communication between firms and consumers through consumers’ strategic decision mitigates the potential free-riding problem. Therefore, to induce greater consumer efforts, an innovator may benefit from inviting competitors by sharing an idea. On the other hand, the firm can achieve competitive advantage and greater market share in the product market by keeping the innovation secret. Even though the innovating firm will bear all the costs for communication and creating the market, it can then enjoy a monopoly situation in the product market.

These two effects are related to the fundamental issue firms often face—whether to focus on maximizing their market share (which can be achieved by keeping the innovation secret) or on increasing the total market size (which can be achieved by sharing their innovation with competitors). Understanding these tradeoffs enables us to understand an important issue of coopetition (Brandenburger and Nalebuff 1996): whether to cooperate with competitors to reach a higher value creation (e.g., market expansion) or struggle to achieve a competitive advantage without cooperation.

In our model, we analyze the strategic interactions not only among firms that compete with each other, but also between firms and consumers who jointly create market demand through communication. The game involves three players (two firms and consumers) and each makes multiple decisions at different stages; these are endogenously interlinked. By analyzing the entire game, we identify conditions related to demand and product market competition under which an innovator should disclose an innovation, inviting competitors into the market.

The remainder of this paper proceeds as follows. In section 2, we relate our paper to the existing literature in marketing and economics. Section 3 presents the model, and section 4 provides our main results and analysis. In section 5, we extend our model and Section 6 concludes.

2 Literature Review

This paper is related to several streams of research. First, it contributes to the literature on a firm’s communication strategy by highlighting the two-sided costly efforts in the communication process. Marketing communication is typically viewed as one-sided costly communication, with firms bearing all the costs. In those models, consumers effortlessly become informed and recognize their needs once they are exposed to a firm’s advertising. In contrast, we recognize that the success of communication requires both a firm’s persuasion efforts and consumers’ learning or elaboration efforts. Wernerfelt (1996) is the first paper that recognizes marketing communication as teamwork between firms and consumers, and he suggests that an efficient communication plan should induce the efficient division of labor between the two parties. Dewatripont and Tirole (2005) formalized this perspective and developed a formal model of communication. They argued that the acts of formulating and absorbing the content of communication are privately costly, and the successful communication depends on the efforts of both sender and receiver together. Furthermore, they suggested that communication
eorts are strategic complements; that is, one side will try harder if the other side also tries harder. We expand Dewatripont and Tirole (2005)’s framework by incorporating the competition between firms. Thus, the communication in our model is characterized not only by strategic complementarity between the consumers’ effort and the firms’ aggregate effort, but also by strategic substitutability between each firm’s individual efforts. Our model explores several interesting tradeoffs between these two key strategic effects in communication, and its implications for secrecy within an innovative firm.

Second, a number of papers examine the impact of consumers’ costs of learning and deliberation. Shugan (1980) is among the earliest to highlight the importance of incorporating consumers’ cost of thinking, and several subsequent papers explore its implications for a firm’s strategy (Guo and Zhang 2012, Kuksov and Villas-Boas 2010, Villas-Boas 2009, Wathieu and Bertini 2007). Wathieu and Bertini (2007) explores the firm’s pricing strategy to spur deeper thinking among consumers about the personal relevance of product benefits; others (Guo and Zhang 2012, Kuksov and Villas-Boas 2010, Villas-Boas 2009) explore the firm’s optimal product line decisions when consumers need to incur costly deliberation eorts to evaluate the product. We also explore the implications of consumers’ costly eorts for deliberation to evaluate a new product, but our focus is on the consumers’ essential role in marketing communication.

Third, the current work also relates to the literature on a firm’s strategies of managing business secrets and information sharing. One stream of research in this area focuses on a firm’s choice between patenting and secrecy. Patents protect but publicize the firm’s proprietary knowledge through the patent process thus enabling it to exploit its private information (secrets) while reducing asymmetry between the firm and its competitor. Competitors can imitate and create differentiated products. Thus, firms may sometimes forgo patenting and keep secrets internally. Several papers have examined when to disclose secrets for patent protection and when to retain trade secrets internally (Kultti et. al. 2007, Denicolo and Franzoni 2004, Horstman et. al. 1985, and Anton and Yao 2004).

Another stream of research in this area deals with the information sharing decision in the context of R&D – whether or not a firm discloses its interim R&D results to the public. If R&D contests are “winner-take-all”, firms disclose pessimistic or no information to discourage a rival’s investment (Hendricks and Kovenock 1989 and Gill 2008). If the appropriability of an innovation is weak, the free-riding incentive arises and firms disclose positive information about their interim R&D results to further encourage a rival’s R&D investment (De Fraja 1993 and Jansen 2010). While the appropriability of an innovation is exogenously given in these papers, the appropriability in ours is endogenously determined by the degree of information disclosure; moreover, this endogenous appropriability affects the rival’s incentive to invest in communication efforts in equilibrium.

Fourth, broadly speaking, our research deals with a firm’s strategies around cooperation and competition with competitors, an important issue of coopetition (Brandenburger and Nalebuff 1996).
A large body of literature examines various forms of co-opetition such as strategic alliances among competing firms (Amaldoss et al. 2000, Bucklin and Sengupta 1993, Luo et al. 2007, Harbison and Pekar 1998),\textsuperscript{13} collaboration with a rival firm through licensing (Inkpen 1996, Steensma 1996), and impacts of these actions on innovation (Bonacorsì and Lipparine 1994, Hamel et al. 1989, Nieto and Santamaria 2007, Tsai 2009). Our paper contributes to this literature by providing one new micro-foundation for co-opetition based on the firm’s communication motives.

Farrell and Gallini (1988) and Shepard (1987) are the two most closely related works in this area. These suggest that inviting competitors might increase the total market demand because competition can serve as a credible commitment to lower price (Farrell and Gallini 1988) or higher quality (Shepard 1987), both of which guarantee a higher \textit{ex post} consumer surplus. Our paper captures a similar insight, but also provides a micro model of how this increased consumer surplus leads to greater market demand based on the communication mechanism. Moreover, our micro model of communication captures not only complementarity between firms and consumers (Farrell and Gallini 1986, and Shepard 1988), but also substitutability between firms.

Finally, the current work is related to a large body of marketing literature on a firm’s preannouncement strategy. The common premise of literature in this area is that the firm always wants to deter the entry of competitors and preannouncement is used to achieve this end through communication with competitors (Bayus et al. 2001, Haan 2003, and Ofek and Turut 2013) or consumers (Farrell and Saloner 1986, Garlach 2004, and Choi et. al. 2005). Our paper primarily differs from these in that we provide another strategic motive of preannouncement – to invite the competitor to enhance the potential market demand through jointly communicating with consumers.\textsuperscript{14}

\section{Model Set-up}

The market consists of two firms, firm 1 and 2, and a continuum of consumers with mass of 1. Firm 1 is an innovator who discovers a radical innovation and firm 2 is a follower. Once firm 1 discovers a new idea, it chooses a level of secrecy by deciding the amount of information to publicly release about its novel product idea. We denote $d$ as the amount of information that firm 1 chooses to disclose, and thereby, $1 - d$ represents the level of secrecy.$^{15}$ The degree of disclosure is a continuous variable, $d \in [0, 1]$, where $d = 0$ means no disclosure (i.e., complete secrecy) and $d = 1$ means full

\textsuperscript{13}Harbison and Pekar (1998) find that more than 50\% of strategic alliances are typically formed among competing firms within the same industry.

\textsuperscript{14}In our model, the market demand is endogenously determined by the communication between firms and consumers. There are several papers that also study the issue of endogenous market demand (in particular, market expansion by the supply side efforts); Agarwal and Bayus (2002) find empirical evidence that the entry of firms in the early stage of the industry can increase consumer awareness and learning, and thus expand the potential market. Shen and Xiao (2014) find that market demand can increase endogenously with the number of firms by facilitating consumer learning in fast food market in China. Similarly, Shen (2014) emphasizes the role of firms’ collective advertising on generating the demand for the new product category. We use secrecy and disclosure interchangeably throughout the paper.

\textsuperscript{15}We use secrecy and disclosure interchangeably throughout the paper.
The innovator discovers a new idea, and decides the degree of disclosure $d \in [0,1]$. The follower decides whether or not to enter the new product market $\chi = \{0,1\}$. If $\chi = 1$, the follower decides whether to enter the market after observing the innovator's disclosure decision. At stage 2, firms and consumers decide their optimal levels of communication efforts to transfer and assimilate the idea. Finally, at stage 3, firms compete with each other in the product market and decide on prices while consumers make a purchase decision.

It is important to note that these decisions are interlinked. For example, when the innovator decides the secrecy in stage 1, it fully takes into account the effects of secrecy (1) on the follower’s decision regarding whether to enter the market (at stage 1), (2) on the equilibrium outcomes of the communication game between firms and consumers (communication subgame in stage 2), and (3) on the equilibrium outcomes of the pricing game in the product market played by the firms (product market subgame in stage 3). Similarly, the communication efforts by firms and consumers endogenously determine the potential market demand, which clearly affects the equilibrium payoffs.

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Figure 1: Sequence of the game

disclosure (i.e., no secrecy). Then, as long as $d > 0$, the follower makes a decision about whether or not to enter the market by following the new idea and developing a similar product, $\chi = \{0,1\}$.16 If the follower decides not to enter ($\chi = 0$), the eventual product market will be a monopoly. If the follower decides to enter ($\chi = 1$), the market becomes a duopoly.

Prior to competition in the product market, consumers and firms communicate with one another about the new product idea. We model this marketing communication process as two-sided costly efforts, a process that involves both firms’ and consumers’ efforts to transmit and assimilate the novel product concept (we will discuss the communication mechanism in greater detail later). The communication’s success endogenously generates consumers’ need recognition and thereby market demand for a novel product. Finally, once the product is launched in the market, the innovator and the follower (if $\chi = 1$) offer differentiated products at prices $\{p_1, p_2\}$, and engage in Bertrand competition whereas if $\chi = 0$, firm 1 monopolizes the market and offer its product at $p_1$.

The entire game involves several distinctive decisions for each firm and consumers at different stages as we can see in Figure 1. At stage 1, the innovator discovers a new idea and decides the amount of information to disclose, $d \in [0,1]$. Then, the follower makes a decision whether to enter the market after observing the innovator’s disclosure decision. At stage 2, firms and consumers decide their optimal levels of communication efforts to transfer and assimilate the idea. Finally, at stage 3, firms compete with each other in the product market and decide on prices while consumers make a purchase decision.

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16Here, we make a simplifying assumption that the competitor can immediately understand the value of the new innovation, and therefore, can always make a similar product once a little piece of the innovation is shared ($d > 0$).
for the firms in the product market at the stage 3. Hence, when the firms choose their optimal effort levels for communication, they fully internalize the future payoffs from the product market, in which they compete in pricing. Furthermore, these considerations affect the follower’s decision regarding whether to enter the market at stage 1. Thus, we use the Subgame perfect Nash equilibrium as our solution concept. We now discuss each of these decisions in detail.

Information disclosure

Managing the secrecy of the innovative idea has strategic implications for the innovator’s competitive advantage. The more innovation the innovator keeps secret (or equivalently, the less the innovator discloses), the more the technological leadership it can possess, and thus the innovator can have greater competitive advantage over the competitor when the product idea is fully developed and launched in the product market. We model the decision of the innovator’s information disclosure as a continuous variable, \( d \in [0, 1] \). An innovator can disclose a piece of proprietary information to its competitor through licensing contracts which can directly affect the relative competitive advantage of the innovator (for example, Tesla and semiconductor companies). Here, we impose a truth-telling assumption such that revealed information is truthful. In this licensing case, \( d \) can be interpreted as the amount of innovation disclosed (no innovation disclosure as \( d = 0 \) and full innovation disclosure as \( d = 1 \)). Alternatively, an innovator can release information about its upcoming product well in advance of actual market availability through preannouncement. By making preannouncement, firms can deliver information on a product while it is still being developed. In this preannouncement case, \( d \) can be interpreted as the length of time prior to actual launch of the product or the actual maturity of an innovative idea upon market announcement; for example, keeping the innovation completely secret and announcing the product on the same time as the product release (\( d = 0 \)) or disclosing its premature idea so early that the follower can fully catch up with the innovator on the time when the actual product is launched (\( d = 1 \)).

We capture the influence of information disclosure on the innovator’s competitive advantage through a cost function. We assume that the innovator can retain its cost advantage stemming from its technical breakthrough if the firm keeps it secret. For example, when Procter & Gamble first introduced disposable diapers in 1969, the company used a synthetic fiber technology that

\[ \text{We assume that there is a fixed cost of } \epsilon \text{ associated with the information disclosure – this } \epsilon \text{ cost is a tie-breaker for the innovator; when it is indifferent between disclosing information and no disclosure, it chooses not to disclose because of this small fixed costs.} \]

\[ \text{The credibility of the announcement is an important issue, and preannouncement can be a pure cheap talk (Bayus et al. 2001, Choi et al. 2005, and Ofek and Turut 2013). However, this cheap-talk analysis is beyond the scope of this paper, and we treat the preannouncement as truthful information sharing. By doing so, we can focus on the effects of information disclosure on market competition, and analyze the strategic motives for a firm to convey its private information on a new innovation in order to change other parties’ behavior in a way that is beneficial to the firm.} \]

\[ \text{Alternatively, one can model the competitive advantage through demand side advantage (consumers may have higher willingness to pay for the innovator’s product, and the sharing of innovation reduces this advantage). We also explored this specification, but the qualitative results remain the same.} \]
helped them to create the diapers at a cheaper, more profitable price over competition (Morrison et al. 2000). We can capture the extent of competitive advantage through secrecy; we fix the innovator’s marginal production cost as \( m_1 = 0 \), and the follower’s marginal production cost \( m_2(d) \) is a decreasing function in \( d \), \( m_2(d) = \bar{m} \cdot (1 - d) \) without loss of generality. Hence, \( m_2(d = 1) = 0 \), and \( m_2(d = 0) = \bar{m} > 0 \). Thus, when \( d = 1 \), the technical innovation is shared with the competitor perfectly, and the innovator has no relative cost advantage \( (m_1 = m_2(d = 1) = 0) \) while the competitive advantage is maximized when \( d = 0 \) \( (m_2(d = 0) = \bar{m} > 0 = m_1) \).

**Communication technology**

We consider a communication technology where both the firms and consumers simultaneously exert elaboration and learning efforts to transfer and assimilate the idea (Dewatripont and Tirole 2005). The probability that the idea is properly communicated to and accepted by consumers is

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\sigma(e_f, e_c) = e_f^k \cdot e_c^{1-k},
\]

where \( e_f \in [0, 1] \) is the aggregate effort of the firms to transmit the innovative idea and persuade consumers to accept it, and \( e_c \in [0, 1] \) is consumer’s efforts to pay attention, deliberate and assess this message. Moreover, \( k \in [0, 1] \) captures the relative importance of two side efforts in communication.

In a monopoly situation \( (\chi = 0) \), the total effort of the firms is \( e_f = e_1 \); in a duopoly situation \( (\chi = 1) \), \( e_f = e_1 + e_2 \).\(^{21}\)

When radical new product categories such as HDTV and recordable DVD format are first introduced in the market, the realization of the market demand is uncertain and, prior to launch, firms need to expend efforts to stimulate greater demand by showing advertisements that mainly promote the benefits and advantages of these new product concepts without touting brand-specific features (Bass et al. 2005). Such advertisements are particularly common in the introduction of new product categories: when satellite radio was first introduced, both Sirius and XM allocated significant portions of their advertising budgets for generic advertising to inform and educate consumers about the product’s uses and benefits to enhance product acceptance (Beardi 2001).\(^{22}\) One can think of these advertising activities as a firm’s communication efforts \( e_f \), and consumers’ costs of learning and deliberation to evaluate these new product idea as \( e_c \), which is time consuming and leads to depletion of cognitive resources (Shugan 1980, Guo and Zhang 2012). Thus, communication efforts involve private costs \( C(e) \) for the firms and consumers. We assume these costs are increasing,

\(^{20}\)This is a generalized version of Dewatripont and Tirole (2005), and their model can be considered as a special case where both parties’ efforts are equally important: \( k = \frac{1}{2} \).

\(^{21}\)To guarantee the probability \( \sigma \leq 1 \), we assume \( v \leq 1 \). The assumption is only for expositional purpose. We will see later that the results rely on the relative ratio of vertical differentiation to horizontal differentiation (i.e., \( \frac{v}{t} \)) not on a specific value of \( v \).

\(^{22}\)XM’s ads will be about continuing to ‘grow the whole category pie,’ rather than competing with Sirius” (Boston and Halliday 2003).
convex such that \( C(e) = \frac{e^2}{2} \).

There are several important properties in this communication technology that we employ. First, \( \sigma(e_f, e_c) = 0 \) when \( e_f = 0 \) or \( e_c = 0 \). This captures the key characteristic that communication requires two-sided efforts, and the communication can never succeed without both sides exerting effort. Second, this communication technology assumes complementarity between the efforts of consumers and the aggregate effort of the firms: \( \sigma_{fc} = \frac{\partial^2 \sigma(e_f, e_c)}{\partial e_f \partial e_c} \geq 0 \). That is, each side is willing to exert more effort if the other side also exerts more efforts, and the return from expending effort increases when the other party explains better or listens better (Dewatripont and Tirole 2005, p. 1224).\(^{23}\) Furthermore, in the duopoly case, we assume that both firms are symmetric in terms of their effects on the effectiveness of communication; that is, \( \sigma_1 = \sigma(e_1 + e_2, e_c) = \sigma(e_1 + e_2, e_c) = \sigma_2. \(^{24}\)

Thus, the efforts between each firm and consumers are strategic complements. Finally, another key characteristic of our communication technology is that the efforts of the two firms are strategic substitutes: \( \sigma_{12} = \frac{\partial^2 \sigma(e_f, e_c)}{\partial e_1 \partial e_2} \leq 0 \). Because communication involves private costs, firms have incentives to reduce their efforts and free ride on the other side’s communication efforts. Hence, the communication stage can be characterized by the strategic complementarity between the consumers’ effort and the firms’ aggregate effort, and the strategic substitutability between each firm’s individual efforts, which are the crux of the current paper’s premise.

Once the idea is successfully communicated (with probability \( \sigma \)), consumers can understand the potential benefits of the new product idea. If the idea fails to be communicated (with probability \( 1 - \sigma \)), consumers cannot be aware of the potential benefits for this new product, and they may think the new innovative idea (such as Google Glass) is only for geeks and not for them. Thus, they will not even consider purchasing the new product. Consumers consider purchasing the product only if the idea is successfully communicated with probability \( \sigma \). Those who become educated about the radical innovation are the potential buyers when the product becomes available in the market.

Here, the role of marketing communication is to generate the consumers’ need recognition and product acceptance. Thus, marketing communication efforts endogenously creates and expands the total demand for the new product; the total number of potential buyers is \( \sigma^T = \sigma(e_f, e_c) \).

**Competition in product market**

The product market competition is modeled as a traditional Hoteling model. The innovator and the follower are located at the two extremes of a Hoteling line, 0 and 1, respectively. They compete in prices by charging \( p_1, p_2 \), respectively when the market is a duopoly. If the market is a monopoly,\(^{23}\) Note that this strategic complementarity arises because \( \sigma \) is endogenous and strategic choices of both firms and consumers. On the other hand, as Dewatripont and Tirole (2005) explained, if \( \sigma \) is fixed and exogenously given, one side can expend less effort as the other side has expended more effort.

\(^{24}\)The firms may have asymmetric influences in persuading consumers. We can capture the asymmetry by setting the total firm-side effort as \( e_F = \beta e_1 + e_2 \). The parameter \( \beta \) affects the exact equilibrium effort levels, but the underlying mechanism that drives the equilibrium outcomes does not change due to the additive form of the specification. Hence, our main results do not change qualitatively.
only the innovator chooses the monopoly price $p_1$.

When communication between firms and consumers succeeds (with probability $\sigma$), consumers become a potential buyer. Potential buyers therefore receives a base utility $v$ from consumption if they purchase a product. Besides the base utility $v$, consumers receive an idiosyncratic match once the idea is developed into a product. Consumers’ preferences ($z_j$) are uniformly distributed along the Hotelling line, $z_j \sim U[0,1]$. Their utility is $u_j = v - t \cdot z_j - p_i$, where $i = 1, 2$, and $t$ is the consumer’s travel (mismatch) cost from the ideal point in Hotelling line. After observing their preferences $z_j$ and the price(s), potential buyers make their purchase decisions and the firms receive their payoffs. And we assume that $t$ is small enough ($t < \frac{v}{3}$) that in equilibrium, all potential buyers purchase from one of two firms in duopoly case.

4 Analysis

The game consists of three subgames: innovation disclosure, communication, and product market competition. We use the Subgame Perfect Nash Equilibrium as our solution concept. We begin our analysis using backward induction to solve the game and all the proofs are presented in the Appendix.

4.1 Stage 3: Product market competition

We first solve the final stage of the game of product market competition. We denote firm 1’s disclosure level as $d$ and the firms’ aggregate communication efforts and consumers’ efforts as $\{e_f, e_c\}$. These will be endogenously determined in equilibrium, but for now we take them as given. There are two different cases we need to consider depending on whether the follower, firm 2, decides to enter the market or not (which again will be endogenously determined in equilibrium).

Duopoly ($\chi = 1$)

As we can see later, there always exists for firm 1 a disclosure level $d^*$ such that for all $d \geq d^*$ firm 2 chooses to follow firm 1’s idea ($\chi = 1$), and thus the market is a duopoly at Stage 3.\footnote{We impose a simplifying assumption of tie-breaker such that when $d = d^*$, firm 2 chooses to enter the market even though it makes exactly zero profit in the product market. This is purely for expositional easiness and it does not affect the results.} Given a fixed $d \geq d^*$ and communication efforts $e_f = e_1 + e_2$ and $e_c$, the total number of consumers who are convinced about the need for the new product is fixed as $\sigma^T = \sigma(e_f, e_c)$. Those potential buyers are uniformly distributed along the Hotelling line $z_j \sim U[0,1]$, and purchase a product either from firm 1 or firm 2. Consumer $j$ obtains utility $u_{1j} = v - t z_j - p_1$ if purchasing from firm 1 and utility $u_{2j} = v - t(1 - z_j) - p_2$ if purchasing from firm 2, where the consumer’s travel cost $t$ captures the extent of product differentiation between two products. Consumer $j$ purchases from
firm 1 if \( u_{1j} \geq u_{2j} \Rightarrow z_j \leq \frac{1}{2} - \frac{p_{1} - p_2}{2t} \). Thus, firm 1’s demand is \( D_1(p_1, p_2) = \sigma^T \left( \frac{1}{2} - \frac{p_{1} - p_2}{2t} \right) \) and firm 2’s demand \( D_2(p_1, p_2) = \sigma^T \left( \frac{1}{2} + \frac{p_{1} - p_2}{2t} \right) \). Here, \( \sigma^T \) defines the total category demand while \( \left( \frac{1}{2} - \frac{p_{1} - p_2}{2t} \right) \) and \( \left( \frac{1}{2} + \frac{p_{1} - p_2}{2t} \right) \) define the selective demand for each firm within the category. The two firms compete on prices \((p_1, p_2)\) that maximize the retail their own profits \( R_1, R_2 \):

\[
\begin{align*}
    p_1^D &= \arg \max_{p_1} R_1(p_1; p_2^D) = \sigma^T (p_1 - m_1) \left( \frac{1}{2} - \frac{p_1 - p_2^D}{2t} \right) \\
    p_2^D &= \arg \max_{p_2} R_2(p_2; p_1^D) = \sigma^T (p_2 - m_2) \left( \frac{1}{2} + \frac{p_2^D - p_2}{2t} \right)
\end{align*}
\]

where firm 1’s marginal cost \( m_1 = 0 \) and firm 2’s marginal cost \( m_2 = \frac{\bar{m}}{2} \cdot (1 - d) \), which is a decreasing function of \( d \).

Then, we have the equilibrium duopoly prices \( p_1^D = t + (1 - d) \frac{\bar{m}}{2}, p_2^D = t + (1 - d) \frac{2m}{3} \). Because the firm 2’s marginal production cost should be low enough to enable it to make positive profit in the product market \( p_2^D \geq m_2 \), we obtain that \( d \geq d^* = 1 - \frac{3t}{m^*} \).

Also, the equilibrium duopoly retail profits are

\[
\begin{align*}
    R_1^D &= \sigma^T \left( \frac{(3t + (1 - d)m^2)}{18t} \right) = \sigma^T \cdot \pi_1^D, \\
    R_2^D &= \sigma^T \left( \frac{(3t - (1 - d)m^2)}{18t} \right) = \sigma^T \cdot \pi_2^D,
\end{align*}
\]

where \( \pi_1^D = \left( \frac{(3t + (1 - d)m^2)}{18t} \right), \pi_2^D = \left( \frac{(3t - (1 - d)m^2)}{18t} \right) \) are average profit that firm 1 and 2 make in the duopoly product market per customer, respectively. It is important to know that this average profit \( \pi_i \) is independent of the total market size, and only captures the marginal benefit to the firm from the increase in the total category demand \( \sigma^T \). This will play an important role in the subsequent analysis.

Therefore, as long as firm 1 discloses sufficient information \((d \geq d^* = 1 - \frac{3t}{m^*})\), firm 2 makes non-negative profit by entering the market. Also, note that firm 1’s profit decreases as it discloses more information (i.e., \( d \) increases) whereas firm 2’s profit increases in \( d \). Next, we analyze the retail market if firm 1 becomes the monopolist (i.e., \( d < d^* \)).

**Monopoly (\( \chi = 0 \))**

As long as firm 1 keeps a sufficient degree of secrecy \((d < d^*)\), firm 2 cannot make a positive profit by entering the market. Therefore, firm 2 chooses not to work on a similar idea (\( \chi = 0 \)), and firm 1 becomes the monopolist in the product market at stage 3. Similar to the duopoly case, potential buyers make purchase decisions once they observe the price and their preferences \( z_j \sim U[0, 1] \). Consumer \( j \)'s utility is \( v - tz_j - p_1 \) if they decides to buy. Consumer \( j \) purchases the product

\[\text{We assume that } \bar{m} > 3t \text{ for the non-emptiness of } d^* = 1 - \frac{3t}{m^*}. \text{ Also, } v > 3t \text{ ensures that even the marginal consumer } (z_j^* = \frac{1}{2} + \frac{\bar{m}}{2} \frac{(1 - d)}{3}) \text{ who is indifferent between purchasing from firm 1 and firm 2, receives a positive utility in equilibrium: } u^* = v - t \left( \frac{1}{2} + \frac{\bar{m}}{2} \frac{(1 - d)}{3} \right) - t - (1 - d) \frac{m^*}{2} > 0.\]
if the utility $v - tz_j - p_1 \geq 0 \Rightarrow z_j \leq \frac{v - p_1}{t}$. Thus, the market demand for firm 1’s product is

$$D_1(v, p_1) = \sigma^T \cdot \max\left\{\frac{v - p_1}{t}, 1\right\}.$$  

Under the market coverage condition in duopoly ($v > 3t$), the equilibrium price is $p_1^M = v - t$, and the market demand is $D_1 = \sigma^T$.

Firm 1’s monopoly retail profit is

$$R_1^M = \sigma^T \cdot (v - t) = \sigma^T \cdot \pi_1^M$$  

where $\pi_1^M = (v - t)$ is the average profit that firm 1 makes in the monopoly product market per customer.

### 4.2 Stage 2: Communication game

The market size $\sigma^T = \sigma(e_f, e_c)$ is the outcome of a two-sided marketing communication, a process in which firms and consumers both exert costly effort. There are three players involved in marketing communication (firm 1 and 2, and consumers), and they are all strategic and forward looking. For a given $d$ of firm 1’s choice in the first stage, each player decides on their own communication efforts anticipating the equilibrium product market outcomes at stage 3. Firm 1 and firm 2 decide on the communication efforts $e_1, e_2$, and consumers also decide on their efforts $e_c$. Since consumers have not observed the idiosyncratic preferences ($z_j$) yet, they form expectation on the consumption utility taking into account the equilibrium prices offered by both firms in the product market competition at the stage 3:

$$E u = \hat{u} = \begin{cases} \hat{u}^M = \int_z \max\{v - tz - p_1^M, 0\} dz & \text{Monopoly case} \\ \hat{u}^D = \int_z \max\{v - tz - p_1^D, v - t(1 - z) - p_2^D\} dz & \text{Duopoly case} \end{cases}$$

When we compare the expected utility of consumers between the monopoly case and duopoly case, it is immediately clear that the competition of duopoly provides a higher expected consumer utility. Thus, competition can serve as a credible commitment to guarantee a higher consumer surplus.

**Lemma 1.** The consumer’s expected utility from purchasing the product is higher under duopoly than monopoly market: $\hat{u}^D > \hat{u}^M$.

### Strategic complementarity and substitutability

When $d < d^*$, the follower (firm 2) chooses to not follow the innovator’s idea ($\chi = 0$) and the innovator (firm 1) engages in marketing communication by itself. The single-firm communication subgame simply displays a strategic complementarity between a firm and consumers. However,

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27 The optimal price from FOC is $p_1 = \arg \max_{p_1} \sigma^T \cdot p_1 \cdot \frac{v - p_1}{t} = \frac{v}{t}$. However, even the consumer at $z_i = 1$ finds the positive utility under this price: $u = v - t - \frac{v}{t} > 0$ because $v > 3t$. Hence, the equilibrium price is determined by $u(z_i = 1) = v - t - p_1 = 0 \Rightarrow p_1^M = v - t$, and the market demand is $D_1 = \sigma^T$. 

---
when \( d \geq d^* \), both firms engage in collective marketing communication, and each firm has an incentive to free ride on the other’s costly efforts. Hence, the communication also displays a strategic substitutability between the two firms, which complicates the analysis.

Let us start from the case of monopoly market \((\chi = 0)\) when \( d < d^* \). The innovator engages in marketing communication by itself. Therefore, firm 1 and consumers choose the optimal communication efforts \( e_1, e_c \) that maximize their total expected payoffs:

\[
EI_1 = (e_1^k \cdot e_c^{1-k}) \pi_1^M - \frac{e_c^2}{2},
\]

\[
EU = (e_1^k \cdot e_c^{1-k}) \tilde{u}^M - \frac{e_c^2}{2},
\]

(6)

Here, the single-firm communication simply displays a strategic complementarity between a firm’s and consumers’ efforts: \( \frac{\partial^2 EI_1}{\partial e_1 \partial e_c} = k (1 - k) e_1^{k-1} e_c^{-k} \pi_1^M \geq 0 \), and \( \frac{\partial^2 EU}{\partial e_1 \partial e_c} = k (1 - k) e_1^{k-1} e_c^{-k} \tilde{u}^M \geq 0 \).

Due to this strategic complementarity effect, increasing effort from one side leads to higher response from the other side. Thus, the equilibrium efforts under monopoly are determined by the following best response functions:

\[
e_1^M = (k \pi_1^M e_c^{1-k})^{\frac{1}{1+k}},
\]

\[
e_c^M = ( (1 - k) \tilde{u}^M e_1^k )^{\frac{1}{1+k}},
\]

(7)

Both equilibrium efforts \( e_1^M(e_c) \) and \( e_c^M(e_1) \) are increasing in the other side’s effort \( e_c \) and \( e_1 \), and its own expected average payoffs from the product market \( \pi_1^M \) and expected utility \( \tilde{u}^M \), respectively.

Next, we consider the case of duopoly market \((\chi = 1)\) when \( d \geq d^* \). In duopoly, both firms engage in collective marketing communication. Thus, firm 1, firm 2 and consumers choose the optimal communication efforts \( (e_1, e_2, e_c) \) that maximize the following payoff functions:

\[
EI_i = (e_1^k \cdot e_c^{1-k}) \pi_i^D - \frac{e_c^2}{2},
\]

\[
EU = (e_1^k \cdot e_c^{1-k}) \tilde{u}^D - \frac{e_c^2}{2},
\]

(8)

where \( i = 1, 2 \), and \( e_f = e_1 + e_2 \) is the firm side total effort.

Same as in the monopoly market case \((\chi = 0)\), there exist a strategic complementarity effect between consumers’ efforts and firms’ aggregate efforts: \( \frac{\partial^2 EI_i}{\partial e_i \partial e_c} = k (1 - k) e_f^{k-1} e_c^{-k} \pi_i^D \geq 0 \), and \( \frac{\partial^2 EU}{\partial e_i \partial e_c} = k (1 - k) e_f^{k-1} e_c^{-k} \tilde{u}^D \geq 0 \), for all \( i = 1, 2 \).

In addition to strategic complementarity, the communication game also shows the strategic substitutability between each firm’s individual effort: \( \frac{\partial^2 EI_i}{\partial e_i \partial e_j = i} = -k (1 - k) (e_1 + e_2)^{k-2} e_c^{1-k} \pi_i^D \leq 0 \), for all \( i = 1, 2 \). This strategic substitutability effect between each firm’s individual effort creates a free-riding problem in communication such that each firm has an incentive to lower its own effort in response to the other firm’s effort.

To measure this free-riding problem more directly, we first examine the equilibrium efforts under
duopoly case for firm \( i \in \{1, 2\} \) and consumers, which are implicitly given by

\[
e^D_i = k\pi^D_1 \left( e^D_c \right)^{1-k} \left( e^D_i + e^D_j \right)^{k-1},
\]

\[
e^D_c = \left( (1-k) \hat{\pi}^D \left( e^D_i + e^D_j \right)^k \right)^{\frac{1}{k+1}}.
\]  

(9)

Similar to the monopoly case (equation 7), the optimal effort of firm \( i \left( e^D_i \right) \) is increasing in consumer side’s effort \( e^D_c \), and its own expected average profit \( \pi^D_i \). However, different from the monopoly case, the firm \( i \)'s effort is now affected by the other rival firm \(-i\)'s effort:

\[
\frac{\partial e_i}{\partial e_{-i}} = \frac{-1}{1 + \frac{e^D_j}{(1-k)\pi^D_i}} \leq 0.
\]  

(10)

This is the direct effect of the rival firm \(-i\)'s effort on its own effort arising from strategic substitutability. The success of communication benefits both firms, but efforts are costly. Therefore, each individual firm has an incentive to free ride on the other firm’s costly effort and lower its own effort in response to increase in the other firm’s effort.

Because of the countervailing effects of strategic complementarity and strategic substitutability, the total effect of an increase in the rival firm’s effort is ambiguous. More precisely, there are two (direct and indirect) effects of the rival firm’s effort on the focal firm’s own efforts. Firm \( i \)'s best response to the increase of the other firm’s effort is to decrease its own because of its incentive to free ride on the other firm’s costly effort (equation 10). Nevertheless, due to the strategic complementarity effect between \( e_c \) and \( e_i \), consumers’ efforts \( e_c \) increases with firm \(-i\)'s efforts \( \left( \frac{\partial e_c}{\partial e_{-i}} \geq 0 \right) \). More importantly, this increased effort in consumers \( (e_c) \) in turn increases firm \( i \)'s efforts \( \left( \frac{\partial e_i}{\partial e_c} \geq 0 \right) \). This is the indirect effect of firm \(-i\)'s effort on the firm \( i \)'s effort. Hence, firm \( i \) may find it profitable to raise the effort with firm \(-i\)'s effort through this indirect effect that arises from complementarity through consumer effort. In other words, complementarity may reduce the firms’ free-riding incentives, which may result in higher total efforts. We explore this interaction between these countervailing effects next.

**Communication incentives and optimal efforts**

We first note that the incentives for all players (firms and consumers) to exert effort increases in the expected payoffs that they can receive from the product market at stage 3 in both monopoly and duopoly cases. Also, because of strategic substitutability, one firm has incentives to free ride off the other firm’s efforts in a duopoly. Such a case, we can see that the ratio of efforts \( \frac{e_1}{e_2} \) between two rival firms is equal to the ratio of their expected average profits \( \frac{\pi^D_1}{\pi^D_2} \).

\( ^{28} \)Also, the optimal level of effort for consumers \( e^D_c \) is again increasing in the other side’s efforts (the firm side’s total effort \( e^D_j \)) and its own expected utility in the product market \( \hat{\pi}^D \).
**Proposition 1.** Equilibrium communication efforts can be characterized as follows:

1. Each player’s equilibrium effort in communication increases in its own expected surplus from the product market: \( \frac{\partial e_i^M}{\partial \pi_i^M} \geq 0 \), \( \frac{\partial e_i^D}{\partial \pi_i^D} \geq 0 \) (for \( i = 1, 2 \)), and \( \frac{\partial e_c}{\partial \tilde{u}} \geq 0 \).

2. Moreover, in duopoly case (\( \chi = 1 \)), the ratio of efforts between two rival firms in duopoly (\( \frac{e_1^D}{e_2^D} \)) is equal to the ratio of their expected average profits from the product market: \( \frac{\pi_1^D}{\pi_2^D} = \frac{\pi_1^D}{\pi_2^D} \).

In a duopoly case, the relative efforts between the two firms is only determined by the split of the total product market profits regardless of levels of consumers efforts. Hence, the innovator may find it optimal to sacrifice market profit by sharing its innovation with competitors. Increasing the competitor’s average profit can induce the competitor to shoulder more communication costs and expand the total market size through cooperation in this stage. In particular, when \( d = 1 \) (full disclosure), the expected average profits are identical (\( \pi_1^D = \pi_2^D \)), and therefore, the equilibrium levels of efforts are identical (\( e_1^D = e_2^D \)). However, the exact level of firms’ efforts is influenced by consumer efforts for communication, which are affected by consumers’ expected payoffs from the product market.

From Lemma 1, we already know that the consumer’s expected utility is higher under a duopoly than under a monopoly market (\( \tilde{u}^D > \tilde{u}^M \)). This proposition implies that consumers always have more incentives to incur costly learning efforts under duopoly than under monopoly (since \( \frac{\partial e_c}{\partial \tilde{u}} \geq 0 \)). This is summarized in the following corollary.

**Corollary 1.** Consumers exert more efforts under a duopoly than a monopoly market for all \( k < 1 \): \( e_c^D > e_c^M \).

This result suggests another important incentive for the company to share its innovation with a competitor. The presence of a competitor serves as a commitment mechanism to assure consumers that they will receive higher utility in the product market, which increases the incentives for consumers to expend more costly effort in deliberating about the new innovation. More importantly, this increased consumer effort further encourages firms to put more efforts in the communication stage through complementarity between firms and consumers (equation 9). Sharing the innovation is a way to invite the competitor to the market, which triggers higher consumer efforts and ensuing firms’ efforts all together. Thus, an innovator may benefit from attracting a competitor.

Next, we compare the equilibrium market size. Under both monopoly and duopoly cases, the total market sizes are given by

\[
\sigma^T = (1 - k)^{1-k} k^k (\tilde{u})^{1-k} (\pi_f)^k,
\]  

where \( \pi_f \) denotes the firm side aggregate average retail profits. If the market is a monopoly, \( \pi_f^M = \pi_1^M \); if the market is a duopoly, \( \pi_f^D = \pi_1^D + \pi_2^D \).
The direct comparison of the total market size $\sigma^T$ under duopoly and monopoly (equation 11 above) allows us to identify the exact condition under which the market expands under duopoly.

**Proposition 2.** For any given $d \in [0, 1]$, there always exists a threshold $\tilde{k}(d) \equiv \frac{1}{1 + \ln \frac{\pi^M_1}{\pi^M_2} / \ln \frac{\pi^D_2}{\pi^D_1}}$ such that for all $k \leq \tilde{k}(d)$, the total market size is greater under duopoly than under monopoly: $\sigma^{TD}(d) \geq \sigma^{TM}$.

This result finds that whether or not the market expands under duopoly simply depends on the consumers’ expected utility $\tilde{u}$ and the firms’ aggregate average retail profits $\pi_f$. More importantly, as consumers play an even more important role or the complementarity effect of consumers becomes larger ($k \leq \tilde{k}$), the market tends to expand under duopoly, and thus, the innovator is more likely to disclose its innovation to invite competitor in the market.

When the competitor’s communication effort increases (which can be induced by sharing more information and thus guaranteeing a sufficient amount of profit in the product market), the innovator has incentives to lower its efforts because of substitutability or free-riding effect (equation 10). At the same time inviting the competitor allows the innovator to commit to provide greater surplus to consumers, which increases the incentives for consumers to expend more costly effort in deliberating about the new innovation. Moreover, as the complementarity effect of consumers becomes larger ($k \leq \tilde{k}$), this increased effort in consumers ($e_c$) in turn increases firm’s efforts ($\frac{\partial e_i}{\partial e_c} \geq 0$). This complementarity in communication between firms and consumers through consumers’ strategic decision is the key to mitigating the potential free-riding incentives of firms.

The next subsection highlights this insight by investigating a benchmark case where consumers do not play a role in communication (i.e., $k = 1$).

### 4.2.1 One-sided Communication

In order to highlight the importance of the role of consumers, we study a benchmark where marketing communication is purely one-sided; that is, only firms’ efforts matter. This is a situation where consumers can passively learn about the potential benefit of the product through firms’ efforts (such as advertising). We study this situation by looking at the extreme case of $k = 1$. In communication, only firms’ efforts affect the success of communication and the size of the potential market. Consumer effort $e_c$ has no effect on the outcome of communication.

First, when $d < d^*$, firm 1 is the monopolist and engages in marketing communication on its own. Firm 1 chooses an effort level that maximizes $E\Pi_1 = (e_1) \pi^M_1 - \frac{e_1^2}{2}$, which is $e_1^M = \pi^M_1 = v - t$. (12)

Then, firm 1’s expected payoff as a monopolist is $E\Pi^M_1 = \frac{1}{2} \cdot (\pi^M_1)^2 = \frac{1}{2} \cdot (v - t)^2$. (17)
When \( d \geq d^* \), firm 2 follows the idea and the market becomes duopoly. Both firms decide on the optimal communication efforts. Since consumers do not play a strategic role in the marketing communication, there only exists the free-riding effect between the two firms. Firm 1 and 2’s efforts \( e_1 \) and \( e_2 \) maximize 
\[
E \Pi_1^D = (e_1 + e_2) \pi_1^D - \frac{e_1^2}{2}, \quad E \Pi_2^D = (e_1 + e_2) \pi_2^D - \frac{e_2^2}{2},
\]
respectively. Then, their optimal efforts are 
\[
e_i^D = \pi_i^D, \quad i = 1, 2.
\]
And the total firm-side effort is 
\[
e_f^D = \pi_1^D + \pi_2^D = \left( t + \frac{(1 - d)m^2}{9t} \right).
\]

Then, their expected payoffs under duopoly are 
\[
E \Pi_1^D = \frac{(\pi_1^D)^2}{2} + \pi_1^D \cdot \pi_2^D, \quad E \Pi_2^D = \frac{(\pi_2^D)^2}{2} + \pi_1^D \cdot \pi_2^D,
\]
where \( \pi_1^D = \left( \frac{(3t+(1-d)m)^2}{18t} \right) \), \( \pi_2^D = \left( \frac{(3t-(1-d)m)^2}{18t} \right) \).

Comparing the equations (12) and (13), we find that whether the market can expand under duopoly is determined by the two firms’ total retail profits.

**Proposition 3.** When marketing communication is one-sided (i.e., \( k = 1 \)), the firm side aggregate effort is always greater under monopoly and so is the total market size: \( e_f^M > e_f^D \) and \( \sigma^{TM} > \sigma^{TD} \).

When consumers do not play a strategic role in marketing communication, the market never expands in duopoly, and a duopoly market always leads to a smaller market size \( (\sigma^{TM} \leq \sigma^{TD}) \). This is a stark contrast to the case of two-sided costly communication case, where the aggregate firm-side effort as well as the total market size can be greater under duopoly.

Moreover, we can show that when the communication is only one-sided such that the market does not expand under duopoly, the innovator has no incentive to disclose innovation and invites competition.

**Proposition 4.** When marketing communication is one-sided, the innovator never shares the innovation and prefers to become the monopolist in the product market.

Without a complementarity effect between consumers and firms, the market never expands under duopoly compared to the monopoly market case. Also, without the market expansion effect, the communication sharing effect itself does not compensate for the loss in profit from the retail market. Hence, the innovator prefers to become the monopolist in the product market and it is never profitable for an innovating firm to invite competition. This suggests that sharing communication costs is not a key driver for the innovator’s information disclosure decision in stage 1.

### 4.3 Stage 1: Innovation Disclosure

In the first stage, the innovator (firm 1) decides on the degree of innovation disclosure, \( d \in [0, 1] \), taking into full consideration the effects of disclosure on (1) the follower’s entry decision \( (\chi \in \{1, 0\}) \),
\[
(13)
\]
(2) equilibrium outcomes of the communication game between firms and consumers \((e_1, e_2, e_c)\), and
(3) the equilibrium profit from the product competition \((\pi_1(d), \pi_2(d))\).

Upon observing \(d\), firm 2 decides to follow the idea and enter the market \((\chi = 1)\) if \(d \geq d^*\), and does not enter the market \((\chi = 0)\) if \(d < d^*\). They both correctly anticipate the equilibrium outcomes in the proceeding communication and product market competition stages given the actions \((d, \chi)\) they play.

First, we analyze firm 1’s optimal disclosure level when \(d \geq d^*\). If under a collective communication mode, firm 1 prefers a certain level of disclosure \(d\) that maximizes its expected payoff:

\[
E\Pi_1(d) = \sigma^{TD}(d)\pi_1^D(d) - \frac{1}{2}(e_1^D(d))^2
\]

Hence, firm 1’s preferred \(d\) solves the following optimality condition:

\[
\frac{\partial E\Pi_1}{\partial d} = \frac{\partial E\Pi_1}{\partial \sigma} \left( \frac{\partial \sigma^{TD}}{\partial e_c} \frac{\partial e_i^D(d)}{\partial d} + \frac{\partial \sigma^{TD}}{\partial e_2} \frac{\partial e_i^D(d)}{\partial d} \right) + \frac{\partial E\Pi_1}{\partial \pi_1^D(d) \frac{\partial \pi_1^D(d)}{\partial d}}
\]

increasing efforts from consumers and firm 2 retail profit loss

To understand the underlying forces that affects \(d\), we transform the above equation as follows:

\[
\frac{\partial E\Pi_1}{\partial d} = \pi_1^D \frac{\partial \sigma^{TD}}{\partial e_c} \frac{\partial e_i^D}{\partial d} + \pi_1^D \frac{\partial \sigma^{TD}}{\partial e_2} \frac{\partial e_i^D}{\partial d} + \pi_1^D \frac{\partial \sigma^{TD}}{\partial e_1} \frac{\partial e_i^D}{\partial d} - \pi_1^D \frac{\partial \sigma^{TD}}{\partial e_1} \frac{\partial e_i^D}{\partial d} + \sigma^{TD} \frac{\partial \pi_1^D}{\partial d}
\]

\[
= \pi_1^D \left( \frac{\partial \sigma^{TD}}{\partial e_c} \frac{\partial e_i^D}{\partial d} + \frac{\partial \sigma^{TD}}{\partial e_2} \frac{\partial e_i^D}{\partial d} + \frac{\partial \sigma^{TD}}{\partial e_1} \frac{\partial e_i^D}{\partial d} \right) - C'(e_1^D) \frac{\partial e_i^D}{\partial d} + \sigma^{TD} \frac{\partial \pi_1^D}{\partial d}
\]

\[
= \frac{\partial E\Pi_1}{\partial \sigma} \cdot \frac{\partial \sigma^{TD}}{\partial d} - \frac{\partial E\Pi_1}{\partial C} \cdot \frac{\partial C}{\partial d} + \frac{\partial E\Pi_1}{\partial \pi_1^D} \cdot \frac{\partial \pi_1^D}{\partial d}
\]

potential market expansion reducing comm. cost retail profit loss

where the second equality follows from the first-order condition: \(\frac{\partial \sigma^{TD}}{\partial e_1} \pi_1^D = C'(e_1^D)\) at \(e_1 = e_1^D\).

The equation (15) clearly suggests that there are three main effects of disclosure in play: (1) the effect on the market expansion through communication \((\frac{\partial E\Pi_1}{\partial \sigma} \cdot \frac{\partial \sigma^{TD}}{\partial d})\), (2) a cost-saving effect on communication through the sharing of communication with competitors \((-\frac{\partial E\Pi_1}{\partial C} \cdot \frac{\partial C}{\partial d})\), and (3) an effect on the retail profit \((\frac{\partial E\Pi_1}{\partial \pi_1^D})\). Clearly, as the innovator discloses more information about its innovation, its own profit decreases due to the increased competition \((-\frac{\partial E\Pi_1}{\partial C} \cdot \frac{\partial C}{\partial d} \leq 0)\), and the cost saving from communication stage can be greater \((-\frac{\partial E\Pi_1}{\partial C} \cdot \frac{\partial C}{\partial d} \geq 0)\). However, as we have seen from the one-sided communication case, the cost saving from the communication can’t fully compensate for the profit loss in the competitive product market (Proposition 4). Hence, the key for firm 1 in deciding whether to share its innovation with the competitor depends on the effect on the market expansion. Only if the market expansion can be sufficiently large such that it can outweigh the loss in the product market (i.e., net profit loss including the cost saving effect), then will the innovator disclose information.
Proposition 5. (Equilibrium) There exists a threshold \( k^e = \frac{\ln u_D^*(d^e)}{\pi^D(d^e)} - \ln \frac{\pi^M}{\pi^D(d^e)} \) such that

1. When \( k > k^e \), the optimal strategy for the innovator is not to disclose any information \( d^e = 0 \), and the market becomes a monopoly.\(^{29}\)

2. Otherwise (i.e., \( k \leq k^e \)), the optimal strategy for the innovator is to disclose sufficient information \( d^e \in \left[ d^* = 1 - \frac{3t}{m}, 1 \right] \), such that the follower decides to enter the market (\( \chi = 1 \)) and the market becomes duopoly (i.e., \( d^* = 1 - \frac{3t}{m} \leq d^e \)).

Proof. See the Appendix.

The market expansion effect \( \left( \frac{\partial E_{\Pi_1}}{\partial \sigma} \cdot \frac{\partial \sigma^D_T}{\partial d} \right) \) increases as consumers’ roles in communication become more important (i.e., \( k \) decreases). Otherwise, inviting competition does not increase the market demand due to the free-riding effect between two rival firms. Only when \( k \) is sufficiently low (or consumers’ roles are sufficiently important), can the market expansion effect more than compensate for the net loss in the product market and a firm be better off inviting its competitor by sharing its innovation. By disclosing sufficient information to the competitor, a firm can convince a sufficient surplus of the competitor in the product market, and therefore it may benefit from joint efforts to create a new category demand with its competitor. Otherwise (i.e., \( k \geq k^e \)), the innovator will prefer to maintain its competitive advantage by keeping its innovation secret, avoiding competition, and extracting monopoly surplus in the product market.

Figure 2 demonstrates the tradeoffs in the firm’s disclosure decision under two different cases \((k = 0.1 \text{ vs. } k = 0.9)\). When the importance of consumers in communication is low \((k = 0.9)\), the innovator’s expected payoff is greater under a monopoly than a duopoly. Thus, the firm is better off avoiding competition by keeping its innovation secret and being a monopolist. On the other hand, when the importance of consumers is high \((k = 0.1)\), the innovator’s expected payoff under a duopoly can be greater than under a monopoly due to the market expansion effect. Hence, the firm can benefit from inviting the competition by disclosing sufficient innovation to the competitor \((d^e \geq d^*)\).

Note that \( d^* = 1 - \frac{3t}{m} \) is the minimum level of information disclosure which can induce the follower to enter the market. Also, we already know that at \( d^e \) the threshold level of \( k \) for the

\(^{29}\) Technically, without any disclosure cost, any amount of information disclosure \( d^e \in [0, d^*] \) can be optimal because the overall equilibrium payoffs for both firms and consumers are independent of the amount of information disclosure \( d^e \) as long as \( d^e < d^* = 1 - \frac{3t}{m} \). This is so because the follower does not enter the market \( \chi = 0 \) when \( d^e < d^* \), and thus, the market is monopoly. As long the market is a monopoly, the amount of the information disclosure does not change consumers’ expected utility, and thus, the optimal communication effort will be the same irrespective of \( d^e \in [0, d^*] \). However, this indifference result is an artifact from the model assumption of zero cost for information disclosure. As long as we allow some small costs associated with information disclosure, the innovator is no longer indifferent to the amount of information disclosure, and the optimal strategy is not to disclose any information \( d^e = 0 \). This no-information disclosure is uniquely optimal under \( \epsilon \) cost of information disclosure.
Figure 2: Firm 1’s expected payoff under $k = 0.1$ and $k = 0.9$

duopoly market to be greater than that the monopoly market (from proposition 2) is

$$
\bar{k}(d^*) = \frac{1}{1 + \left( \ln \frac{\pi^M_1}{\pi^D_1(d^*)} / \ln \frac{\tilde{u}^D(d^*)}{\tilde{u}^M} \right)}
\Rightarrow \frac{\ln \frac{\pi^M_1}{\pi^D_1(d^*)}}{\ln \frac{\tilde{u}^D(d^*)}{\tilde{u}^M}} = \frac{1}{k(d^*)} - 1.
$$

(16)

We can further understand the threshold $k^e$ in Proposition 5 in terms of further $\bar{k}(d^*)$ as follows:

$$
k^e = \frac{\ln \frac{\tilde{u}^D(d^*)}{\tilde{u}^M} - \ln \frac{\pi^M_1}{\pi^D_1(d^*)}}{\ln \frac{\pi^M_1}{\pi^D_1(d^*)} + \ln \frac{\tilde{u}^D(d^*)}{\tilde{u}^M}} = \frac{1 - \ln \frac{\pi^M_1}{\pi^D_1(d^*)} / \ln \frac{\tilde{u}^D(d^*)}{\tilde{u}^M}}{1 + \ln \frac{\pi^M_1}{\pi^D_1(d^*)} / \ln \frac{\tilde{u}^D(d^*)}{\tilde{u}^M}} = \frac{2 - \frac{1}{k(d^*)}}{1 - \frac{1}{k(d^*)}}
= 2\bar{k}(d^*) - 1,
$$

(17)

where $k^e = 2\bar{k}(d^*) - 1 \leq \bar{k}(d^*)$ always holds for $\bar{k}(d^*) \in [0, 1]$. 

This relationship ($k^e \leq \bar{k}(d^*)$) implies that larger market size under the duopoly than the monopoly is a necessary condition for information sharing. The market expansion effect from the increased role of consumers in communication (i.e., low $k$) should be much greater such that it can offset the retail profit loss from sharing its innovation. For example, when $k^e \leq k \leq \bar{k}(d^*)$, the market size can be larger under a duopoly than under a monopoly. However, the innovator still finds it optimal not to disclose any innovation. Only when $k < k^e$, does firm 1 find it optimal to disclose innovation.

Interestingly, the innovator sometimes finds it optimal to share its innovation more than the minimum level of information disclosure ($d^*$) if the role of consumers becomes even greater.
Proposition 6. When \( k \) becomes even smaller (i.e., \( k \leq \bar{k} = \frac{2v - 7t}{3t - 2v} \)), the innovator discloses information \( d^e > d^* \), so that the follower makes positive profit from the product market competition, and therefore exerts positive communication effort \( e_2^D > 0 \). Moreover, the threshold \( \bar{k} > 0 \) exists only when \( t > \frac{2v}{7} \).

As consumer efforts become more important for the success of marketing communication, the innovator can be better off by disclosing more information than necessary for inviting competition \((d^e > d^*)\). In particular, when the horizontal differentiation becomes sufficiently large \((t > \frac{2v}{7})\), and consumers’ roles in marketing communication are increasingly significant \((k < \bar{k})\), creating a market environment where consumers have sufficient incentives to exert more learning efforts is crucial. As \( t \) increases, consumers have lower expected utility from consuming a product and thereby less incentives to incur upfront learning costs. In this case, firm 1 has to convince consumers to exert more upfront communication efforts by committing to even more intense competition and lower prices. Disclosing more information to the competitor can serve as a commitment mechanism for higher consumer utility in this case.

On the other hand, when horizontal differentiation is not large enough \((t \leq \frac{2v}{7})\), even if firm 1 prefers to invite the competitor to the market, firm 1 keeps the maximal secrecy \( d^e = d^* \) (since there does not exist \( \bar{k} > 0 \)). In this case, firm 2 does not exert any communication effort (since \( d^* \) is the level of information disclosure that makes firm 2 zero profit by entering the market). Given the low \( t \), the expected utility of consumers is already sufficiently high such that the existence of competition itself is sufficient to convince consumers to expend learning efforts in communication. The purpose of information disclosure is not to use competitors for expanding the market together; rather competition serves as a commitment to not hold up consumers in the product market.

In summary, there are two key mechanisms through which a firm may benefit from revealing information to its competitor in equilibrium: First, a direct effect on the competitor’s entry decision and communication efforts. More information disclosure increases the follower’s profit, which facilitates the competitor’s entry and makes a rival engage in more communication efforts. Second, there is another indirect effect of information disclosure on consumer efforts: the existence of competition (triggered by the information disclosure) can serve as commitment mechanism for higher consumer surplus, which accelerates consumer efforts in communication. This increased consumer effort can contribute to market expansion through the complementarity of consumers and firms in communication.

5 Extension: sequential two-sided communication

In our main model, we consider a case where firms and consumers simultaneously decide their communication efforts without observing the other’s decisions. In our first extension, we relax this assumption and study a realistic case where communication decisions are sequential such that
consumers adjust their learning efforts after observing firms’ marketing efforts. We find that the effect of complementarity is amplified in the sequential communication, which results in a higher level of equilibrium communication efforts and disclosure compared to simultaneous communication.

The last stage of retail competition remains the same as the simultaneous communication case. However, at the communication stage, the sequentiality would affect the equilibrium results. Under this sequential communication case, consumers adjust their efforts $e_c$ in reaction to the firm’s choice of efforts. Consumers maximize their expected utility, $EU = (e_f^k \cdot e_c^{1-k}) \tilde{u} - \frac{e_c^2}{2}$, after observing a given effort $e_f$. Hence, the best reply function for consumers is still the same as the simultaneous case and given by

$$e_c = \left((1-k) \tilde{u} e_f^k\right)^{\frac{1}{k+1}}. \quad (18)$$

However, the firm is now a Stackelberg leader. Anticipating the consumers’ best response (Equation 18), the firm solves the following maximization problem:

$$\max_{e_i} E\Pi_i = \sigma(e_f, e_c) \cdot \pi_i^D - \frac{e_i^2}{2}, \quad (19)$$

where $\sigma(e_f, e_c) = e_f^k \cdot e_c^{1-k}$, and $e_f = e_1 + e_2$ when $\chi = 1$, and $e_f = e_1$ when $\chi = 0$.

The first order condition for the optimal efforts for firm $i$ ($i = 1$ under monopoly while $i \in \{1, 2\}$ under duopoly) is given by

$$\left(\frac{\partial \sigma}{\partial e_i} + \frac{\partial \sigma}{\partial e_c} \cdot \frac{\partial e_c}{\partial e_i}\right) \pi_i^D = e_i \quad (20)$$

Compared to a simultaneous communication case, there is an additional incentive $\left(\frac{\partial \sigma}{\partial e_c} \cdot \frac{\partial e_c}{\partial e_i} \cdot \pi_i^D > 0\right)$ because of complementarity $\frac{\partial \sigma}{\partial e_c} > 0$ for firm $i$ to exert an effort. Consequently, firm $i$ expends more efforts in a sequential communication case, which in turn leads to greater effort for consumers in a sequential communication than simultaneous case.

At the last stage, firm 1 chooses the optimal disclosure level $d^c \in [0, 1]$. Similarly, we can find that the amount of disclosure increases under sequential communication compared to the simultaneous communication case. Under sequential communication, both firms and consumers incur more communication efforts and thereby the market expansion effect through communication is amplified, which increases the firm 1’s incentive to disclose more information.

**Proposition 7.** The threshold $k^e$ at which firm 1 is indifferent between inviting competition and becoming a monopolist does not change. Nevertheless, in the case in which firm 1 intends to invite the competitor (i.e., $k \leq k^e$), the optimal disclosure level $d^c$ is higher in the sequential communication case than in the simultaneous communication case.
6 Conclusion

In this research, we study the strategy for introducing revolutionary products and try to explain the rationale for different information disclosure strategies among innovating firms: whether firms disclose innovation to competitors or keep it secret. Our explanation is based on the persuasion motive of firms in communication and the strategic role of consumers. Specifically, we model marketing communication as two-sided costly efforts and the key determinant for market demand of a novel product. Not only does a firm exert costly effort during the communication process, but consumers also have to expend effort in assimilating the idea and deliberating their needs. Hence, a key challenge for firm is how to persuade consumers to expend more communication efforts in assimilating the new idea, and clarifying their needs by examining an unproven product category.

Our analysis suggests that there are interesting tradeoffs facing the innovating firms in choosing their level of secrecy. Disclosing innovation can increase the follower’s expected profit from the product market, which encourages entry of competition and makes a rival engage in more communication efforts. Thus, the innovating firm can benefit from disclosing innovation, which serves as an invitation for competitors, because the presence of the competitor can help to expand the market through cooperation in communication stage.

This communication stage is characterized by strategic complementarity between consumers’ efforts and the firms’ aggregate effort, and strategic substitutability between each firm’s individual efforts. Our analysis further reveals that without consumers playing a complementary role in marketing communication, free-riding problem arising from strategic substitutes between each individual firm’s efforts causes the market size to shrink under a duopoly. Only when communication shows a strong strategic complementarity between consumers and firms, can an innovating firm benefit from sharing innovation; strategic consumers alleviate the potential free-riding effects of rival firms.

More precisely, the presence of competition induced by sharing the innovation, can serve as a commitment to guarantee greater surplus for consumers, which helps firms to convince consumers to expend more communication efforts. Consequently, the increased consumer effort can result in market expansion through the complementarity of consumers and firms in communication. Therefore, to induce greater consumer efforts, an innovator may benefit from inviting competitors by sharing an idea.

Of course, inviting competitors to jointly expand the market by sharing its innovation involves significant costs. In particular, we find that the relative profit of the product market determines the division of efforts between the two firms. The follower has an incentive to exert more effort only when it would get sufficient profit in the product market. To incentivize the follower to expand the market, the innovator has to pay costs of giving part of market share to the follower. But the innovating firm can achieve competitive advantage and greater market share in the product market.
by keeping the innovation secret. Even though the innovating firm should bear all the costs for communication and creating the market, it can still enjoy the monopoly situation in the product market. It is thus not obvious that it will be profitable to share innovation and invite competitors into the market. Nevertheless, we find conditions that lead a firm to reveal information to increase its profitability in the product market despite the additional competition.

Essentially, the issue that any innovator faces is trade-offs in “coopetition” – whether to increase market size with competitors or capture market share. In this sense, we believe our research can shed light on understanding this important issue of coopetition by providing one micro-mechanism about when to cooperate with competitors to reach a higher value creation (such as market expansion) or struggle to achieve a competitive advantage without cooperation.

Finally, our results critically depend on the key features of communication: strategic complementarity between consumers’ efforts and the firms’ aggregate effort, and strategic substitutability between each firm’s efforts. Although we believe that imposing those characteristics in marketing communication is reasonable in many situations, we would not wish to claim that marketing communication always takes on strategic complementarity between firms and consumers. In certain situations, it is possible that communication takes on strategic substitutability between firms and consumers. For example, many inbound marketing firms encourage consumers to search product information on their own instead of bearing all communication costs if this can be more efficiently done by consumers (see a seminal paper by Wernerfelt 1994 for the conceptual argument for efficiency criterion on who should bear communication costs between firm and consumers). Nonetheless, communication between firms and consumers can be better captured through strategic complementarity in many situations like radical innovation, in which consumers are unsure of both the uses and the benefits of a product and their needs. In these situations, firms’ efforts cannot totally substitute for consumers’ efforts, because consumers have to incur private costs in learning about the product and clarifying their needs. In this research, we recognize that the acts of formulating and absorbing the content of a communication are privately costly, and we provide a new perspective on marketing communication (as two-sided costly efforts) and its implications.

\footnote{Also, Mayzlin and Shin (2011) formalize this perspective by considering firms’ and consumers’ efforts in communication as substitutes. They show that inducing consumers’ efforts in the communication is sometimes more efficient for firms.}
Appendix A

Proof of Lemma 1.

Proof. Under monopoly, firm 1 offers its monopoly price $p_1^M = v - t$ such that all consumers purchase the product, thus we have

$$\hat{u}^M = \int_z (v - t \cdot z - v + t) \, dz = \frac{t}{2}$$

Under duopoly,

$$\hat{u}^D = \int_z \max \{ v - tz - p_1^D, v - t (1 - z) - p_2^D \} \, dz$$

$$= \int_0^1 \frac{1}{2} \cdot \frac{v - t - \frac{p_1^D + p_2^D}{2}}{\pi} \cdot (v - t (1 - z) - \frac{p_1^D + p_2^D}{2}) \, dz$$

$$= \frac{1}{36t} ((1 - d) \bar{m})^2 - \frac{1}{2} (1 - d) \bar{m} + v - \frac{5}{4} t$$

Hence, $\hat{u}^D \geq \hat{u}^M \iff d \leq \bar{d} = 1 - \frac{1}{\bar{m}} \left( 9t + 6 \sqrt{t(4t - v)} \right)$ or $d \geq \bar{d} = 1 - \frac{1}{\bar{m}} \left( 9t - 6 \sqrt{t(4t - v)} \right)$.

Note that the market is duopoly ($\chi = 1$) only when $d \geq d^* = 1 - \frac{3t}{\bar{m}}$. Also, it is immediate that $d^* > \bar{d}$ because $v > 3t$. Hence, when the market is duopoly (i.e., $d \geq d^*$), $\hat{u}^D > \hat{u}^M$. \qed

Proof of Proposition 1.

Proof. The first part of the proposition can be easily checked by examining the equilibrium efforts. Consumers’ equilibrium effort is $e_c = ((1 - k) \hat{u})^{1 - \frac{k}{2}} (k \pi_f)^{\frac{k}{2}}$, where $\pi_f$ denotes the total firm profit, i.e., $\pi_f = \pi_1^M$ if monopoly or $\pi_f = \pi_1^D + \pi_2^D$ if duopoly. For all $k \in [0, 1]$, $\frac{\partial e_c}{\partial u} = (1 - \frac{k}{2}) \left( (1 - k) \hat{u} \right)^{\frac{1}{2}} \left( k \pi_f \right)^{\frac{k}{2}} \geq 0$. Next, we check the firms’ efforts. In the monopoly case, firm 1’s equilibrium effort is $e_1^M = ((1 - k) \hat{u}^M)^{1 + \frac{k}{2}} (k \pi_1^M)^{\frac{k}{2}}$. Also, it is immediate that $\frac{\partial e_1^M}{\partial \pi_1^M} = ((1 - k) \hat{u}^M)^{1 + \frac{k}{2}} \cdot \frac{(1 + k) (k \pi_1^M)^{\frac{k}{2}}}{2} \geq 0$ holds for all $k \in [0, 1]$. In the duopoly case, the equilibrium effort of firm $i$, $i \in \{1, 2\}$, is $e_i^D = ((1 - k) \hat{u}^D)^{1 + \frac{k}{2}} (k (\pi_1^D + \pi_2^D))^{\frac{1 + k}{2}} \left( \frac{1}{\pi_1^D + \pi_2^D} \right)$. So $\frac{\partial e_i^D}{\partial \pi_i^D} = ((1 - k) \hat{u}^D)^{1 + \frac{k}{2}} \cdot \frac{(1 + k) (k (\pi_1^D + \pi_2^D))^{\frac{k}{2}}}{2} \left( \frac{\pi_1^D}{\pi_1^D + \pi_2^D} \right) \geq 0$ holds for all $k \in [0, 1]$.

The second part of the proposition is straightforward by comparing the first-order conditions. For firm $i$, $i \in \{1, 2\}$, its equilibrium effort $e_i^D$ solves the following first-order condition:

$$\sigma_i (e_i^D + e_j^D, e_c) \pi_i^D = e_i^D \Rightarrow k (e_i^D + e_j^D)^{k-1} e_c^{1-k} \pi_i^D = e_i^D$$

Hence, $e_i^D = \frac{k (e_i^D + e_j^D)^{k-1} e_c^{1-k} \pi_i^D}{k (e_i^D + e_j^D)^{k-1} e_c^{1-k} \pi_i^D} = \pi_i^D$. \qed

26
Proof of Corollary 1.

Proof. If consumers do not play a role in the communication, i.e., \( k = 1 \), consumers’ effort has no effect. Hence, in the proceeding analysis, we consider the case when \( k \in [0,1) \). Consumers’ equilibrium effort is \( e_c = ((1-k)\tilde{u})^{\frac{1}{1-k}}(k\pi_f)^{\frac{k}{1-k}} \), where \( \pi_f \) denotes the total firm profit, i.e., \( \pi_f = \pi_1^M \) if monopoly or \( \pi_f = \pi_1^D + \pi_2^D \) if duopoly. So consumers exert more effort in the duopoly situation when \( \frac{c_D}{c_M} = \left( \frac{\tilde{u}M}{\tilde{u}D} \right)^{1-k} \left( \frac{\pi_1^D + \pi_2^D}{\pi_1^M + \pi_2^M} \right)^{\frac{k}{2}} > 1 \). Taking logarithm of both sides, we have the equivalent condition as \((1-k)\ln \frac{\tilde{u}M}{\tilde{u}D} + \frac{k}{2} \ln \frac{\pi_1^D + \pi_2^D}{\pi_1^M + \pi_2^M} > 0 \) \( \Rightarrow \ln \frac{\tilde{u}M}{\tilde{u}D} - \frac{k}{2} \left( \ln \frac{\pi_1^M}{\pi_1^1 + \pi_2^M} + \ln \frac{\pi_2^D}{\tilde{u}D} \right) > 0 \). We will show next that the aforementioned condition always holds for all \( v > 3t \) and \( d \in [d^*, 1] \). First, by Lemma 1, \( \ln \frac{\tilde{u}D}{\tilde{u}M} > 0 \). Second, we find that for all \( v > 3t \) and \( d \in [d^*, 1] \),

\[
\pi_1^D + \pi_2^D = t + \frac{(1-d)^2}{9t} < v - t = \pi_1^M
\]

So \( \ln \frac{\pi_1^M}{\pi_1^1 + \pi_2^M} > 0 \).

Moreover, for all \( v > 3t \) and \( d \in [d^*, 1] \),

\[
\ln \frac{\tilde{u}D}{\tilde{u}M} > \ln \frac{\pi_1^M}{\pi_1^1 + \pi_2^M} > 0
\]

because \( \frac{\tilde{u}D}{\tilde{u}M} = \frac{1}{\ln((1-d)^2-\frac{1}{2})} > \frac{v - t}{t + \frac{(1-d)^2}{9t}} = \frac{\pi_1^M}{\pi_1^1 + \pi_2^M} \).

Hence, we know the following inequality should hold: \( \ln \frac{\tilde{u}D}{\tilde{u}M} - \frac{k}{2} \ln \frac{\pi_1^D + \pi_2^D}{\pi_1^1 + \pi_2^M} > \frac{k}{2} \ln \frac{\pi_1^M}{\pi_1^1 + \pi_2^M} \). Lastly, for all \( k \in [0,1) \), \( \ln \frac{\tilde{u}D}{\tilde{u}M} - \frac{k}{2} \left( \ln \frac{\pi_1^D + \pi_2^D}{\pi_1^1 + \pi_2^M} + \ln \frac{\pi_1^M}{\pi_1^1 + \pi_2^M} \right) > 0 \).

\( \square \)

Proof of Proposition 2.

Proof. According to equation (11), the relative market size \( \sigma_{TM} \) is given by \( \left( \frac{\tilde{u}D}{\tilde{u}M} \right)^{1-k} \left( \frac{\pi_1^D + \pi_2^D}{\pi_1^M + \pi_2^M} \right)^k \), where \( \tilde{u}D \) and \( \pi_1^D + \pi_2^D \) are functions of \( d \). Hence, \( \sigma_{TM} \geq \sigma_{TM} \) if and only if \( \left( \frac{\tilde{u}D}{\tilde{u}M} \right)^{1-k} \left( \frac{\pi_1^D + \pi_2^D}{\pi_1^M + \pi_2^M} \right)^k \geq 1 \). We can reduce the condition \( \left( \frac{\tilde{u}D}{\tilde{u}M} \right)^{1-k} \left( \frac{\pi_1^D + \pi_2^D}{\pi_1^M + \pi_2^M} \right)^k \geq 1 \) to \( (1-k) \ln \frac{\tilde{u}D}{\tilde{u}M} + k \ln \frac{\pi_1^D + \pi_2^D}{\pi_1^M + \pi_2^M} \geq 0 \). Next, we show \( k (d) \) belongs to \((0,1)\). From equation (21), we know that \( \pi_1^M > \pi_1^D + \pi_2^D \), for all \( v > 3t \) and \( d \in [d^*, 1] \). So \( \ln \frac{\pi_1^M}{\pi_1^1 + \pi_2^M} > 0 \). And Lemma 1 shows that \( \tilde{u}D > \tilde{u}M \), for all \( v > 3t \) and \( d \in [d^*, 1] \). So \( \ln \frac{\tilde{u}D}{\tilde{u}M} > 0 \). Hence, \( \ln \frac{\pi_1^M}{\pi_1^1 + \pi_2^M} / \ln \frac{\tilde{u}D}{\tilde{u}M} \) is also positive. Then we know

\[
\frac{1}{1+\ln \frac{\pi_1^M}{\pi_1^1 + \pi_2^M} / \ln \frac{\tilde{u}D}{\tilde{u}M}} \in (0,1).
\]

\( \square \)
Proof of Proposition 3.

Proof. The inequality (21) in the proof of Corollary 1 has shown that \( \pi_1^D + \pi_2^D < \pi_1^M \) for all \( v > 3t \) and \( d \in [d^*, 1] \). It follows immediately that \( e_1^M > e_1^D \). In the benchmark case, the market size is only determined by the firm-side effort. Therefore, \( e_1^M = \sigma_{TM} > \sigma_{TD} = e_1^D \). \( \square \)

Proof of Proposition 4.

Proof. In the monopoly case, firm 1’s expected profit is: \( E\Pi_1^M = \pi_1^M \cdot \frac{1}{2} \pi_1^M \); whereas in the duopoly case, its expected payoff becomes \( E\Pi_1^D = \pi_1^D \cdot \left( \frac{1}{2} \pi_1^D + \pi_2^D \right) \). The inequality (21) in the proof of Corollary 1 implies that the following inequality should hold as well. That is, \( E\Pi_1^M = \pi_1^M \cdot \frac{1}{2} \pi_1^M > \frac{1}{2} (\pi_1^D + \pi_2^D)^2 = \frac{1}{2} (\pi_1^D)^2 + \pi_1^D \pi_2^D + \frac{1}{2} (\pi_2^D)^2 = E\Pi_1^D + \frac{1}{2} (\pi_1^D)^2 > E\Pi_2^D \). \( \square \)

Proof of Proposition 5.

Proof. We first suppose \( E\Pi_1^D (d) \) monotonically decreases in the range \([d^*, 1]\) so that \( d^* \) is the optimal disclosure level in the duopoly case. With that supposition, we find the condition on \( k \) under which \( E\Pi_1^M > E\Pi_1^D (d^*) \). Then, we verify the supposition is indeed true under that condition of \( k \).

At \( d^* \), firm 1 does not leave any surplus to firm 2, i.e., \( \pi_2^D (d^*) = 0 \). Firm 1’s expected payoff becomes:

\[
E\Pi_1^D (d^*) = \left( 1 - \frac{k}{2} \right) (1 - k)^{-k} k^k (\tilde{u}^D (d^*))^{1-k} (\pi_1^D (d^*))^{1+k}
\]

In comparison, as a monopolist, its expected payoff is:

\[
E\Pi_1^M = \left( 1 - \frac{k}{2} \right) (1 - k)^{-k} k^k (\tilde{u}^M)^{1-k} (\pi_1^M)^{1+k}
\]

Firm 1 finds a monopoly market more profitable if:

\[
E\Pi_1^M > E\Pi_1^D (d^*) \iff \frac{E\Pi_1^M}{E\Pi_1^D (d^*)} = \left( \frac{\tilde{u}^M}{\tilde{u}^D (d^*)} \right)^{1-k} \left( \frac{\pi_1^M}{\pi_1^D (d^*)} \right)^{1+k} > 1
\]

Take the logarithm of the both sides, we have

\[
(1 - k) \ln \frac{\tilde{u}^M}{\tilde{u}^D (d^*)} + (1 + k) \ln \frac{\pi_1^M}{\pi_1^D (d^*)} > 0
\]

\[
\iff k \left( \ln \frac{\pi_1^M}{\pi_1^D (d^*)} - \ln \frac{\tilde{u}^M}{\tilde{u}^D (d^*)} \right) > \ln \frac{\tilde{u}^D (d^*)}{\tilde{u}^M} - \ln \frac{\pi_1^M}{\pi_1^D (d^*)} \quad (23)
\]

Since the inequality (22) suggests that \( \ln \frac{\pi_1^M}{\pi_1^D (d^*)} - \ln \frac{\tilde{u}^M}{\tilde{u}^D (d^*)} = \ln \frac{\pi_1^D}{\pi_1^D (d^*)} + \ln \frac{\tilde{u}^D (d^*)}{\tilde{u}^M} - \ln \frac{\pi_1^M}{\pi_1^D (d^*)} > 0 \) hold, we can reduce the inequality (23) to the condition \( k > \frac{\ln \frac{\tilde{u}^D (d^*)}{\tilde{u}^M} - \ln \frac{\pi_1^M}{\pi_1^D (d^*)}}{\ln \frac{\pi_1^D}{\pi_1^D (d^*)} + \ln \frac{\tilde{u}^D (d^*)}{\tilde{u}^M}} \equiv k^e \)
and \( k^e \in (0, 1) \). Therefore, under the condition \( k > k^e \), \( E \Pi^M \succ E \Pi^P \) (\( d^* \)).

We proceed to verify that \( E \Pi^P_1 (d) \) is indeed monotonically decreasing in the range \([d^*, 1]\) when \( k > k^e \). Firm 1’s expected duopoly payoff is:

\[
E \Pi^P_1 (d) = (1 - k)^{-k} k^k \left( \hat{u}^P (d) \right)^{1-k} \left( \pi_1^P (d) + \pi_2^P (d) \right)^k \pi_1^P \left( 1 - \frac{k}{2} \frac{\pi_1^P (d)}{\pi_1^P (d) + \pi_2^P (d)} \right)
\]

The sign of \( \frac{\partial E \Pi^P_1}{\partial d} \) is equivalent to that of its log-transformation, which is

\[
\frac{\partial \log E \Pi^P_1}{\partial d} = (1 - k) \frac{\partial \hat{u}^P}{\partial d} + k \frac{\partial \left( \pi_1^P + \pi_2^P \right)}{\partial d} < 0
\]

\[
+ \frac{1}{\pi_1^P} \frac{\partial \pi_1^P}{\partial d} + \frac{1}{\pi_2^P} \frac{\partial \pi_2^P}{\partial d} < 0
\]

We further reduce the above formula to a quadratic function of \( k \). Denote that function as \( G (k) \), which is

\[
G (k) = k^2 9t (\hat{m} - 3t)^2 (\hat{m}^2 - 9t^2 + 2\hat{m} (3t - 2v)) - 8 (\hat{m}^2 + 9t^2)^2 (\hat{m}^2 - 12\hat{m}t + 18t(v - 2t))
\]

\[
+ k (\hat{m} + 3t) (2\hat{m}^5 + 15\hat{m}^4 t + 36\hat{m}^3 t(4t - 3v)) - 216\hat{m}^2 t^3 + 162\hat{m}t^3(t - 6v) - 43t^4(25t - 8v)
\]

where \( \hat{m} = (1 - d) \hat{m} \).

For all \( v > 3t \) and \( d \in [d^*, 1] \), \( E \Pi^P_1 (d) \) monotonically decreases as long as \( G (k) < 0 \). Since \( G (k) \) is a convex function and there exists a point, e.g., \( k = 1 \), such that \( G (k) < 0 \), we are able to find two roots \( k_1 \) and \( k_2 \) such that \( G (k) < 0 \) when \( k \in (k_1, k_2) \). Notice that \( k_2 > 1 \) for all \( v > 3t \) and \( d \in [d^*, 1] \), all we need to show is that \( k \in (k^e, 1) \) is a subset of \((k_1, 1)\) for all \( v > 3t \) and \( d \in [d^*, 1] \).

First, when \( v > \frac{7}{2} t \), \( k_1 < 0 \) for all \( d \in [d^*, 1] \). Thus, when \( v > \frac{7}{2} t \) and \( d \in [d^*, 1], k \in (k^e, 1) \subset (0, 1) \subseteq (k_1, 1) \) and \( G (k) < 0 \). Next, when \( v \leq \frac{7}{2} t \), we only need to verify that \( k^e \geq k_1 \). This is so because \( k^e \geq k_1 \) always holds for \( d \in [d^*, 1] \). Therefore, when \( v \leq \frac{7}{2} t \) and \( d \in [d^*, 1], k \in (k^e, 1) \subseteq (k_1, 1) \).

\[\square\]

Proof of Proposition 6.

Proof. As we have already shown in the above proof, when \( k > k_1 \), \( G (k) < 0 \). As long as \( k \leq \max_d k_1 \), we can find the condition on \( k \) such that \( G (k) \geq 0 \) hold for some \( d \). Notice that \( k_1 (d) \) monotonically decreases in \( d \), and thus, \( \max_d k_1 = k_1 (d^*) = \frac{2v - 7t}{3t - 2v} \equiv k \). \( k \) exists in \([0, 1]\) when \( v \leq \frac{7}{2} t \).

For any given \( k \leq k \), we can find a unique disclosure amount \( d^e \) such that \( G (k; d^e) = 0 \). Equivalently speaking, \( k = k_1 (d^e) \). Next, we need to verify \( d^e \) indeed maximizes firm 1’s expected payoff.
Consider a lower disclosure amount $d' < d^e$. Since $k_1(d)$ decreases in $d$, a lower $d' < d^e$ corresponds to a higher $k_1(d')$ such that $G(k_1(d'); d') = 0$. Then $G(k; d') > 0$ when $k < k_1(d')$. Similarly, consider a higher disclosure amount $d'' > d^e$. That $d''$ corresponds to a lower $k_1(d'')$ such that $G(k_1(d''), d'')$ equals zero. $G(k; d'') < 0$ when $k > k_1(d'')$. Therefore, $G(k; d) > 0$ ($\Pi^D_k(d)$ increases) when $d \in [d', d^e]$; whereas $G(k; d) < 0$ ($\Pi^D_k(d)$ decreases) when $d \in (d^e, 1]$. And $d^e > d^e$ is the optimal disclosure amount.

\[\square\]

**Proof of Proposition 7.**

Proof. Following equation (20), firm $i$’s optimal effort $e_i$ in the duopoly case is:

\[
\left(ke_i^{k-1}\left((1 - k)\hat{u}D e_i^k\right)\frac{1-k}{1+k} + (1 - k)e_i^k\left((1 - k)\hat{u}D e_i^k\right)\frac{k}{e_i(k + 1)}\left((1 - k)\hat{u}D e_i^k\right)\frac{1-k}{1+k}\right)\frac{1}{\pi_i^D} = e_i
\]

The total firm-side effort $e_f = e_1 + e_2$ is determined by adding up the first-order conditions of the two firms:

\[
\left(ke_i^{k-1}\left((1 - k)\hat{u}D e_i^k\right)\frac{1-k}{1+k} + (1 - k)e_i^k\left((1 - k)\hat{u}D e_i^k\right)\frac{k}{e_i(k + 1)}\left((1 - k)\hat{u}D e_i^k\right)\frac{1-k}{1+k}\right)\left(\frac{\pi_i^D + \pi_2^D}{\pi_i^D + \pi_2^D}\right) = e_f \Rightarrow
\]

\[
\left(k\left((1 - k)\hat{u}D\right)\frac{1-k}{1+k} + (1 - k)e_i^k\left((1 - k)\hat{u}D e_i^k\right)\frac{k}{e_i(k + 1)}\left((1 - k)\hat{u}D e_i^k\right)\frac{1-k}{1+k}\right)\left(\frac{\pi_i^D + \pi_2^D}{\pi_i^D + \pi_2^D}\right) = e_f \Rightarrow
\]

\[
e_f^{\frac{k+1}{k+2}}\left((1 - k)\hat{u}D\right)\frac{1-k}{1+k} + \frac{k}{(k + 1)}\left((1 - k)\hat{u}D e_i^k\right)\frac{k}{e_i(k + 1)}\left((1 - k)\hat{u}D e_i^k\right)\frac{1-k}{1+k} = e_f
\]

Therefore, we have the equilibrium efforts:

\[
e_i^D + e_2^D = \left(\frac{2}{k + 1}\right)^{\frac{k+1}{2}} \times \left(k\left(\pi_i^D + \pi_2^D\right)\right)^{\frac{1+k}{2k}} \left((1 - k)\hat{u}D\right)^{\frac{1-k}{2k}}
\]

multiplier effect in sequential comm.

eq\text{eqn. effort in simultaneous comm.}

Firm $i$’s equilibrium effort is simply $e_i^D = \frac{\pi_i^D}{\pi_i^D + \pi_2^D} \left(e_1^D + e_2^D\right)$, $i = 1, 2$. And consumers’ equilibrium effort is:

\[
e_c^D = \left((1 - k)\hat{u}D (e_1^D + e_2^D)^k\right)^{\frac{1}{1+k}}
\]

\[
= \left(\frac{2}{k + 1}\right)^{\frac{k}{2}} \times \left((1 - k)\hat{u}D\right)^{\frac{1-k}{2k}} \left(k\left(\pi_i^D + \pi_2^D\right)\right)^{\frac{k}{2}}
\]

multiplier effect in sequential comm.

eq\text{eqn. effort in simultaneous comm.
Then, we can compute firm 1’s expected payoff in the duopoly market:

\[
E\Pi_1^D = \sigma (e_1^D + e_2^D + e_c^D) \pi_1^D - \frac{1}{2} (e_1^D)^2 \\
= \left( \frac{2}{1+k} \right)^k (1-k)^{1-k} k^k \left( \tilde{\varphi}^D \right)^{1-k} \left( \pi_1^D + \pi_2^D \right)^k \pi_1^D \left( 1 - \frac{k}{1+k} \pi_1^D + \pi_2^D \right)
\]

Similarly, we will get equilibrium efforts in the monopoly case as follows:

\[
e_1^M = \left( \frac{2}{k+1} \right)^{\frac{k+1}{2}} (1-k)^{\frac{1-k}{2}} \left( \tilde{\varphi}^M \right) \left( \pi_1^M \right)^{\frac{k+1}{2}}
\]

and

\[
e_c^M = \left( \frac{2}{k+1} \right)^{\frac{k}{2}} (1-k)^{\frac{1-k}{2}} \left( \tilde{\varphi}^M \right) \left( \pi_1^M \right)^{\frac{k}{2}}
\]

Firm 1’s expected payoff in the monopoly market is

\[
E\Pi_1^M = \sigma (e_1^M, e_c^M) \pi_1^M - \frac{1}{2} (e_1^M)^2 = \left( \frac{2}{1+k} \right)^k (1-k)^{1-k} k^k \left( \tilde{\varphi}^M \right)^{1-k} \left( \pi_1^M \right)^{1+k} \left( 1 - \frac{k}{1+k} \right)
\]

We follow the reasoning in the proof of Proposition 5 and 7. First, we find that the cutoff \(k^e\) at which firm 1 is indifferent between becoming a monopolist and inviting competition remains unchanged in the sequential setting. Because the relative ratio in the sequential setting \(\frac{E\Pi_1^M}{E\Pi_1^D(d^*)} = \left( \frac{\varphi^M}{\varphi^D(d^*)} \right)^{\frac{k}{2}} \left( \frac{\pi_1^M}{\pi_1^D(d^*)} \right)^{\frac{k+1}{2}}\) is the same as that in the simultaneous setting and we will get the same cutoff point \(k^e\) by applying the same reasoning in the proof of Proposition 5.

Again, we will check \(E\Pi_1^D(d)\) indeed monotonically decreases in \([d^*, 1]\). Similarly, we reduce \(\frac{\partial \log E\Pi_1^D}{\partial d}\) to a quadratic function of \(k\). Denote that function as \(\tilde{G}(k)\), which is

\[
\tilde{G}(k) = k^2 9t(\tilde{m} - 3t)^2 (\tilde{m} + 3t) (\tilde{m}^2 + 6\tilde{m} - 4\tilde{m} - 9t^2) - 4 (\tilde{m}^2 + 9t^2)^2 (\tilde{m}^2 - 12\tilde{m}t + 18t(v-2t)) \\
- k \left( 2\tilde{m}^6 - 51\tilde{m}^5t - 9\tilde{m}^4(11t - 12v) - 108\tilde{m}^3t^2(7t - v) \\
- 810\tilde{m}^2t^3(7t - v) + 243\tilde{m}^4t(13t - 4v) + 2187t^6 \right)
\]

where \(\tilde{m} = (1 - d) \tilde{m}\).

For all \(v > 3t\) and \(d \in [d^*, 1]\), \(E\Pi_1^D(d)\) monotonically decreases in \([d^*, 1]\) as long as \(\tilde{G}(k) < 0\). At \(d^*\), \(\tilde{G}(k)\) is a downward-sloping function and \(\tilde{G}(k) = 0\) at \(k = \frac{2v-7t}{3t-2v}\). When \(k > \frac{2v-7t}{3t-2v}\), \(\tilde{G}(k) < 0\). We have already shown that \(k^e > \frac{2v-7t}{3t-2v}\) in the proof of Proposition 6. For all \(d \in (d^*, 1]\), \(\tilde{G}(k)\) is a concave function and \(G(k) < 0\) when \(k < k_1\) or \(k > k_2\), where \(k_1\) and \(k_2\) are the two roots at which \(\tilde{G}(k) = 0\). Observing \(k_1 < 0\) for all \(v > 3t\) and \(d \in (d^*, 1]\), then we are left to show \(k \in (k^e, 1]\) is a subset of \((k_2, 1]\) for all \(v > 3t\) and \(d \in (d^*, 1]\). This is so because \(k^e > k > k_2\) always holds.
The fact that \( k_2 \) increases in \( d \) and the relation \( k > k_2 \) also indicate that \( \tilde{G}(k) \geq 0 \) for some \( d \) when \( k \leq \tilde{k} \). The cutoff \( \tilde{k} \) in the sequential setting remains unchanged compared to that in the simultaneous setting.

When \( k \leq \tilde{k} \), the optimal disclosure \( d^e \) solves the following equation:

\[
\frac{\partial \log E\Pi_1}{\partial d} = (1 - k) \frac{1}{\tilde{u}^D} \frac{\partial \tilde{u}^D}{\partial d} + k \frac{1}{\pi_1^D + \pi_2^D} \frac{\partial (\pi_1^D + \pi_2^D)}{\partial d} + \frac{1}{\pi_1^D} \frac{\partial \pi_1^D}{\partial d} + \frac{1}{\pi_1^D + \pi_2^D} \frac{\partial \pi_1^D}{\partial d} > 0
\]

In comparison with equation (24), for \( k \in [0, 1) \),

\[
(1 - k) \frac{1}{\tilde{u}^D} \frac{\partial \tilde{u}^D}{\partial d} + k \frac{1}{\pi_1^D + \pi_2^D} \frac{\partial (\pi_1^D + \pi_2^D)}{\partial d} + \frac{1}{\pi_1^D} \frac{\partial \pi_1^D}{\partial d} + \frac{1}{\pi_1^D + \pi_2^D} \frac{\partial \pi_1^D}{\partial d} > 0
\]

Hence, the optimal disclosure \( d^e \) is higher in the sequential setting. \qed
References


