Market Share Evolution, Size Spillovers and Organizational Forgetting in Retail Chain Dynamics

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Abstract

A firm’s decision to expand or contract has long term strategic implications for not only its market outcomes but also industry structure and evolution. A “Chandlerian” view suggests that once firms become large they persist in being dominant. An alternative “Schumpeterian” view emphasizes transience in market shares. This paper examines the link between firm size and market performance by developing a dynamic oligopoly model in which firm size is endogenous to decisions to add or subtract retail stores, and market share transitions result from strategic interaction between firms. Through a firm specific unobservable that incorporates the spillovers of current firm size on future chain profitability, the model allows for heterogeneity in the dynamic link between firm size and future market outcomes. Furthermore, it allows for heterogeneity in how well firms retain such advantages via organizational forgetting. This seemingly intractable dynamic game is estimated by extending the Bajari, Benkard and Levin (2007) two-step estimation method to incorporate a particle filter procedure to account for such serially correlated unobservables with endogenous feedback. The empirical analysis is based on data for Canadian hamburger retail chains from their inception in 1970 to 2005. There is evidence not only of spillovers of firm size and organizational forgetting but more importantly of heterogeneity in these across firms. The heterogeneity in the dynamic linkage between firm size and profitability helps explain why some firms become dominant and others falter as they evolve, with consequences for market structure and concentration.

Keywords: Dynamic discrete choice, market structure, industry dynamics, firm size spillovers, organizational forgetting, particle filter, serial correlation.

*The data on Canada’s fast food industry were collected with the aid of patient staff from the Toronto Reference Library. We thank Joshua Lewis for providing us details about municipal smoking by-laws in Canada. We also thank Philip Gayle, Avi Goldfarb, Matthew Lewis, Nathan Miller, Mar Reguant, and K. Sudhir, as well as participants at the International Industrial Organization Conference, Johns Hopkins University, Marketing Dynamics Conference, Marketing Science Conference, New York University, and North American Econometric Society Summer Meetings for helpful discussions. All remaining errors are our own.
1 Introduction

The strategic decision of a firm to expand or contract is inherently dynamic with long term implications for not only its market outcomes but also industry structure and evolution. A “Chandlerian” view suggests that once firms become large they persist in being dominant (e.g., Chandler, 1990). An alternative “Schumpeterian” view emphasizes that dominance is transient (e.g., Fisher, McGowan, and Greenwood, 1983, Sutton 2007a).¹ There is ample evidence of big firms such as Walmart, Amazon or Starbucks getting even bigger, see e.g., (The Economist, 2012). In fact, sometimes expansion in itself is a key performance index for firms, e.g., recently a key priority for the Four Seasons hotel chain has been to adopt a location growth strategy (The Economist, 2013).

On the other hand, there are many examples of firms that were once dominant but have since experienced various stages of decline, e.g., Kmart, Circuit City, and Blockbuster. More interestingly, there is also evidence of firms that continue to be dominant but have found that rapid firm expansion may come at the cost of quality, and thus, profitability. Indeed, Toyota’s focus on size and growth has been blamed as one of the main reasons behind recent overlooked safety issues with its cars (see e.g., BBC News, 2010, Cole, 2011).² More recently, Target’s expansion in to Canada has been unprofitable and problematic (Austen, 2014).³

The key to understanding market share persistence is analyzing market shares dynamics within an industry (Sutton, 2007a). This suggests that a natural approach to study the process driving market share transitions is through an oligopoly model of industry dynamics that has the potential

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²Akio Toyoda, president and CEO of Toyota in testimony before the U.S. Congress stated, “Toyota has, for the past few years, been expanding its business rapidly. Quite frankly, I fear the pace at which we have grown may have been too quick. I would like to point out here that Toyota’s priority has traditionally been the following: First; Safety, Second; Quality, and Third; Volume. These priorities became confused, and we were not able to stop, think, and make improvements as much as we were able to before, and our basic stance to listen to customers’ voices to make better products has weakened somewhat. We pursued growth over the speed at which we were able to develop our people and our organization, and we should sincerely be mindful of that. I regret that this has resulted in the safety issues described in the recalls we face today, and I am deeply sorry for any accidents that Toyota drivers have experienced (The Guardian, 2010).”

³Although the jury is probably still out on whether Target will succeed in Canada, this example indicates that retail success in the U.S. doesn’t necessarily guarantee success in Canada, much less translate across more distant borders and cultures. Another such example is KFC’s experience in the Japanese market in the 70s. Our data includes retail chains entering the Canadian market after success in the U.S. These examples serve as a reminder that a priori one should not presume that because a firm was successful in the U.S. it would be successful in Canada. In fact, the KFC experience suggests that the very factors that drive success in the U.S. can be detrimental and obstacles to success in a different culture and market (Bartlett and Rangan, 1992).
to generate these transitions as a consequence of strategic interaction between firms.\(^4\) This paper develops and estimates such a dynamic oligopolistic model in which firms choose to expand or contract and firm size is endogenous. Size, in turn, has spillovers on a firm’s future market outcomes, and thereby on its relative dominance and market structure. Additionally, the model allows for heterogeneity in the dynamic link between firm size and future market outcomes either due to inter-temporal spillovers of size or organizational retention of past profitability shocks. The model is motivated by the objective to examine the nature of the dynamic relationship between firm size, market performance and industry evolution. For example, do firms continue to grow and remain profitable as they get larger, and whether there is heterogeneity across firms in this linkage. Furthermore, whether heterogeneity in these effects leads to some firms becoming dominant, while others are relegated to the fringe, and the consequences of this for industry concentration and evolution.

A critical challenge in estimating a dynamic oligopoly model that also allows for firm level persistence in market outcomes is that it will by construction also contain a firm specific state variable that is time varying, serially correlated and potentially unobserved (to the researcher). Given the current state of econometric methods it remains very difficult to estimate such a dynamic game (see e.g., Ackerberg, Benkard, Berry, and Pakes, 2007). Thus, in estimating the model the paper makes a methodological innovation by extending the two step procedure proposed by Bajari, Benkard and Levin (2007) to allow for firm specific serially correlated unobservable variables that are endogenous to past actions. Using data on Canadian hamburger retail chains from their inception in 1970 to 2005, we study the decision to expand or contract for A&W, Burger King, Harvey’s, McDonald’s, and Wendy’s across all Canadian cities. This setting provides us a suitable laboratory for studying the relationship between firm size and market dominance. First, the time period we study captures comprehensive dynamics on the extensive margin, as this was the time period in which the chains’ strategic focus was revenue growth via store numbers, as opposed to improvements in existing store sales. Secondly, each outlet that is constructed by a retail chain is nearly identical, in terms of outlet size and product offerings. Therefore, the volatility in expansion and contraction helps map out the spillovers of each firm’s operational size over time.

Our model is also motivated by the observations that (i) the dynamics of firm entry, exit, expansion and contraction are very rich and complex, and there is evidence that that changes in retail market structure are driven more by expansion and contraction and less by de novo entry

\(^4\)In contrast, Sutton (2007a) uses a statistical Markovian approach to study market share dynamics.
or exit (Hanner et al., 2011), and (ii) that firms change over time based on their experience, and so capturing unobserved firm specific heterogeneity requires going beyond a time invariant fixed effect.

As a key step to understanding the dynamic linkages between firm size, and market structure, our work builds on various literatures. The cornerstone of our work is the literature on estimating static and dynamic games of entry (e.g., Bresnahan and Reiss, 1991a, 1991b, Berry, 1992, Scott Morton, 1999, Seim, 2006, Pakes, Ostrovsky, and Berry 2007, Pesendorfer and Schmidt-Dengler, 2008, Aguirregabiria and Mira, 2007, Ellickson and Misra, 2008, Datta and Sudhir, 2009, Zhu and Singh, 2009, Shen, 2010; Vitorino, 2012, Igami and Yang, 2013, Orhun, 2013) that has used entry and exit decisions of firms to estimate the primitive parameters of the payoffs, and hence, infer industry conduct. In particular, our research contributes to the growing body of applied empirical work on dynamic oligopoly models (e.g., Ericson and Pakes, 1995, Collard-Wexler, 2013). In particular, we extend the work-horse model of entry (e.g., Bresnahan and Reiss, 1991a, Berry, 1992) to allow for firm expansion or contraction and endogenous evolution of firm size.

The model allows for an inter-temporal size spillover that affects profitability and is endogenous to past firm size thereby providing a mechanism that may link firm size with profitability and market dominance. In order to do this we borrow from the literature on firm size and industry dynamics (e.g., Lucas, 1978; Jovanovic, 1982; Hopenhayn, 1992; Boulding and Staelin, 1990). In particular, as in Jovanovic (1982) and Hopenhayn (1992), we model firm level spillovers shocks as a stochastic element in the profit function that follows a Markov process. Further, we endogenize this shock to firm decisions as in, e.g., Ericson and Pakes (1995) and Benkard (2004). These papers treat the shocks as endogenous to firm R&D whereas we treat the spillover shock as a function of firm size. This is predicated on the fact that firm size expansion via store proliferation is a key investment and strategic decision for retail chains, and a proxy for their experience and familiarity with the market. Although, given our data we cannot disentangle the underlying sources of the firm size spillovers, such as learning by doing or economies of scale, our empirical implementation of the link between profitability and size spillovers is related to the long literature on learning by doing.

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6See e.g., Benkard (2004) for an example of a firm that evolves based on learning by doing.

7Collard-Wexler (2013) also allows for firm size to be endogenous but in contrast to his model where firms may choose to be “small,” “medium,” or “large.” Our model allows for a much finer choice of firm size.

8A common practice now for retail chains is to purchase the land that houses their stores (Love, 1995). In that sense, expansion or contraction can be seen as strategic real estate investment decisions.
e.g., Arrow (1962), Bass (1980), Dolan and Jeuland (1981), Benkard (2000, 2004), Besanko et al. (2010). Moreover, as in Benkard (2000) and Besanko et al. (2010) we allow for organizational forgetting (or retention) in the form of persistence (or depreciation) of the past profitability shocks. Our empirical specification of the controlled stochastic process that defines the firm level spillover can also be broadly linked to the literature on profitability dynamics (e.g., Hall and Weiss, 1967; Schmalensee, 1989). However, in contrast, our controlled stochastic spillover process generates an *endogenous* serially correlated unobserved state variable in the dynamic game.

We estimate the model by extending the two step procedure proposed by Bajari, Benkard and Levin (2007) to allow for serially correlated unobservable variables that are endogenous to past actions by using a particle filter based procedure. Our work is related to the literature on using particle filters or sequential Monte Carlo to control for serially correlated unobserved heterogeneity in dynamic models. A special case of this is the popular GHK simulator (see e.g., Keane 1994, Erdem and Keane 1995) that arises for a particular choice of the Gaussian importance sampling density. More recently, particle filters have also been used to estimate dynamic equilibrium models (e.g., Fernandez-Villaverde and Rubio-Ramirez 2007, Blevins 2011, Gallant, Hong and Khwaja 2013a, 2013b). We depart from this literature in two important ways. In contrast with Erdem and Keane (1996), and Fernandez-Villaverde and Rubio-Ramirez (2007) we allow for explicit strategic interaction among agents in the form of a dynamic game. Our two-step estimation method extends Blevins (2011) to allow for endogenous feedback from past actions in the serially correlated firm specific unobservable state. Moreover, our dynamic oligopoly model of firm size dynamics is estimated in a way that is computationally easier to implement than the “full solution” nested fixed point approach adopted by Gallant, Hong and Khwaja (2013a, 2013b). Our paper also provides a bridge between the literature applying Kalman or particle filters to account for serially correlated unobservable variables in either dynamic linear or reduced form models of firm decisions (e.g., Naik, Raman, and Winer, 2005, Sriram, Chintagunta, and Neelamegham, 2006, Jap and Naik, 2008; Bass et al., 2007, Bruce, 2008), and dynamic oligopoly models that explicitly incorporate strategic interactions between forward-looking firms. Our method is also related to the pioneering work of Arcidiacono and Miller (2011). However, there are some key differences: (i) our method

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9Previous research has generated mixed findings about organizational forgetting. For instance, Argote, Beckman, and Epple (1990), Darr, Argote, and Epple (1995), Epple, Argote and Devadas (1991), and Epple, Argote and Murphy (1996) provide evidence in favor of depreciation with data from shipbuilding, pizza chains, truck production, and automotive assembly respectively. In contrast, Thompson (2007) shows a weaker depreciation effect in the shipbuilding once sufficient controls are accounted for. Sorenson (2003) provides evidence from the computer workstation manufacturing industry on heterogeneity in organizational learning depending on internal firm and external market structure.
allows for the serially correlated unobservable to have continuous support, (ii) be firm (or agent) specific as opposed to market specific, and (iii) endogenous to past agent actions.\textsuperscript{10}

The estimates from our model highlight the importance of both firm size spillovers and organizational forgetting among Canadian hamburger retail chains. There is evidence not only of spillovers of firm size and organizational forgetting but more importantly of heterogeneity in these across firms. The heterogeneity in the dynamic linkage between firm size and profitability helps explains why some firms become particularly dominant and others falter as they evolve, with consequences for market structure and concentration. Simulations with the estimated model produce three main insights. First, we demonstrate that our baseline model that incorporates firm size spillovers in the serially correlated unobserved profitability component fits the data better than alternative models that ignore such effects. Second, we illustrate the strength of McDonald’s organizational retention of past profitability through the persistence of unobserved profitability shocks over time. Finally, we show that the market share advantage McDonald’s enjoys from its unobserved profitability is robust to various demand and supply shocks.

2 Data

2.1 Market Definition

For our analysis, we study the retail chain outlet expansion and contraction patterns across all Canadian Census Metropolitan Areas (i.e., cities) from 1970 to 2005. The hamburger retail chains we focus on are the main players in the industry during that time period: A&W, Burger King, Harvey’s, McDonald’s, and Wendy’s. In total, our panel covers 31 cities over 36 years for each of the 5 retail chains. We interpret each city as an isolated market. Cities are separated by distances of at least 60 km.\textsuperscript{11} We choose this definition of market for two main reasons. First, one could make a similar argument as Toivanen and Waterson (2005) in that demand in one city’s fast food is unlikely driven by residents in another city at least 60 km away. Secondly, while individual managers within each chain’s real estate division have well-defined geographic jurisdictions, each city’s headquarters has a real estate manager that is in charge of the chain’s overall growth strategy.

\textsuperscript{10}Conceivably, Arcidiacono and Miller’s method may be extended to allow of agent specific unobservables but this might lead to a proliferation of parameters, e.g., if an agent specific transition matrix for the unobservables is required. Similarly, the transition matrix could potentially be allowed to depend on lagged choices of agents but we are unaware of such an implementation. Conversely, incorporating market specific unobservables instead of firm (or agent) specific unobservables is a straightforward special case of our set up.

\textsuperscript{11}In terms of driving time between two cities in our sample that are in closest proximity to one another (Toronto and Oshawa), one would need to drive at least 40 minutes.
Figure 1: Evolution of Market Structure Over Time

for that city. With this definition of the market, we can see for each year, how many new stores were added, how many existing stores there are, and how many existing stores were closed.

2.2 Data Sources

The retail store entry and exit data in raw form was originally collected by Yang (2012), who using historical archives of phone directories tracked each and every outlet that was ever in operation in Canada. This data is supplemented with market characteristics obtained from the Canadian Census. In particular, we have characteristics that affect revenue, such as population and income, and characteristics that may affect the fixed costs, such as property value (as many retail chains purchase the land that houses their restaurants). We also include region-specific minimum wage levels over time from the Human Resources and Skills Development Canada online database. This provides an additional variable that controls for cost, as fast food chains often hire workers at or near minimum wage.

2.3 Market Characteristics

Table 1 provides a snapshot of the main variables used for estimation. Most retailers have about three existing outlets on average (across markets and over time). McDonald’s is an outlier with about 12 outlets on average. Figure 1 displays the growth of the fast food industry as measured by the annual total number of outlets.

We augment Yang’s (2012) data by including information about whether a city hosted the
Table 1: Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>S.D.</th>
<th>Min.</th>
<th>Max.</th>
<th>Obs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual number of A&amp;W outlets per city</td>
<td>4.5</td>
<td>6.9</td>
<td>0</td>
<td>50</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of Burger King outlets per city</td>
<td>2.6</td>
<td>3.7</td>
<td>0</td>
<td>33</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of Harvey's outlets per city</td>
<td>2.9</td>
<td>5.9</td>
<td>0</td>
<td>54</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of McDonald's outlets per city</td>
<td>12.3</td>
<td>21.7</td>
<td>0</td>
<td>164</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of Wendy's outlets per city</td>
<td>2.3</td>
<td>3.5</td>
<td>0</td>
<td>23</td>
<td>1116</td>
</tr>
<tr>
<td>Annual change in number of A&amp;W outlets</td>
<td>0.2</td>
<td>0.9</td>
<td>-7</td>
<td>13</td>
<td>1116</td>
</tr>
<tr>
<td>Annual change in number of Burger King outlets</td>
<td>0.2</td>
<td>0.7</td>
<td>-2</td>
<td>10</td>
<td>1116</td>
</tr>
<tr>
<td>Annual change in number of Harvey's outlets</td>
<td>0.2</td>
<td>1.0</td>
<td>-14</td>
<td>18</td>
<td>1116</td>
</tr>
<tr>
<td>Annual change in number of McDonald's outlets</td>
<td>0.8</td>
<td>2.1</td>
<td>-7</td>
<td>29</td>
<td>1116</td>
</tr>
<tr>
<td>Annual change in number of Wendy's outlets</td>
<td>0.2</td>
<td>0.6</td>
<td>-4</td>
<td>6</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities entered by A&amp;W</td>
<td>1.0</td>
<td>1.9</td>
<td>0</td>
<td>8</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities entered by Burger King</td>
<td>0.9</td>
<td>1.6</td>
<td>0</td>
<td>7</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities entered by Harvey's</td>
<td>0.9</td>
<td>1.2</td>
<td>0</td>
<td>4</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities entered by McDonald's</td>
<td>0.9</td>
<td>1.9</td>
<td>0</td>
<td>8</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities entered by Wendy's</td>
<td>0.9</td>
<td>1.5</td>
<td>0</td>
<td>6</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities exited by A&amp;W</td>
<td>0.2</td>
<td>0.5</td>
<td>0</td>
<td>2</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities exited by Burger King</td>
<td>0.1</td>
<td>0.3</td>
<td>0</td>
<td>1</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities exited by Harvey's</td>
<td>0.1</td>
<td>0.4</td>
<td>0</td>
<td>2</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities exited by McDonald's</td>
<td>0.0</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
<td>1116</td>
</tr>
<tr>
<td>Annual number of cities exited by Wendy's</td>
<td>0.1</td>
<td>0.2</td>
<td>0</td>
<td>1</td>
<td>1116</td>
</tr>
<tr>
<td>HHI (based on number of outlets)</td>
<td>0.4</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>1076</td>
</tr>
<tr>
<td>Population (millions)</td>
<td>0.4</td>
<td>0.5</td>
<td>0.1</td>
<td>2.9</td>
<td>1116</td>
</tr>
<tr>
<td>Income (millions)</td>
<td>0.1</td>
<td>0</td>
<td>0</td>
<td>0.1</td>
<td>1116</td>
</tr>
<tr>
<td>Property value (millions)</td>
<td>0.3</td>
<td>0.1</td>
<td>0.1</td>
<td>0.8</td>
<td>1116</td>
</tr>
<tr>
<td>Minimum wage</td>
<td>4.6</td>
<td>1.9</td>
<td>1.1</td>
<td>8</td>
<td>1116</td>
</tr>
</tbody>
</table>

Canadian Football League’s (CFL) Grey Cup championship tournament. Each year, a city is selected to host the Grey Cup by a board of governors at the CFL. While this process is done through a bidding process to ensure that a certain level of revenue can be generated, the board tries to rotate the event across all member cities. The Grey Cup event draws in fans from all provinces, and is said to generate significant revenues for the host city (Johnstone, 2012). There may also be some long-run benefits in the form of improved infrastructure and construction of new facilities, as these investments are often conditions of the submitted bids. Table 2 lists each of the cities that have hosted the Grey Cup at any point between 1970 to 2005. In our sample, while all cities that have CFL teams had an opportunity to host at some point, Montreal, Toronto, and Vancouver had the most opportunities to host the event. The table also shows us the average number of total new fast food outlets for those cities for years that they hosted the Grey Cup, and years that they did not host the Grey Cup. For most cities, fast food expansion is greater during the years they host the event.

Our data also includes additional information about the roll-out of anti-smoking regulations across Canadian municipalities. We obtain this information from Shields (2007), which gives us the precise years in which each municipality introduced smoking by-laws that prohibit people from

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12 This event is the Canadian equivalent to the National Football League’s (NFL) Super Bowl.
13 Refer to [http://cfldb.ca/faq/league/](http://cfldb.ca/faq/league/) for more details.
smoking in public places. In Canada, these regulations were first introduced in some cities, and then applied generally at a provincial level. From Table 3, we see that there is some variation in terms of the timing of the regulation. This additional data is included in our analysis as past work has shown that smoking by-laws have an impact on the amount of food consumed in restaurants (Lewis, 2012). From the table below, it is not obvious whether the smoking by-laws had a positive or negative effect on fast food expansion. In some cities, there was more expansion after the policy, while in others, there was less.

### 2.4 Expansion and Contraction Patterns

To motivate our decision to focus on expansion and contraction in the dynamic oligopoly model, we present total counts of entry, exit, re-entry, and re-exit. Table 4 illustrates that while there is
variation in these entry, exit, re-entry, and re-exit events across retail chains, there is dispropor-
tionately more expansion and contraction than pure entry and exit. In fact, this is consistent with
recent evidence that changes in market shares and industry structure among retailers are largely
due to expansion and contraction by incumbents and less due to de novo entry or exit (Hanner et
al., 2011).

To focus on a concrete example, let us consider A&W’s experience in Abbotsford, British
Columbia in more detail (see footnote 5). It first entered the city in 1972, left in 1975, re-entered
in 1976, and left again in 1984. Not only can expansion and contraction decisions perfectly capture
these entry and exit patterns, expansion and contraction patterns will identify dynamics otherwise
left out by entry and exit. Continuing with our example, in 1972, A&W enters by expanding via
one new store, exits by contraction in 1975, re-enters by expanding via one new store in 1976,
and re-exits in 1984. But expansion and contraction information tells us more; after the re-exit in
1984, A&W expands via one store in 1988, and then expands again via one store per year in 1991
and 1992, followed by a contraction of 2 stores in 1993, expansion via one store per year in 1994
and 1995, contraction of 2 stores in 1996, expansion of 2 stores in 1997, contraction of 1 store in
1999, and expansion of 1 store in 2002. Essentially, a lot of A&W’s decisions after 1984 would not
be captured using entry and exit observations alone. Overall, there are 207, 144, 157, 354, and
146 cases for A&W, Burger King, Harvey’s, McDonald’s, and Wendy’s respectively, in which entry
observations do not capture variation in expansion decisions. Similarly, for exit observations, there
are 71, 42, 39, 14, and 19 cases.

2.5 Market Structure and Size Dynamics

Our aim is to understand the relationship between firm size and future market share, which ulti-
mately generates persistence in market power. To look more closely at the relationship between
firm size and subsequent market power, we regress market share on last period’s own-brand and
other-brand firm size with fixed effects and time dummies. For these regressions, we calculate a
chain’s market share as the proportion of stores in a given market-time that belong to that chain. We consider 12 possible specifications. Specifications 1 and 2 run the regressions using the pooled observations from all of the chains, and include chain fixed effects on top of city and time fixed effects. Specifications 3 to 12 run the regressions separately for each of the chains.

From Table 5, we see in column 1 that increasing the size of the chain in a market is associated with an increase in subsequent market shares. Column 2 demonstrates how the size effect changes if we include also the size of competitors. After the inclusion of competitors, we see that the size effect of a chain’s own stores remains at roughly the same value. Slightly different patterns emerge if we look at columns 3 to 12, where the regressions are run separately for each chain. The own chain size effects as displayed in columns 3 to 7 are smaller than in columns 8 to 12. Therefore, the inclusion of competition from other chains leads to estimated own chain size effects that are a bit larger. Also notice from columns 8 to 12 that there are a number of instances in which a chain’s size of a competitor has a negative effect on its future market share. If we compare the firm size effects across the different chains, we see that they are largest for McDonald’s. McDonald’s past size is associated with a subsequent growth of stores. For its competitors, we don’t see such a relationship. Additional description of the data and reduced form analysis is reported in the Appendix.

3 Model

We consider a model of \( i = 1, \ldots, I \) forward-looking firms in a retail industry that make decisions about operating in market \( m \) in every time period \( t \). For a given market \( m \), at the beginning of each time period \( t \), firm \( i \) must decide whether to expand or contract operations, or make no changes to the number of outlets, i.e., \( n_{imt} \in \{-K, -(K-1), \ldots, -1, 0, 1, \ldots, K-1, K\} \). Based on this decision the total stock of active outlets in city\(^{14} \) \( m \) at time \( t \) for firm \( i \) evolves according to:

\[
N_{imt} = N_{im,t-1} + n_{imt}.
\]

It should be noted that our formulation allows for entry if \( (N_{im,t-1} = 0 \& N_{imt} > 0) \) and exit if \( (N_{im,t-1} > 0 \& N_{imt} = 0) \). Each forward-looking firm \( i \), whether incumbent or potential entrant, make a decision \( n_{imt} \) to maximize its expected discounted stream of profits for each market \( m \) in each time period \( t \). In the tradition of discrete choice models of entry and exit (e.g., Bresnahan

\(^{14}\)Since we define each market to be a city we use these interchangeably.
Table 5: Market Share Regressions

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged own outlets</td>
<td>0.00520***</td>
<td>0.00520***</td>
<td></td>
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Standard errors in parentheses; ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.
and Reiss, 1991a, 1991b, Berry, 1992), we define the reduced form one-shot payoff function as

\[
\Pi_i(n_{imt}, n_{-imt}, N_{im,t-1}, N_{-im,t-1}, X^R_{mt}, X^C_{mt}, Z_{imt}, \nu_{imt})
\]
\[
\quad = R(n_{imt}, n_{-imt}, N_{im,t-1}, N_{-im,t-1}, X^R_{mt}, Z^R_{imt})
- C(n_{imt}, n_{-imt}, N_{im,t-1}, N_{-im,t-1}, X^C_{mt}, Z^C_{imt}) + \nu_{imt},
\]

where \( R \) is a revenue function, \( C \) is a cost function, and \( \nu_{imt} \) is a private payoff shock. Here, \( n_{-imt} \) is a vector of the number of outlets \( i \)'s rivals choose to open or close in city \( m \) at time \( t \). Similarly, \( N_{-imt} \) is a vector of the total number of outlets \( i \)'s rivals have in city \( m \) at time \( t \). The revenue and cost shifters are \( X^R_{mt} \) and \( X^C_{mt} \), respectively; for instance, population and income would be included in \( X^R_{mt} \), while real estate costs would be in \( X^C_{mt} \). The variables \( Z^R_{imt} \) and \( Z^C_{imt} \) are, respectively, the unobserved (to the researcher) revenue and cost components of profitability of the retailers. Furthermore, \( \theta^R \) and \( \theta^C \) are vectors of model parameters. Finally, \( \nu_{imt} \) is a privately known i.i.d. profit shock (i.e., a structural error) drawn from a distribution \( F(\nu_i|S_{imt}) \) with support \( \mathcal{V}_i \subset \mathbb{R} \), where \( S_{imt} \) is the payoff relevant state defined below.

Let \( S_{imt} = (N_{im,t-1}, N_{-im,t-1}, X^R_{mt}, X^C_{mt}, Z_{imt}) \) denote the current payoff-relevant state from the perspective of firm \( i \) in market \( m \) at time \( t \). To be more precise, we specify the one-shot payoff function parametrically as follows:

\[
\Pi_i(n_{imt}, n_{-imt}, S_{imt}, \nu_{imt}) = \theta^R_1 X^R_{mt} - \theta^C_1 X^C_{mt} - \theta_2 \sum_{j \neq i} N_{jmt} + \theta_3 N_{imt}
\]
\[
- \psi_1 \cdot 1\{N_{im,t-1} = 0, n_{imt} > 0\} - \psi_2 \cdot 1\{n_{imt} > 0\} \cdot n_{imt}
\]
\[
- \psi_3 \cdot 1\{N_{im,t-1} > 0, n_{imt} < 0\} \cdot n_{imt} + \gamma Z_{imt} + \nu_{imt}.
\]

The one-shot payoff represents revenue net of costs. Revenue is a function of the size of the market, as determined by \( \theta^R_1 X^R_{mt} \). However, if retailers face competition, their revenue is reduced by \( \theta_2 \sum_{j \neq i} N_{jmt} \). Each store brings in additional revenue totaling \( \theta_3 N_{imt} \). In addition to the costs \( \theta^C_1 X^C_{mt} \), the retailers face an entry cost, \( \psi_1 \), expansion cost, \( \psi_2 \), and salvage or scrap value \( \psi_3 \). The salvage or scrap value is net of any contraction costs for the firm if these exist, e.g., penalties for breaking a real estate rental lease, severance payments to workers, etc.

We assume that the composite unobserved profitability, \( Z_{imt} \), is additive in \( Z^R_{imt} \) and \( Z^C_{imt} \) and that this composite profitability is a firm-specific, autoregressive process which evolves according to

\[
Z_{imt} = \mu_i + \delta_i Z_{im,t-1} + \beta_{i1} N_{im,t-1} + \beta_{i2} N^2_{im,t-1} + \beta_{i3} N^3_{im,t-1} + \eta_m + \epsilon_{imt},
\]
where \( \epsilon_{imt} \sim N(0, \omega_i^2) \) is i.i.d. Therefore, the parameters \((\mu_i, \delta_i, \beta_{i1}, \beta_{i2}, \beta_{i3}, \omega_i, \eta_m)\) fully characterize the evolution of firm \(i\)'s unobserved profitability in market \(m\), net of the i.i.d. shock \(\nu_{imt}\).

This unobserved profitability measure has three main components. The first component, captured by \(\delta_i\), is the persistence of profitability, or in other words, the extent to which retailers retain or conversely forget what they have learned through experience (i.e., organizational forgetting). A second component, captured by the parameters \(\beta_{i1}, \beta_{i2}, \text{and } \beta_{i3}\), accounts for the changes in profitability as the chain’s size in a given city changes over time (i.e., intertemporal size spillover). This empirical specification of inter-temporal size spillovers and organizational retention (or forgetting) in unobserved profitability is similar to the learning by doing process in Benkard (2000, 2004), who in turn builds on Argote, Beckman and Epple (1990).15 Finally, there is the drift component consisting of a firm-specific fixed effect, \(\mu_i\), which represents the long run average profitability for firm \(i\) and a city specific fixed effect, \(\eta_m\). These effects also account for any time trends as the city fixed effect propagates over time as a result of the recursive structure. Finally, \(\epsilon_{imt}\) is a normally distributed i.i.d. innovation to unobserved profitability with standard deviation \(\omega_i\).

This specification allows for heterogeneity across firms since the parameters are chain-specific. Therefore, this specification captures firm-market-specific unobserved heterogeneity that is potentially serially correlated. Incorporating this time-varying endogenous profitability in the model is a critical aspect when studying the inter-linkages between firm size, evolution of market share and profitability. Although, this component of firm-specific profitability is unobserved to the researcher, we make the crucial assumption that \(Z_{imt}\) is observed by all firms. However, the model allows for some elements of a firm’s profitability to be private information, e.g., proprietary technology or processes for supply chain management or service operations, or manufacture of products. This is incorporated in \(\nu_{imt}\), which is known to firm \(i\) when making a decision but unknown to its rivals. This is the key difference in the structural interpretation of the two components of profitability that are unobserved (to the researcher).

In summary, the model’s structural parameters can be represented as \(\alpha = \{\alpha_i\}_{i=1}^L\), where

\[
\alpha_i = (\theta_{i1}^R, \theta_{i1}^C, \theta_2, \theta_3, \psi_1, \psi_2, \psi_3, \gamma, \delta_i, \beta_{i1}, \beta_{i2}, \beta_{i3}, \omega_i, \eta_m).
\]

15As is conventional in the literature, Benkard (2000) estimates the relationship between cumulative output (as a proxy for experience) and lagged cumulative output (allowing for depreciation to measure organizational forgetting), current output, and output of a related product (to capture spillovers across products). Our specification of size spillovers and organizational retention is more reduced form than e.g., Argote, Beckman and Epple (1990), since it cannot disentangle the underlying sources of these effects like economies of scale or learning by doing or employee turnover. This is primarily because of the nature of our data set. On the other hand, we embed this specification in fully specified dynamic oligopoly model with strategic interaction among firms.
The one-shot payoff functions $\Pi_i$ depend implicitly on these parameters. Given an initial state $S_{int}$ at time $t$, the firm’s expected present discounted profit, prior to the private shock $\nu_{int}$ being realized, is

$$E \left[ \sum_{\tau=t}^{\infty} \rho^{\tau-t} \Pi_i(n_{int}, n_{-int}, S_{int}, \nu_{int}) \mid S_{int} \right],$$

where $\rho$ is the discount factor, $0 \leq \rho < 1$. The firm’s objective is to maximize the present discounted value of its profit at each time period $t$ taking as given the equilibrium action profiles of other firms. The expectation here is taken over the firms’ actions and private shocks in the current period as well as the future evolution of the state variables, private shocks, and actions of all firms.

We follow the literature in specifying a dynamic game of incomplete information and focusing on Markov perfect equilibria (MPE) in pure strategies (e.g., Ackerberg, Benkard, Berry, and Pakes, 2007; Bajari, Benkard and Levin, 2007). In order to define the MPE strategies for the game we employ the following notation. Recall that $S_{int}$ is the payoff-relevant state for firm $i$ in market $m$ at time $t$. Let $S_i$ denote the state space containing all feasible values of $S_{int}$ for firm $i$ and let $N_i$ denote firm $i$’s choice set (i.e., $n_{int} \in N_i$). Also, define $S$ to be the collection of $S_i$. For simplicity, we let $S_{mt}$ denote the market state, defined as the collection of the state variables $S_{int}$ of all firms in market $m$ at time $t$. Similarly, let $\nu_{mt}$ denote the collection of the i.i.d. private shocks $\nu_{int}$ of all firms in market $m$ at time $t$. A Markov strategy for firm $i$ is a function $\sigma_i : S \times \mathbb{R} \rightarrow N_i$ mapping payoff-relevant state variables and private information to the set of possible actions. We denote a profile of Markov strategies by $\sigma = (\sigma_1, \ldots, \sigma_I)$.

The ex-ante value function $V_i(S_{mt}; \sigma)$ gives the expected present discounted value of profits obtained by firm $i$ when players use strategies $\sigma$ and the market state is $S_{int}$. Dropping the market and time indices here, we define the ex-ante value function recursively as

$$V_i(S; \sigma) = E \left[ \Pi_i(\sigma(S, \nu), S, \nu) + \rho E \left[ V_i(S'; \sigma) \mid S, n = \sigma(S, \nu) \right] \mid S \right].$$

Here, $\sigma(S, \nu)$ denotes the action profile $(\sigma_1(S, \nu_1), \ldots, \sigma_I(S, \nu_I))$ and $n$ denotes the same profile represented as $(n_1, \ldots, n_I)$. The outer expectation is over current values of the private shocks, $\nu$, and hence current actions of rivals, given $S$. The inner expectation is with respect to the state variable next period, $S'$, conditional on the state $S$ and the actions of all firms in the current period $(n_i, n_{-i})$.

A MPE is defined as a Markov strategy profile $\sigma$ such that no firm $i$ has an incentive to unilaterally deviate from its strategy $\sigma_i$ while its rivals are playing according to their strategies in $\sigma_{-i}$. Thus, for any firm $i$ there is no alternative Markov strategy $\tilde{\sigma}_i$ that yields higher expected
discounted profits (in terms of $V_i(\cdot)$) than $\sigma_i$ while its rivals are using their strategies in $\sigma_{-i}$. Formally, $\sigma$ is an MPE if for all firms $i$, all market states $S$, and for all alternative Markov strategies $\tilde{\sigma}_i$ the following condition holds:

$$V_i(S; \sigma_i, \sigma_{-i}) \geq V_i(S; \tilde{\sigma}_i, \sigma_{-i}).$$

(6)

Note that for the alternative Markov strategy profile $(\tilde{\sigma}_i, \sigma_{-i})$, when the realized private information is $\nu$, the realized actions are $\tilde{n}_i = \tilde{\sigma}_i(S, \nu_i)$ and $n_{-i} = \sigma_{-i}(S, \nu_{-i})$. Therefore, the recursive expression for the ex-ante value function is

$$V_i(S; \tilde{\sigma}_i, \sigma_{-i}) = \mathbb{E}\left[ \Pi_i(\tilde{\sigma}_i(S, \nu_i), \sigma_{-i}(S, \nu_{-i}), S_i, \nu_i) + \rho \mathbb{E}\left[ V_i(S'; \tilde{\sigma}_i, \sigma_{-i}) \mid S, \tilde{n}_i = \tilde{\sigma}_i(S, \nu_i), n_{-i} = \sigma_{-i}(S, \nu_{-i}) \right] \mid S \right].$$

Inside the outer expectation, the first argument of the payoff function $\Pi_i$ is $\tilde{n}_i = \tilde{\sigma}_i(S, \nu_i)$, which is the implied action by firm $i$ under strategy $\tilde{\sigma}_i$ when the state is $S$ and the realized private information is $\nu_i$. Similarly, the second argument is $n_{-i} = \sigma_{-i}(S, \nu_{-i})$, which is a profile of rival actions given the state $S$ and the vector of private information shocks $\nu_{-i}$.

### 4 Estimation

Solving for even a single equilibrium of the game is both intractable analytically and prohibitively expensive computationally, therefore we take a two-step approach to estimation. Our estimation procedure is an augmented version of the method proposed by Bajari, Benkard, and Levin (2007). In the first step, we estimate transition equations for the state variables, which characterize how the state variables evolve, along with the reduced form policy functions for each of the firms, which map state variables to actions and approximate the observed equilibrium behavior. In the second step, we use these quantities to simulate the ex-ante value functions which are in turn used to impose the equilibrium conditions in (6) via a minimum distance criterion function. This allows us to estimate the structural parameters without ever directly solving the model. As with the original method, we assume the state variables follow a first-order Markov process. The key difference in our approach, however, is in introducing latent firm-specific and time-varying state variables to control for unobserved, but possibly persistent differences in the spillovers and organizational forgetting. To accomplish this, we build on the sequential Monte Carlo approach of Blevins (2011) and Gallant, Hong and Khwaja (2013a, 2013b).
4.1 First Stage Estimation

In the first stage we jointly estimate the posterior distributions of firm-specific latent state variables in each period (conditional on observed actions and states), parameters of the transition equations for the latent states, and policy functions that condition on the levels of the latent states. We describe the estimation of state transition equations and policy functions in turn below, before turning to the second stage.

Let $X_{R}^{m,t}$ be the vector of state variables related to revenue in market $m$ at time $t$, such as population, income and whether the city is hosting the CFL Grey Cup. Similarly, let $X_{C}^{m,t}$ be the vector of state variables related to costs, such as property value. We summarize these exogenous state variables by collecting them in a vector $X_{mt} = (X_{R}^{m,t}, X_{C}^{m,t})$. We use similar notation for the latent, endogenous state variables $Z_{imt}$. Let $Z_{mt} = (Z_{1mt}, \ldots, Z_{Jmt})$ denote the vector of all firm-specific latent state variables in market $m$ in period $t$. The variables $Z_{imt}$ are endogenous. The evolution of these variables is influenced by the lagged actions of each firm as well as the lagged values $Z_{im,t-1}$, according to (4). As such, we estimate the parameters of the law of motion of $Z_{imt}$ for each $i$ jointly with the policy functions as described below. On the other hand, the variables in $X_{mt}$ are exogenous and so we estimate the parameters of the transition equations for these variables separately from the other parameters.

Collectively, let $\phi$ denote the vector of all first-stage parameters: the coefficients of the reduced form policy functions, the coefficients for the transition functions for the exogenous state variables, and the parameters $(\delta, \beta_{1i}, \beta_{2i}, \beta_{3i}, \omega_i)$ and $\eta_m$ for the law of motion for $Z_{imt}$ for each $i$ and $m$ as specified in (4). The Bajari, Benkard and Levin (2007) method requires that in the first step we obtain consistent estimates of $\phi$. Given data $\{n_{mt}, X_{mt}\}_{t=0}^{T}$ for the entire sample of $m = 1, \ldots, M$ markets, consistent estimates of $\phi$ can be obtained by maximizing the following likelihood function,

$$L_{M}(\phi) = \prod_{m=1}^{M} \prod_{t=1}^{T} L_{m}(n_{mt}, X_{mt} \mid N_{m,t-1}, X_{m,t-1}, \phi)$$

$$= \prod_{m=1}^{M} \prod_{t=1}^{T} \int l_{m}(n_{mt} \mid X_{mt}, Z_{mt}, \phi) p(X_{mt} \mid X_{m,t-1}, \phi) p(Z_{mt} \mid N_{m,t-1}, X_{m,t-1}, \phi) dZ_{mt}. \quad (7)$$

Here, the $M$ subscript indicates the dependence of the likelihood function on the entire sample, and the $m$ subscript denotes dependence on the parameters for market $m$. It should be noted that

---

16Note that although the state transition parameters are structural parameters, they are being estimated in the first stage, so there is some overlap between the parameters in $\alpha$ (described in Section 3) and $\phi$. Recall also that $N_{mt}$ is a vector of the total number of outlets each firm $i$ has in market $m$ at time $t$, and $n_{mt}$ is the analogue of firm choices.
the second equality follows from using the structure of the model to decompose the likelihood for
the observed data into three components after conditioning and integrating out the unobservable
firm specific profitability components $Z_{mt}$. The three components in the likelihood are: (i) the
firm-specific choice probabilities $l_m$ conditional on $Z_{mt}$ that represent the reduced form policy
functions, (ii) the joint transition density of the observable state variables, and (iii) the posterior
distribution of $Z_{mt}$ given the data. It should be noted that although the posterior distribution
of $Z_{mt}$ is a reduced form component underlying it is the structural transition function for these
unobserved state variables (4). It is estimated in the process of computing the posterior distribution
as we describe below. We next discuss the specification and estimation of each component starting
with the second and third components and then finally the reduced form policy function. We
pay particular attention to the efficient calculation of the posterior distribution, which is the main
technical innovation in our estimation approach.

We begin with the second component as it is the easiest to describe. The joint transition of the
observable state variables density arises in this form because all variables in $X_{mt}$ are exogenous and
independent of both $Z_{m,t-1}$ and $N_{m,t-1}$. It is for this reason that the transitions of the observed
state variable can be estimated separately, whereas the other two components of the likelihood
$L_m$ are estimated jointly together. Specifically, the process by which the exogenous $X_{mt}$ evolve is
determined by a seemingly unrelated regressions (SUR) model described in the Appendix. Since
major sporting events such as the CFL Grey Cup are announced to the public far ahead of time, we
assume that retailers have perfect foresight about whether a city will host the event in the future.
A similar assumption is made for the roll-out of smoking by-laws, as these regulatory changes are
often announced (and debated) well in advance of the enactment date (Lewis, 2012).\(^\text{17}\)

The posterior distribution $p(Z_{mt} | N_{m,t-1}, X_{m,t-1}, \phi)$ is the third main component of the like-
lihood function (7) and is the implied distribution of $Z_{mt}$ given the parameters and the data from
last period. The main complication in estimating this model is that for each firm $i$ the $Z_{imt}$ is
serially correlated and depends on lagged firm size through (4). In turn, the reduced form policy
functions depend on this unobserved profitability state. Given this recursive relationship between
the firm choices about stores, and hence size, and the the unobserved $Z_{imt}$, we estimate the reduced
form policy functions jointly with the transition process for $Z_{imt}$ by integrating with respect to the
posterior distribution of $Z_{imt}$ given the observed data.

\(^{17}\)The assumption is invoked when forward simulating these market characteristics in estimating the second stage
parameters as described below.
Although, this posterior is not a model primitive, it can be calculated recursively using a sequential Monte Carlo or particle filtering procedure, that requires three pieces of information: (i) the initial distribution\(^{18}\) of unobserved \(Z_{mt}\), (ii) the observation likelihood \(l_m\) from (7) that relates the unobserved \(Z_{mt}\) to the observed \(n_{mt}\), and (iii) the law of motion for the unobserved \(Z_{mt}\) given by (4). Once the process has been initialized using a draw from the initial distribution, the recursive procedure for obtaining the posterior \(p(Z_{mt} | N_{m,t-1}, X_{m,t-1}, \phi)\) can be described in two steps, starting with the lagged posterior \(p(Z_{m,t-1} | N_{m,t-2}, X_{m,t-2}, \phi)\). First, the updating step applies Bayes’ rule to obtain the filtering distribution given by (8) for period \(t-1\),

\[
p(Z_{m,t-1} | N_{m,t-1}, X_{m,t-1}, \phi) = \frac{l_m(n_{m,t-1}, X_{m,t-1} | X_{m,t-2}, Z_{m,t-1}, \phi)p(Z_{m,t-1} | N_{m,t-2}, X_{m,t-2}, \phi)}{\int l_m(n_{m,t-1}, X_{m,t-1} | X_{m,t-2}, Z_{m,t-1}, \phi)p(Z_{m,t-1} | N_{m,t-2}, X_{m,t-2}, \phi) dZ_{m,t-1}}.
\] (8)

Second, the prediction step yields the posterior for \(Z_{mt}\) with period \(t-1\) information by integrating with respect to the transition density for \(Z_{mt}\) given \(Z_{m,t-1}\) and \(N_{m,t-1}\) which we denote by \(q\) and which is implied by (4):

\[
p(Z_{mt} | N_{m,t-1}, X_{m,t-1}, \phi) = \int q(Z_{mt} | Z_{m,t-1}, N_{m,t-1}, \phi)p(Z_{m,t-1} | N_{m,t-1}, X_{m,t-1}, \phi) dZ_{m,t-1}.
\] (9)

Thus, the transition parameters for the \(Z\) processes enter the likelihood through the transition density \(q\).

The fundamental problem is that this distribution is difficult to evaluate analytically because the firm’s choices \(n_{int}\) are a complicated non-linear (best-response) function of \(Z_{int}\) based on a Markov Perfect Equilibrium of the dynamic game. This is seen in particular in (7) and (8) that include the observation likelihood \(l_m\) incorporating the reduced form policy function \(l_m(n_{m} | X_{mt}, Z_{int}, \phi)\). To solve this problem, we extend the approach of Blevins (2011) to estimate the model using a particle filter that allows for endogenous feedback from the lagged size of firms. A non-linear particle filter makes it possible to approximate continuous distributions by a finite collection of weighted point masses. Thus, we replace the integral over the continuous distribution of \(Z_{mt}\) by a summation over a finite number of support points with weights. These points and weights are selected to incorporate all available information about the \(Z_{mt}\) given the model, the data up to time \(t\), and

\(^{18}\)The initial distribution \(p(Z_{m,0})\) was taken to be \(N(0, 1)\) in the empirical application.
a vector of parameters to efficiently approximate the posterior distribution, which is then used to integrate the likelihood sequentially in every time period \( t \). In this procedure, the points are given by the draws from the distribution of \( Z_{mt} \) and the weights by the observation density \( l_m \) evaluated appropriately. More details about this algorithm are provided in the Appendix. We next describe the empirical specification adopted in estimating the reduced form policy functions jointly with the transitions of \( Z_{mt} \).

There are two issues in estimating the reduced form policy functions. The first is that we do not know the true reduced form because finding it would involve finding the choice-specific value functions and projecting them onto exogenous state variables. In our application, firms choose each year how many stores by which to expand or contract, denoted by \( n_{imt} \). Since the choices are naturally ordered and the costs are linear, in practice we approximate the reduced form policy functions by estimating an ordered probit model where the latent index is a suitably flexible, linear-in-parameters function of the exogenous state variables \( X_{imt} \), their interactions and squares, and the unobserved profitability \( Z_{imt} \) (which is unobserved and can be regarded here as an error term).\(^{19}\)

The second complication in estimating the first stage is that \( Z_{imt} \) is a serially correlated unobservable. To deal with this, we evaluate the integrated likelihood function of the ordered probit model that approximates the reduced form policy function using draws from the particle filtering procedure described above (see (8) and (9)).

In particular, we estimate an ordered probit model where the choice set is \( \mathcal{K} = \{ k_1, k_2, \ldots, k_K \} \) with \( k_1 < k_2 < \cdots < k_K \). These values may range from negative to positive, representing expansion and contraction decisions by firms. In our application, we choose \( \mathcal{K} = \{-10, -5, -1, 0, 1, 5, 10\} \). We motivate this discretization with Table 6, which provides us the proportion of observations we see for each expansion or contraction decision. The table shows that there are many instances where the stock of outlets do not change, increase or decrease by one to ten outlets.

Each firm’s decision depends on the value of a latent index, \( y^*_{imt} \), which can be flexibly specified to depend on the relevant state variables. Let \( W_{mt} \) denote the vector of exogenous state variables \( X_{mt} \) along with all distinct pairwise interactions and squares.\(^{20}\) Following the literature, we use the following simple, but flexible linear specification for \( y^*_{imt} \) that includes higher-order terms and

\(^{19}\)Bajari, Benkard and Levin (2007) suggest that the first stage reduced form policy functions should be estimated using flexible nonparametric methods such as kernel regressions or sieve estimators. However, in practice this is usually infeasible and the convention in the literature is to employ some form of parametric approximation, e.g., Ellickson and Misra (2008) use a multinomial logit.

\(^{20}\)We omit certain interactions with indicator variables such as Grey Cup hosting, since including the square of such an indicator variable would introduce perfect multicollinearity. For the precise list of variables contained in \( W_{mt} \), see the reduced form estimates in Table 16 in the Appendix.
interactions:

\[ y^*_{int} = \phi^W \cdot W_{mt} + \phi^Z Z_{int} + \zeta_{int}. \]

As stated earlier, \( Z_{int} \) is a serially correlated variable representing unobserved, firm-specific profitability which follows the law of motion defined in (4), and that is integrated out using the particle filter. The final term, \( \zeta_{int} \), is an independent and normally distributed error term with mean zero and unit variance.\(^{21}\) Also note that firm and city fixed effects are captured by \( Z_{int} \) as specified in (4).

Decisions are related to the latent variable by a collection of threshold-crossing conditions:

\[
  n_{int} = \begin{cases} 
  k_1 & \text{if } y^*_{int} \leq \vartheta_1 \\
  k_2 & \text{if } \vartheta_1 < y^*_{int} \leq \vartheta_2 \\
  \vdots & \vdots \\
  k_K & \text{if } \vartheta_{K-1} \leq y^*_{int} \leq \vartheta_K 
  \end{cases}
\]

The values \( \vartheta_1, \ldots, \vartheta_K \) are the \( K \) cutoff parameters corresponding to each outcome. These cutoffs are estimated using sieve maximum likelihood along with the index coefficients, and the parameters in the law of motion for \( Z_{int} \).

### 4.2 Second Stage Estimation

Once we have estimated policy functions that condition on \( Z_{int} \), along with estimated parameters for the laws of motion of these variables, then the second stage is conceptually identical to that proposed by Bajari, Benkard, and Levin (2007). As such, we briefly summarize the steps below and reserve the full description for the Appendix. In effect, since we can simulate values of \( Z_{int} \)

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\(^{21}\)We normalize the variance of the error term to one because the coefficients in the payoff function are only identified up to scale. Moreover, since there is only a scalar unobservable there are no covariances to estimate or report. Finally, to normalize the scale of \( Z_{int} \), which is also included in the second stage, we fix \( \phi^Z = 1 \).
we can treat these latent variables the same way as the observable variables for the purposes of the second stage.

Recall that we have estimated firm policy functions and state transition equations in hand from the first stage. Each firm’s estimated policy function describes how it will act given a particular state and market structure. In equilibrium, each firm’s strategy must agree with rival firms’ beliefs. We assume that firms have rational expectations about state transitions, so firm beliefs and the state transition equations also agree. Therefore, the quantities estimated in the first stage can be used to simulate many sample paths, or alternative realizations, of the game in each market. Each such path is a sequence of firm actions and state transitions.

These simulated paths are used to estimate ex-ante value functions. We can then use the equilibrium conditions to form a minimum distance objective function to estimate the structural parameters of the model. Although the second stage is unmodified with respect to Bajari, Benkard, and Levin (2007), the introduction of the latent state variables in the first stage means that one can use the estimated structural parameters to obtain best-response functions for firms that depend not only on the exogenous state variables, but also on the actions of the firm’s rivals and the latent states, both of which are endogenous.

The second stage uses the estimated parameters \( \hat{\phi} \) to approximate the Markov-perfect-equilibrium policy profile \( \sigma \) for the dynamic game. Let \( \hat{\sigma} \) denote the estimated policies corresponding to \( \hat{\phi} \). Using these estimated policies, we can then generate the sequence of latent state vectors \( Z_t \), and subsequently, the sequence of profits. Discounting these profits and summing then yields an estimate of the valuations under the policies used.

Therefore, given policies \( \sigma \) and structural parameters \( \alpha \), we can simulate the \textit{ex ante} value function for a particular firm \( i \) in any possible initial state \( S_1 = (N_0, X_1, Z_1) \),

\[
\hat{V}_i(S_1; \sigma, \alpha) = \mathbb{E} \left[ \sum_{\tau=1}^{\infty} \rho^{\tau-1} \Pi_i(\sigma(S_\tau, \nu_\tau), S_\tau, \nu_\tau; \alpha) \mid S_1, \sigma \right]
\]

\[
\approx \frac{1}{S} \sum_{s=1}^{S} \sum_{\tau=1}^{T} \rho^{\tau-1} \Pi_i(\sigma(S^s_\tau, \nu^s_\tau), S^s_\tau, \nu^s_\tau; \alpha),
\]

where we simulate \( \hat{S} \) paths of length \( T \). Variables with superscript \( s \) are for simulation \( s \) with \( s = 1, \ldots, \hat{S} \). In particular, \( \sigma(S^s_\tau, \nu^s_\tau) \) denotes a vector of simulated expansion or contraction actions from the policy profile \( \sigma \). The other variables are simulated according to their laws of motion, given the parameters \( \alpha \).

With this formulation, we can then repeat the same procedure using each of \( B \) different ini-
tial states, each under both the estimated optimal policies $\hat{\sigma}_i$ and when one firm uses a random alternative policy $\tilde{\sigma}_i$. Each alternative policy is generated by randomly perturbing the subvector of policy function parameters in $\phi$ by adding a random vector $\varrho$, where $\varrho \sim N(0, \sigma^2 \varrho I)$. Rather than simply additively perturbing the latent index used in the threshold crossing rules, these perturbations interact with the state variables. As discussed in Srisuma (2010), such alternative policies can contain additional information that is helpful in identifying the structural parameters of interest. We note that because the profit function is linear in parameters, as in Bajari, Benkard, and Levin (2007), many values needed in these simulations can be pre-computed and re-used as $\alpha$ changes.

Under the true parameters $\alpha$, by revealed preference, the estimated policy $\hat{\sigma}_i$ for firm $i$ must yield a higher ex-ante valuation for that firm than any other policy $\tilde{\sigma}_i$, given that the other firms are using policies $\hat{\sigma}_{-i}$. Therefore, we can use the difference in these two ex-ante valuations as a basis for estimating $\alpha$. Each initial condition and alternative policy yields an separate inequality condition.

Let $b$ index the individual inequalities, with each inequality consisting of an initial market structure and state $S^b_{1} = (N^b_0, X^b_1, Z^b_1)$, an index for the unilaterally deviating firm $i$, and an alternative policy $\tilde{\sigma}_i$ for firm $i$. We denote this difference in valuations for firm $i$ using inequality $b$ by

$$g_b(\hat{\sigma}, \alpha) = \bar{V}_i(S^b_{1}; \hat{\sigma}, \alpha) - \bar{V}_i(S^b_{1}; \tilde{\sigma}_i, \hat{\sigma}_{-i}, \alpha).$$

(10)

In equilibrium, for the true parameter values, this difference should be positive based on a revealed preference argument. Hence, the minimum distance estimator of Bajari, Benkard, and Levin (2007) chooses $\hat{\alpha}$ to minimize the violations of the equilibrium requirement in (10), i.e., minimizes the sum of squared deviations from positivity in the function $Q(\alpha)$ defined as follows,

$$Q(\alpha) = \frac{1}{B} \sum_{b=1}^{B} (\min\{g_b(\hat{\sigma}, \alpha), 0\})^2.$$

4.3 Identification

An important component of our model is the presence of serially correlated firm-specific profitability shocks $Z_{imt}$ that arise due to size spillovers. These are integrated out of the likelihood in order to estimate and identify the structural payoff function (see, e.g., Pakes, 1986). We use particle filtering or sequential Monte Carlo simulation, which is fundamentally a Bayesian procedure, to perform

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22 Although, a particle filter is in essence a Bayesian procedure, there is nothing that precludes it from being used as part of frequentist estimation routine as in our case.
this integration and is therefore predicated on some form of parametric assumptions. Given this backdrop we next discuss identification much in line with with the heuristic style of Keane (2010) and Ching, Erdem and Keane (2013).

The main difference in our model from a typical dynamic oligopoly model for which the Bajari, Benkard and Levin (2007) procedure might be applicable is that in the first stage we jointly estimate the policy functions that condition on the latent profitability state $Z_{int}$, along with the parameters of the transition equations (4) and the posterior distribution $p(Z_{int} | N_{int,t-1}, X_{m,t-1}, \phi)$ for the firm specific latent profitability states. After the serially correlated unobservable states are integrated out, identification of the primitives in the one-shot payoff follows very much from the Bresnahan and Reiss (1991a) framework based on exclusion restrictions (see e.g., Tamer 2003). The distinction is that in our case the one-shot payoff includes the effect ($\gamma$) of the latent profitability shock $Z_{int}$. A key exclusion restriction for identifying this effect is that the competitors’ lagged firm size $N_{-i,m,t-1}$ affects the current period payoffs $\Pi_{int}$ in (3) but not the transition of the firm-$i$-specific unobserved profitability state $Z_{int}$ in (4). Alternatively, the exclusion restriction is that controlling for market fixed effects, there is no direct market wide profitability spillover of each firm’s lagged size. The identification assumption put another way is that a firm’s lagged size only directly affects its own latent profitability state $Z_{int}$. Of course, indirectly, in equilibrium through the actions of the firm,

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23See e.g., Fernandez, Villaverde, and Rubio (2007), Norets (2009), and Fang and Kung (2010) for other applications of such Bayesian methods for integration. Hu and Shum (2012a, b), provide conditions on needed for non-parametric identification of dynamic models with serially correlated unobservables, although, they do not consider the case with endogenous feedback.

24As stated by Ching, Erdem and Keane (2013, p. 10) identification has multiple meanings: (1) “showing that the parameters of a model are identified given the assumed model structure” (italics from authors). This may involve formal proof as well as intuitive discussion of what data patterns drive the estimates,” (2) “analysis of which assumptions are necessary (italics from authors) to estimate a model or just convenient” (i.e., nonparametric identification analysis), and (3) examining “fragile identification,” i.e., whether some “parameters may be formally identified but difficult to pinpoint in finite samples.” Our discussion is in the spirit of (1) above. Regarding (2) they further state (p. 10, fn. 16), “Unfortunately, this literature has been misinterpreted by many researchers as suggesting that it may be possible to obtain ‘model free evidence’ about behavior. In fact, the approach of the nonparametric identification literature is to make a priori assumptions about certain parts of a model and then show that some other part (e.g., the functional form of utility or an error distribution) is identified without further assumptions.” Moreover, they also state (p. 23), “It is important to remember that truly model-free evidence cannot exist. The simple empirical work that promises to deliver such evidence always relies on some assumptions. These assumptions are often left implicit as a result of failure to present an explicit model. Often these implicit assumptions are (i) not obvious, (ii) hard to understand, and (iii) very strong.”

25One could consider a more general model of market-wide spillovers from, say, R&D or advertising of each firm that makes the entire category more profitable over and above what the market fixed effect can capture. We abstract from that situation. In order to do this one approach could be to include the sum of the lagged sizes of all firms in (4). However, in that case one could not estimate heterogeneous firm specific spillovers. An alternative would be to include the vector $N_{int,t-1}$, with the firm-specific effects being potentially heterogeneous. This would make the model not only more computationally burdensome, but would also make identification more difficult by eliminating the exclusion restriction just described. Our current approach lies somewhere inbetween these two extremes. We allow for heterogeneity in spillovers across firms but restrict attention to internal firm specific spillovers.
there is an effect on the actions and outcomes of its rivals. Additional excluded variables are the exogenous state variables $X_{imt}$ that affect the current payoffs but not the evolution of the firm specific unobservable profitability component. Finally, in the forward simulation process, $Z_{imt}$ can also be thought of as a pre-determined state variable that is excluded from the payoffs of the rival firms thus providing variation in payoffs across firms (see e.g., Pesendorfer and Schmidt-Dengler, 2003).\textsuperscript{26}

The identification of organizational forgetting ($\delta_i$) and intertemporal size spillovers ($\beta_{i1}, \beta_{i2}, \beta_{i3}$) in (4) may be considered in the following way. For firm $i$ in market $m$ at each time $t$, the particle filter starting with a draw from an initial distribution $p(Z_{im,0})$ uses (8) and (9) to recursively compute the sequence $(Z_{im,t}, Z_{im,t-1}, \ldots, Z_{im,0})$.\textsuperscript{27} Each $Z_{im,t}$ is projected on its lag value ($Z_{im,t-1}$) and the corresponding lagged firm size ($N_{im,t-1}$). Using this projection the autocovariance between $Z_{im,t}$ and $Z_{im,t-1}$ provides a measure of organizational forgetting for a chain. At the same time the variation over time within a market and across markets for given chain in the predetermined lagged firm size helps to pin down the intertemporal size spillover for that chain through the projection.

Further basis for identification is provided by variation exhibited in the data for $\{N_{im,t-1}\}_{i,m,t}$. The summary statistics displayed in Table 1 and discussed in Section 2 confirm that there is substantial in the number of outlets, ranging from 0 to as large as 164. Furthermore, variation in the observable market characteristics and demographics over time serve as important exclusion restrictions that have short- and long-term effects on the stock of outlets. The assumption here is that these characteristics move independently of a firms’ unobserved profitability levels. For these exclusion restriction to have identifying power, we need at least one exogenous variable that shifts the current and future shock of outlets. To ensure that we have such exogenous variables, we apply these identification arguments with data we have collected on the CFL Grey Cup hosting across cities, smoking regulation, and minimum wage policies across cities. For instance, cities tend to experience greater fast food expansion during the years in which they host the CFL Grey Cup versus years in which they do not. Furthermore, the fact that the CFL board of governors desires to rotate the event across all cities in a fair manner adds some randomness to the assignment

\textsuperscript{26}Recall, the assumption is that the profitability components $\{Z_{imt}\}_{i=1}^I$ are observed by all firms but are unobserved to the researcher. Thus, in the forward simulation process the firms know the draws of the $\{Z_{imt}\}_{i=1}^I$ from the particle filter when making choices about stores.

\textsuperscript{27}As an analogy, Berry, Levinsohn and Pakes (1995, pp. 853-854), estimate unobserved marginal costs by computing the residual of the inverted first order condition for the Bertrand-Nash equilibrium, and then projecting this on to a vector of product characteristics that account for costs. In our case we don’t have first order conditions to invert since the firm’s choice set is discrete but the $Z_{imt}$ are computed recursively under the assumption that the firm is making optimal decisions about adding or subtracting stores. This optimality is incorporated through the observation likelihood $l_m$ in (7) and (8)).
of host cities. A similar argument can be made for the use of smoking regulation roll-out across municipalities over time, as this regulatory change is permanent once implemented, and will thus have an impact on the firms’ future profitability gains from the firm size spillovers. Minimum wage regulation serves as another exclusion restriction; changes to the minimum wage have long-run cost implications for retail chain expansion decisions.

5 Results

Table 7 contains estimates of the structural parameters of interest. This includes both the payoff parameters, which are estimated in the second stage, and the parameters from the law of motion of \( Z_{imt} \) for each firm, which are estimated in the first stage. We report bootstrap standard errors.\(^{28}\)

To avoid reporting very small numbers, coefficients on terms involving \( N \) are reported as coefficients on \( N/100 \), with corresponding scaling factors being used for variables involving \( N^2 \) and \( N^3 \).

In the first stage, we estimate an ordered probit via sieve maximum likelihood, including all exogenous variables and their interactions up to second order. Because the ordered probit coefficients and cutoffs are not of direct interest, we report these estimates in Table 16 in the appendix.

To prepare for the minimum distance estimator in the second stage, we generated \( B = 3000 \) random inequalities. Each inequality consists of an alternative policy function, which we generate by randomly perturbing the coefficients and cutoffs of the estimated reduced form policy function, and an initial state, which we draw randomly from the sample.

First, the coefficients on \( Z_{imt} \) in the payoff function are positive in both specifications that include it. The main parameters of interest are those related to the law of motion of \( Z_{imt} \). Recall that \( Z_{imt} \) is an AR(1) process with a drift term which also depends on the level, square, and cube of the lagged number of own outlets. The parameters of this process differ across firms. Thus, for each firm we can think of increments to \( Z_{imt} \) being decomposed into four components: a trend component (\( \mu_i \)), an autoregressive component (\( \delta_i \)), a size spillover component (\( \beta_{i1}, \ldots, \beta_{i3} \)), and an i.i.d. Gaussian innovation (with standard deviation \( \omega_i \)). The main specification, “\( Z \) (with spillovers)”, contains all four components, the second specification, “\( Z \) (no spillovers)”, omits the size spillover component, and the specification with no persistence, “\( \text{No } Z \)” omits all four.\(^{29}\)

The estimates indicate that there is substantial heterogeneity across chains. Since the drift terms

\(^{28}\)Given the computational burden we bootstrapped the standard errors using 96 replications with replacement from the sample of 31 markets for 36 years.

\(^{29}\)The “\( \text{No } Z \)” specification does, however, include firm and city fixed effects which serve a similar role as the drift terms in the \( Z \) processes. We do not, however, report these fixed effects as drift terms in Table 7 to avoid confusion, since they are not directly comparable.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Z (with spillovers)</th>
<th>Z (no spillovers)</th>
<th>No Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population ($\theta_{1,1}$)</td>
<td>-0.0075 (0.0142)</td>
<td>-0.0232 (0.0147)</td>
<td>-0.1169 (0.1197)</td>
</tr>
<tr>
<td>Income ($\theta_{1,2}$)</td>
<td>-0.0178 (0.0080)</td>
<td>-0.0117 (0.0089)</td>
<td>0.0319 (0.0348)</td>
</tr>
<tr>
<td>Property Value ($\theta_{1,3}$)</td>
<td>0.0199 (0.0147)</td>
<td>0.0018 (0.0141)</td>
<td>-0.2162 (0.0616)</td>
</tr>
<tr>
<td>Grey Cup Host ($\theta_{1,4}$)</td>
<td>0.5212 (0.3606)</td>
<td>0.6763 (0.3320)</td>
<td>0.8161 (1.4234)</td>
</tr>
<tr>
<td>Smoking Regulation ($\theta_{1,5}$)</td>
<td>-0.0313 (0.0323)</td>
<td>-0.0376 (0.0397)</td>
<td>0.0036 (0.3305)</td>
</tr>
<tr>
<td>Minimum Wage ($\theta_{1,6}$)</td>
<td>0.0417 (0.0116)</td>
<td>0.0427 (0.0140)</td>
<td>0.1940 (0.0552)</td>
</tr>
<tr>
<td>$Z$ ($\gamma$)</td>
<td>0.0693 (0.0174)</td>
<td>0.1050 (0.0217)</td>
<td>-</td>
</tr>
<tr>
<td>Rival $N$ ($\theta_{2}$)</td>
<td>-0.0186 (0.1271)</td>
<td>0.0088 (0.1376)</td>
<td>-0.2988 (0.3672)</td>
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<tr>
<td>Own $N$ ($\theta_{3}$)</td>
<td>0.7278 (0.3289)</td>
<td>0.6229 (0.5165)</td>
<td>0.5261 (1.0946)</td>
</tr>
<tr>
<td>Entry Cost ($\psi_{1}$)</td>
<td>-0.5540 (0.1112)</td>
<td>-0.6606 (0.1057)</td>
<td>0.2116 (0.2540)</td>
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<tr>
<td>Expansion Cost ($\psi_{2}$)</td>
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<td>-0.1721 (0.0939)</td>
<td>-0.4106 (0.1838)</td>
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<tr>
<td>Scrap Value ($\psi_{3}$)</td>
<td>-0.0033 (0.0486)</td>
<td>-0.1255 (0.1065)</td>
<td>-0.2610 (0.1812)</td>
</tr>
<tr>
<td>A&amp;W</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drift ($\mu_{AW}$)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-</td>
</tr>
<tr>
<td>AR(1) ($\delta_{AW}$)</td>
<td>0.1052 (0.0010)</td>
<td>0.2200 (0.0334)</td>
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<tr>
<td>$N$ ($\beta_{AW,1}$)</td>
<td>0.0111 (0.0001)</td>
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<td>$N^2$ ($\beta_{AW,2}$)</td>
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<td>S.D. ($\omega_{AW}$)</td>
<td>1.1616 (0.0397)</td>
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<td>-</td>
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<tr>
<td>Burger King</td>
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<tr>
<td>Drift ($\mu_{BK}$)</td>
<td>0.1275 (0.0018)</td>
<td>0.0129 (0.0013)</td>
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<td>AR(1) ($\delta_{BK}$)</td>
<td>0.0262 (0.0003)</td>
<td>0.0728 (0.0046)</td>
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<td>-</td>
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<td>-</td>
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<tr>
<td>$N^3$ ($\beta_{BK,3}$)</td>
<td>-0.0475 (0.0006)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S.D. ($\omega_{BK}$)</td>
<td>0.6585 (0.0095)</td>
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<td>-</td>
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<td>Harvey's</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Drift ($\mu_{HARV}$)</td>
<td>0.0770 (0.0009)</td>
<td>0.0084 (0.0003)</td>
<td>-</td>
</tr>
<tr>
<td>AR(1) ($\delta_{HARV}$)</td>
<td>0.0468 (0.0004)</td>
<td>0.0958 (0.0149)</td>
<td>-</td>
</tr>
<tr>
<td>$N$ ($\beta_{HARV,1}$)</td>
<td>0.1752 (0.0014)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$N^2$ ($\beta_{HARV,2}$)</td>
<td>2.3189 (0.0231)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$N^3$ ($\beta_{HARV,3}$)</td>
<td>-0.0274 (0.0003)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S.D. ($\omega_{HARV}$)</td>
<td>0.8204 (0.0237)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>McDonald's</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drift ($\mu_{MCD}$)</td>
<td>0.5082 (0.0058)</td>
<td>0.4333 (0.0246)</td>
<td>-</td>
</tr>
<tr>
<td>AR(1) ($\delta_{MCD}$)</td>
<td>0.4133 (0.0090)</td>
<td>0.3729 (0.0230)</td>
<td>-</td>
</tr>
<tr>
<td>$N$ ($\beta_{MCD,1}$)</td>
<td>0.0646 (0.0067)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$N^2$ ($\beta_{MCD,2}$)</td>
<td>0.1319 (0.0018)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$N^3$ ($\beta_{MCD,3}$)</td>
<td>-0.0510 (0.0004)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S.D. ($\omega_{MCD}$)</td>
<td>0.3804 (0.0053)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Wendy’s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drift ($\mu_{WEND}$)</td>
<td>0.0493 (0.0006)</td>
<td>-0.0032 (0.0001)</td>
<td>-</td>
</tr>
<tr>
<td>AR(1) ($\delta_{WEND}$)</td>
<td>0.1226 (0.0013)</td>
<td>0.2297 (0.0171)</td>
<td>-</td>
</tr>
<tr>
<td>$N$ ($\beta_{WEND,1}$)</td>
<td>0.0024 (0.0001)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$N^2$ ($\beta_{WEND,2}$)</td>
<td>1.2775 (0.0104)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$N^3$ ($\beta_{WEND,3}$)</td>
<td>-0.1446 (0.0018)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>S.D. ($\omega_{WEND}$)</td>
<td>0.5599 (0.0053)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
are similar to firm fixed effects, we normalize $\mu_{AW}$ to be zero and estimate the relative differences for the other firms. In the main specification with spillovers, we expected the coefficients on the linear and quadratic $N$ terms to be positive and the cubic coefficient to be negative, reflecting initially increasing, then decreasing returns in the increments to the latent payoff as a function of the lagged number of stores. Otherwise, ceteris paribus, firms would have the incentive to expand by building an infinite number of outlets. As shown in Table 7, this is indeed the case for all firms with the exception of Burger King.

Overall, the persistence in spillovers is strongest for McDonald’s, which has by far the largest drift term and autoregressive coefficient and the least amount of noise. Among all five retailers, the evolution of A&W’s latent state has the smallest trend and also the most unexplained variation. For Burger King, although it has the second largest drift term, it has the lowest autoregressive coefficient and the coefficient on the quadratic spillover term is negative, reflecting a very small window of increasing returns before decreasing returns dominate. Compared to the other firms, Harvey’s and Wendy’s have relatively large quadratic spillover coefficients, relatively low drift and autoregressive coefficients, and moderately large standard deviations for the i.i.d. innovations. This indicates a more transient payoff benefit from having built more outlets in previous periods when compared, for example, to McDonald’s.

The retailers appear to be sensitive to competition, but not in a statistically significant way, and earn higher profits as they build additional outlets. The insignificant competitive effect indicates that consumers may view these chains as being relatively differentiated. Among the cost estimates, for the main specification the estimated initial cost of entry ($\psi_1$) is more than twice the cost of building a single store ($\psi_2$). The estimated scrap values ($\psi_3$) are not significantly different from zero, indicating that liquidating outlets is not lucrative.

### 5.1 Model Fit Comparison

Having estimated our model with three different specifications, we now seek to determine which specification best fits the observed data. We first carry out simulations using each of the estimated specifications and plot the average number of outlets predicted by each. We then compare the fit in a statistical sense by evaluating the mean bias and mean squared error in the simulated predictions. To implement the model simulations and counterfactuals, we employ a similar forward simulation approach as in Benkard, Bodoh-Creed, and Lazarev (2010), which does not require one to solve a computationally intractable dynamic model. We provide additional technical details about the
simulations that follow in the appendix.

The main findings from our model fit simulations are displayed in Figures 2 and 3. Figure 2 plots the evolution of store counts over time across these different scenarios, with the actual dynamics found in the data serving as a benchmark. For each firm, we plot the average number of outlets (averaged across markets) each period observed in the data and the average simulated number of outlets with each of three model specifications (averaged over 250 simulations for each market, then across markets). The simulation runs are initialized using the observed market characteristics and number of outlets at the beginning of our sample. Since we are essentially forecasting 36 years ahead, differences relative to the observed number of outlets are expected. However, in that sense all three models perform quite well.

Our preferred specification, “Z (with spillovers)”, includes persistent unobserved profitability via $Z$ process and allows for size spillovers. This specification has the smallest mean squared error (MSE) of the predicted number of stores, as discussed in more detail below. The “No Z” specification without the $Z$ process has the smallest mean bias (MB) of the predicted number of stores, but the higher variance of the predicted number of stores results in this specification having the largest MSE. Additionally, the estimated entry cost for this specification is positive, as shown in Table 7, which is contrary to economic theory and therefore underscores the choice of “Z (with spillovers)” as our preferred specification.

Returning to the results of our simulations, Figure 2 plots the mean predicted number of outlets for all three specifications. This graph can be used to inspect the bias of each specification, but it does not speak to the variance of the simulations which is another important consideration. In terms of bias at the end of the sample period, the simplest specification with no $Z$ process and only firm and market fixed effects appears to have the smallest bias for all firms except for A&W and Wendy’s. It is nearly tied with the specification with $Z$ but with no spillovers for A&W, Burger King, Harvey’s, and Wendy’s. There is a near three-way tie for Burger King, with all specifications fitting equally well. The specification with $Z$ and size spillovers fits best for McDonald’s and Wendy’s. The bias, however, is only one component of the MSE. To achieve the smallest MSE, a specification must achieve a balance between having a small bias and a small variance.

Since we carry out many simulations, we can evaluate the fit in statistical terms by comparing the mean bias (MB) and mean squared error (MSE) of the predictions for each specification. We define the mean bias in the number of outlets for a particular firm $i$ and market $m$ to be the average difference between the simulated values $\{N_{r_{imt}}\}_{r=1}^{R}$ and the observed value $N_{imt}$. Then,
Figure 2: Comparison of Average Number of Outlets by Model Specification
in the left panel of Figure 3 we plot the mean bias averaged over firms and markets: \( \text{MB}_t = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{I} \sum_{i=1}^{I} \frac{1}{R} \sum_{r=1}^{R} (N_{imt}^* - N_{imt}). \) As expected, there is some accumulation of bias as time progresses, but the fit of all specifications improves by the end of the sample period. Overall, the “No Z” specification has the smallest bias, followed by “Z (with spillovers)” and “Z (no spillovers)”.

In the right panel of Figure 3 we plot the mean squared error averaged over firms and markets: \( \text{MSE}_t = \frac{1}{M} \sum_{m=1}^{M} \frac{1}{I} \sum_{i=1}^{I} \frac{1}{R} \sum_{r=1}^{R} (N_{imt}^* - N_{imt})^2. \) The “Z (with spillovers)” specification achieves the lowest mean squared error, followed by “Z (no spillovers)” and “No Z”. The mean squared error can be decomposed into the bias and the square of the variance. Despite having the smallest bias, the “No Z” specification has by far the largest mean squared error. This is due to having higher variance than the other specifications. Therefore, the “Z (with spillovers)” specification achieves the best balance of bias and variance in terms of the predicted number of outlets.

5.2 Drivers of McDonald’s Dominance

With the estimated structural model of retail chain evolution, we can better understand the role that firm capability, in the form of firm specific serially correlated unobserved profitability, plays in market share dynamics. In particular, we compare the unobserved profitability processes in two ways: (i) comparing the statistical properties of the processes and their stationary distributions, and (ii) carrying out an impulse response analysis wherein we analyze the responses, in terms of number of new stores built, following one-time unanticipated shocks to the \( Z \) process for each firm.

First, Table 8 reports the means and variances of the stationary distributions for each firm. Since the mean values vary across markets, due to the inclusion of market-specific drift parameters, we report averages across all markets. The mean for McDonald’s is over 3.5 times larger than that
Table 8: Means and Variances of Stationary Distributions of Unobserved Profitability

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A&amp;W</td>
<td>0.2311</td>
<td>1.3645</td>
</tr>
<tr>
<td>Burger King</td>
<td>0.3433</td>
<td>0.4339</td>
</tr>
<tr>
<td>Harvey’s</td>
<td>0.2978</td>
<td>0.6745</td>
</tr>
<tr>
<td>McDonald’s</td>
<td>1.2187</td>
<td>0.1745</td>
</tr>
<tr>
<td>Wendy’s</td>
<td>0.2920</td>
<td>0.3183</td>
</tr>
</tbody>
</table>

Figure 4: Impulse Response Functions for Expansion and Contraction Decisions

of Burger King, which has the second-highest mean. Furthermore, the variance of the process is smallest for McDonald’s. The next largest variance is hat of Wendy’s, which is almost twice as large as that of McDonald’s. A&W has both the smallest mean, being just behind Wendy’s, and the largest variance, being twice as large as that of Harvey’s and 7.8 times higher than that of McDonald’s.

Next, to better understand what the estimated processes parameters reported mean in terms of market outcomes, we consider the impact of one-unit shocks to the unobserved profitability of each firm. Figure 4 displays the impulse response functions for each firm when that firm receives a one-time, one-unit shock to $Z_{it}$ in the year 2000. The plotted net expansion and contraction values are relative to the mean simulated values in absence of the shock. By far, the effect of this temporary shock affects McDonald’s decisions for more periods than the other firms. In other words, they retain unobserved profitability to a larger degree and for longer than any of their rivals.

Finally, to compare the persistence across firms we can compare the autocovariance functions of $Z_{it}$ for each $i$. For $k$ periods ahead, the autocovariance for firm $i$ is $\text{Cov}(Z_{it}, Z_{i,t+k}) = \delta_i^k \omega_i^2 / (1 - \delta_i^2)$. 
Table 9: Autocovariances of Unobserved Profitability Processes by Firm

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A&amp;W</td>
<td>0.1435</td>
<td>0.0151</td>
<td>0.0016</td>
<td>0.0002</td>
<td>0.0000</td>
</tr>
<tr>
<td>Burger King</td>
<td>0.0114</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Harvey’s</td>
<td>0.0315</td>
<td>0.0015</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>McDonald’s</td>
<td>0.0721</td>
<td>0.0298</td>
<td>0.0123</td>
<td>0.0051</td>
<td>0.0021</td>
</tr>
<tr>
<td>Wendy’s</td>
<td>0.0390</td>
<td>0.0048</td>
<td>0.0006</td>
<td>0.0001</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Therefore, the persistence in the $Z_{it}$ process is determined by both the autocorrelation coefficient, $\delta_i$, and the standard deviation of the i.i.d. innovations, $\omega_i$. Larger values of either parameter will tend to increase the time until the process reverts back to the mean following a shock. Table 9 reports the autocovariances for $k = 1, \ldots, 5$ periods ahead for each firm $i$. McDonald’s has by far the most persistence, due largely to its large autoregressive parameter. For A&W the effects of a shock one period after is larger than for McDonald’s, due to the large variance parameter for A&W, but the effect decays more quickly than for McDonald’s. These autocovariances explain the trajectories of the impulse response functions in plotted in Figure 4.

These analyses illustrate the drivers of McDonald’s dominance. These show that even if the absolute levels of inter-temporal spillovers are relatively small, organizational retention of these spillovers over a long enough period of time can lead to drastic cumulative effects.

5.3 Robustness of McDonald’s Dominance

Our next set of counterfactual simulations evaluates the robustness of McDonald’s dominance in light of shocks to its initial conditions and shocks to the economy.

First, we test the importance of initial conditions and look at the impact of imposing a handicap on McDonald’s initially. We posit a series of scenarios in which each of McDonald’s rivals are endowed with 0 to 10 more outlets than they actually had in the first time period. These simulations illustrate what sort of advantage rivals would need in order to overtake McDonald’s for market dominance. To succinctly summarize our results, we present them in terms of discounted profit shares, i.e., profit shares calculated using the present discounted value of profits for each firm as opposed to the period-by-period profits. Table 10 shows us that even if each of McDonald’s rivals were endowed with two additional outlets in the first year, McDonald’s would still capture nearly 20% of discounted profits. These results illustrate that McDonald’s rivals would need some initial advantage in order to compete on a level playing field.
Table 10: Discounted Profit Shares When McDonald’s Rivals Have Additional Outlets

<table>
<thead>
<tr>
<th>Additional Rival Outlets</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>A&amp;W</td>
<td>0.0436</td>
<td>0.0347</td>
<td>0.0337</td>
<td>0.0343</td>
<td>0.0361</td>
<td>0.0362</td>
<td>0.0379</td>
<td>0.0383</td>
<td>0.0383</td>
<td>0.0405</td>
<td>0.0413</td>
</tr>
<tr>
<td>Burger King</td>
<td>0.2185</td>
<td>0.2609</td>
<td>0.2601</td>
<td>0.2651</td>
<td>0.2587</td>
<td>0.2569</td>
<td>0.2619</td>
<td>0.2601</td>
<td>0.2611</td>
<td>0.2671</td>
<td>0.2751</td>
</tr>
<tr>
<td>Harvey’s</td>
<td>0.2316</td>
<td>0.2479</td>
<td>0.2505</td>
<td>0.2500</td>
<td>0.2557</td>
<td>0.2547</td>
<td>0.2547</td>
<td>0.2566</td>
<td>0.2592</td>
<td>0.2592</td>
<td>0.2622</td>
</tr>
<tr>
<td>McDonald’s</td>
<td>0.2922</td>
<td>0.2085</td>
<td>0.1964</td>
<td>0.1939</td>
<td>0.1843</td>
<td>0.1835</td>
<td>0.1766</td>
<td>0.1727</td>
<td>0.1738</td>
<td>0.1640</td>
<td>0.1599</td>
</tr>
<tr>
<td>Wendy’s</td>
<td>0.2141</td>
<td>0.2480</td>
<td>0.2594</td>
<td>0.2566</td>
<td>0.2652</td>
<td>0.2687</td>
<td>0.2689</td>
<td>0.2723</td>
<td>0.2666</td>
<td>0.2723</td>
<td>0.2615</td>
</tr>
<tr>
<td>HHI</td>
<td>0.2345</td>
<td>0.2339</td>
<td>0.2374</td>
<td>0.2374</td>
<td>0.2379</td>
<td>0.2381</td>
<td>0.2384</td>
<td>0.2389</td>
<td>0.2392</td>
<td>0.2396</td>
<td>0.2401</td>
</tr>
</tbody>
</table>

Table 11: Discounted Profit Shares When McDonald’s Rivals Have Better Initial Draws of Unobserved Profitability

<table>
<thead>
<tr>
<th>Initial Draw Increase</th>
<th>0%</th>
<th>100%</th>
<th>200%</th>
<th>300%</th>
<th>400%</th>
<th>500%</th>
<th>1000%</th>
<th>2000%</th>
<th>3000%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A&amp;W</td>
<td>0.0436</td>
<td>0.0363</td>
<td>0.0328</td>
<td>0.0297</td>
<td>0.0297</td>
<td>0.0294</td>
<td>0.0312</td>
<td>0.0369</td>
<td>0.0382</td>
</tr>
<tr>
<td>Burger King</td>
<td>0.2185</td>
<td>0.2359</td>
<td>0.2488</td>
<td>0.2534</td>
<td>0.2585</td>
<td>0.2569</td>
<td>0.2560</td>
<td>0.2706</td>
<td>0.2770</td>
</tr>
<tr>
<td>Harvey’s</td>
<td>0.2316</td>
<td>0.2380</td>
<td>0.2479</td>
<td>0.2469</td>
<td>0.2513</td>
<td>0.2552</td>
<td>0.2593</td>
<td>0.2652</td>
<td>0.2737</td>
</tr>
<tr>
<td>McDonald’s</td>
<td>0.2922</td>
<td>0.2560</td>
<td>0.2330</td>
<td>0.2230</td>
<td>0.2181</td>
<td>0.2079</td>
<td>0.1908</td>
<td>0.1611</td>
<td>0.1260</td>
</tr>
<tr>
<td>Wendy’s</td>
<td>0.2141</td>
<td>0.2338</td>
<td>0.2374</td>
<td>0.2471</td>
<td>0.2435</td>
<td>0.2506</td>
<td>0.2567</td>
<td>0.2663</td>
<td>0.2851</td>
</tr>
<tr>
<td>HHI</td>
<td>0.2345</td>
<td>0.2338</td>
<td>0.2351</td>
<td>0.2368</td>
<td>0.2376</td>
<td>0.2380</td>
<td>0.2384</td>
<td>0.2417</td>
<td>0.2503</td>
</tr>
</tbody>
</table>

We now investigate how well McDonald’s maintains its market dominance if its rivals have initial draws of their unobserved profitability components, Z_i, taken from the stationary distribution for McDonald’s instead of their own. For this experiment, rather than drawing each simulated Z_im from the stationary distribution we draw them from the stationary distribution for McDonald’s, in effect giving the rival firms an initial advantage. We also simulate the market dynamics when inflating the rival draws by a factor (1+α) where α ∈ {0, 1, 2, 3, 4, 5, 10, 20, 30}, with α = 0 denoting the baseline case of 0% inflation. Table 11 describes the main results from this counterfactual in terms of discounted profit shares. From the table, we see that McDonald’s can maintain at least 20% market share even when its competitors are endowed draws from it’s own stationary distribution that are also then inflated by as much as 500%. Furthermore, it is only when all rival firms are endowed with initial draws inflated by 3000% that any one chain (Wendy’s) achieves a market share as large as the baseline market share of McDonald’s (29%).

In our next simulations, we look at how well McDonald’s dominance withstands economic downturns through demand and supply shocks. For this analysis we first simulate the impact of a sudden drop in demand through a fall in income in 2006, around the time of a major economic downturn in North America. We consider five cases, all of them presented in Table 12. The first case captures the event in which income in 2006 drops 10% from the 2005 level, the second
Table 12: Discounted Profit Shares in Response to Changes in Economic Conditions

<table>
<thead>
<tr>
<th>Change in Income</th>
<th>-10%</th>
<th>-5%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A&amp;W</td>
<td>0.0443</td>
<td>0.0458</td>
<td>0.0460</td>
<td>0.0467</td>
</tr>
<tr>
<td>Burger King</td>
<td>0.2084</td>
<td>0.2137</td>
<td>0.2088</td>
<td>0.2032</td>
</tr>
<tr>
<td>Harvey’s</td>
<td>0.2190</td>
<td>0.2141</td>
<td>0.2106</td>
<td>0.2102</td>
</tr>
<tr>
<td>McDonald’s</td>
<td>0.3231</td>
<td>0.3304</td>
<td>0.3397</td>
<td>0.3549</td>
</tr>
<tr>
<td>Wendy’s</td>
<td>0.2052</td>
<td>0.1961</td>
<td>0.1949</td>
<td>0.1850</td>
</tr>
<tr>
<td>HHI</td>
<td>0.2399</td>
<td>0.2412</td>
<td>0.2434</td>
<td>0.2478</td>
</tr>
</tbody>
</table>

Table 13: Discounted Profit Shares in Response to Changes in Minimum Wage

<table>
<thead>
<tr>
<th>Change in Minimum Wage</th>
<th>-10%</th>
<th>-5%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A&amp;W</td>
<td>0.0455</td>
<td>0.0463</td>
<td>0.0455</td>
<td>0.0453</td>
</tr>
<tr>
<td>Burger King</td>
<td>0.2065</td>
<td>0.2129</td>
<td>0.2096</td>
<td>0.2058</td>
</tr>
<tr>
<td>Harvey’s</td>
<td>0.2186</td>
<td>0.2128</td>
<td>0.2101</td>
<td>0.2104</td>
</tr>
<tr>
<td>McDonald’s</td>
<td>0.3269</td>
<td>0.3321</td>
<td>0.3384</td>
<td>0.3513</td>
</tr>
<tr>
<td>Wendy’s</td>
<td>0.2024</td>
<td>0.1959</td>
<td>0.1964</td>
<td>0.1872</td>
</tr>
<tr>
<td>HHI</td>
<td>0.2404</td>
<td>0.2414</td>
<td>0.2432</td>
<td>0.2471</td>
</tr>
</tbody>
</table>

case captures 5% drop relative to the 2005 level, the third case captures 5% increase from the 2005 level, and the fourth case captures a 10% increase from the 2005 level. Notice that these shocks have the effect of diminishing market power (again, in terms of discounted profit share) of McDonald’s, but not by very much. Thus, a major economic downturn does not affect McDonald’s overall leadership position in the hamburger retail industry. These results are consistent with anecdotes that McDonald’s success is recession proof (Bandyk, 2009). The resilience of McDonald’s is especially consistent with the common belief dictates that fast food is counter-cyclical with the economy (Kowitt, 2011).

In a similar manner as the previous counterfactual, we consider a supply side cost shock in the form of changes to minimum wage levels. We examine how market structure, as measured by discounted profit shares, changes in light of these shocks. We consider the effect of sudden decreases or increases in wage by 5 or 10 percent in 2006. These five cases are presented in Table 13. As in the previous counterfactual, McDonald’s remains the market leader following downward and upward shocks to the minimum wage.

5.4 Discussion: Sources of Size Spillovers and Organizational Learning?

Our results indicate that dynamic link between size and unobserved profitability through the components of size spillovers, organizational retention and firm-specific drift is key to McDonald’s
dominance. Although, identifying the underlying mechanisms and specific elements of firm capabilities that constitute these these components is beyond the scope of our paper, we offer some thoughts about the sources that may generate such effects in our empirical setting.\footnote{One may broadly link the nature of this discussion about the components of unobserved profitability to the extensive research that attempts to identify and decompose Total Factor Productivity (TFP), where this measure can be thought of as the residual (and unobserved) component that explains variation in output after relevant inputs have been taken into account. TFP (or the Solow residual) is often considered to be a measure of long run technological change or technological productivity of an economy. See Syverson (2011) for a recent, comprehensive survey of this literature.}

Experience may allow a retailer to expand and operate in markets at lower fixed costs. As indicated in Love (1995), its real estate portfolio can serve as collateral for better financing (i.e., lower interest rate for borrowing); furthermore, its experience may allow it to lock-in the best and cheapest local suppliers ultimately lowering operating costs. A retailer with greater recognition and presence may also become better at negotiating rent with land-developers, as many shopping centers internalize the externalities that well established chain brands bring via increased consumer traffic (Gould, Pashigian, and Prendergast, 2005). Finally, some McDonald’s are known to enforce strict managerial practices and standards they have learned to be efficient; such matters are important if firms face the possibility of organizational forgetting due to employee turnover (Benkard, 2000).

The presence of existing outlets may help develop consumer goodwill in a given market (Shen and Xiao, 2012), where the stock of outlets may have a similar impact as advertising on consumer goodwill. Alternatively, a retailer’s experience may facilitate information gathering from their customers which can ultimately be used to improve their product quality (Jovanovic and Rob, 1987). Furthermore, there may be an interaction between the retailer’s economies of scale and scope, which ultimately generate a complementarity that consumers wish to exploit (Basker, Klimek, and Van, 2012). A retailer’s size may actually act to deter entry,\footnote{We refer the reader to Shen and Villas-Boas (2010) for a theoretical treatment about entry deterrence in growing markets.} and thus, augmenting its future expected payoffs (Igami and Yang, 2013). Finally, an experienced retailer may be able to achieve exclusive contracts with shopping centers so as to preclude competitors from entering.

Heterogeneity across firms in their ability to retain these size spillover advantages may be attributed to turnover rates among employees, as past research on restaurant chains has uncovered connections between turnover and store performance (Shaw et. al., 2005). Motivated by a desire to maintain low turnover rates, fast food chains, such as McDonald’s, have increasingly started to provide not just their managers but also hourly wage employees with a number of benefits such as educational, health, and retirement plans, and opportunities for growth within the organization.
More generally, research about the (restaurant chain) service industry has posited instruments that would mitigate turnover rates among the overall population of workers (e.g., Ghiselli, La Lopa, and Bai, 2001).

Firms may be inherently more profitable in unobserved ways through demand or cost-side channels. As an example of a demand-side channel, global brands may be perceived to sell products of a higher quality. Yu (2003) argues that McDonald’s enjoys such a reputation across countries. In terms of cost-side advantages, note that there is heterogeneity in terms of when each of the chains incorporated. While they all entered Canada around the same time, it is well understood that McDonald’s is a pioneer in this restaurant format. It has been argued that pioneers have advantages in terms of supplies, costs, and information (Golder and Tellis, 1993).

Finally, it should be noted that our findings complement those of Sutton (2007a), who finds heterogeneity across industries in terms of “persistence of leadership.” In particular, using data from Japan on 45 industries for a 23 year period from 1974 to the late nineties, he finds that a subset of industries is characterized by “Chandlerian” persistence in market shares, whereas the majority exhibit “Markovian” dynamics, i.e., neither “Schumpeterian” nor “Chandlerian” tendencies. In contrast, we focus on one industry and examine persistence in market shares or lack thereof that may arise as a result strategic interaction between firms. We offer the insight that in addition to heterogeneity across industries there may also be heterogeneity across firms within an industry in terms of persistence in market shares. Furthermore, this heterogeneity in persistence may be generated due to varying degrees of inter-temporal spillovers and organizational retention of these spillovers across firms.

6 Conclusions

This study presents a new empirical model of retail chain dynamics that allows for endogenous firm size, and its consequences for evolution of market structure. Moreover, it analyzes the dynamic link between firm size and profitability that may arise from inter-temporal firm size spillovers and organizational retention of these spillovers. Through a firm specific unobservable the model allows for heterogeneity in the dynamic link between firm size and future market outcomes. The primary methodological innovation is to extend the Bajari, Benkard and Levin (2007) two-step estimation procedure to incorporate time varying firm specific heterogeneity subject to endogenous feedback using a particle filter based method.

Using data on Canadian hamburger retail chains, our estimated model reveals a link between
current size, future profitability and market dominance. Furthermore, there is heterogeneity in this link across firms which implies that some firms can become more dominant while others less profitable as they grow. In fact, our estimated model and subsequent simulations demonstrate that McDonald’s market power in Canada can be attributed to such effects. We also show that McDonald’s advantage via this spillover and organizational retention process is robust to hypothetical scenarios in which McDonald’s experiences unexpected demand and supply shocks, and faces a competitive handicap resulting from arbitrary increases in rival outlets or unobserved profitability during the initial year.

This paper’s empirical and estimation framework could be applied to other retail industries in which key decisions revolve around expansion and contraction via stores. We believe that our framework may uncover similar firm size spillovers in settings in which other studies have demonstrated a growing wedge between large and small enterprises (e.g., Jia, 2008, Basker, Klimek, and Van, 2012).

A potential caveat is that we abstract away from potential national level expansion strategies in our analysis. Such incentives may be important to consider if the retail chains are concerned about geographic risk. However, there are two reasons why we believe such a concern may be mitigated in our context. First, hamburger retail store expansion or contraction decisions are almost always made at the level of city headquarters. Second, although, a likely strategy borne out of risk aversion may involve diversification of outlets across cities this would be counteracted by an incentive to avoid losing the potential benefits of city-wide firm size spillovers. Finally, estimating expansion as a retail network decision is currently infeasible, in the form of a fully dynamic game with a rich state space, heterogeneous players, and serial correlation in unobservables with endogenous feedback. On the other hand, this suggests a very challenging but ambitious avenue for future research.

References


A Web Appendix (Not for Publication)

A.1 First Stage Estimation

In the first stage, we maximize the log likelihood function with respect to the first-stage parameters, denoted \( \phi \). For each trial value of \( \phi \), the basic algorithm for evaluating the log likelihood function is as follows:

1. For each market \( m = 1, \ldots, M \):
   
   (a) Draw \( R \) particles, denoted \( \tilde{Z}_{m,0}^r \), from the initial distribution of firm-specific spillover levels. Each draw \( \tilde{Z}_{m,0}^r \) is a vector of length \( I \), with one component for each firm.
   
   (b) For each \( t = 1, \ldots, T \):
      
      i. Transition each of the \( R \) particles \( \tilde{Z}_{m,t-1}^r \) from period \( t - 1 \) through the joint transition equation as determined by (4), given the current value of \( \phi \), to obtain a new collection of particles denoted\(^{32} Z_{mt}^r \).
      
      ii. Calculate and store the joint likelihood (the product of the firm-specific choice probabilities) associated with each particle, given the values of \( n_{mt} \) and \( X_{mt} \) from the data. Denote these values by \( p_{mt}^r \) for \( r = 1, \ldots, R \).
      
      iii. Calculate and store the log likelihood value, given by log of the averaged joint particle-specific likelihood values: \( l_{mt} = \ln \left( \frac{1}{R} \sum_{r=1}^{R} p_{mt}^r \right) \).
      
      iv. Assign the likelihood \( p_{mt}^r \) as an importance weight for each corresponding particle \( Z_{mt}^r \).
      
      v. Draw a collection of \( R \) new particles, denoted \( \check{Z}_{mt}^r \) for \( r = 1, \ldots, R \) by sampling with replacement from the particles \( \{ Z_{mt}^r \}_{r=1}^{R} \) in proportion to the assigned importance weights.\(^{33} \)

2. Return the value of the log-likelihood function for the trial value of \( \phi \), \( L_M(\phi) = \sum_{m=1}^{M} \sum_{t=1}^{T} l_{mt} \).

A.2 Second Stage Estimation

In the second stage, we choose structural parameters \( \alpha \) in order to maximize the minimum distance objective function \( Q(\alpha) \). Initially, we construct a collection of \( B \) “inequalities” as follows:

\(^{32}\)The distinction between \( \tilde{Z}_{mt}^r \) and \( Z_{mt}^r \) is intentional. The former are draws from the period \( t - 1 \) filtering distribution (for the latent states at time \( t - 1 \) given period \( t - 1 \) information) while the latter are draws from the period \( t \) prediction distribution (for the latent states at time \( t \) using period \( t - 1 \) information).

\(^{33}\)Again, the tilde denotes that these are draws for period \( t \) updated with period \( t \) information.
1. Randomly draw $B$ initial market structures, denoted $(N^b_1, X^b_1, Z^b_1)$ for $b = 1, \ldots, B$, consisting of exogenous state variables, firm-specific observable variables, and firm-specific unobservable spillover levels.

2. For each initial market structure indexed by $b = 1, \ldots, B$, choose a single firm $i$ and draw a random alternative policy function for that firm by adding a random vector $\varphi^b \sim N(0, \sigma^2_{\varphi}I)$ to the subvector of parameters in $\dot{\phi}$ related to the first-stage policy functions. Let $\tilde{\sigma}^b$ denote the policy profile where firm $i$ is using the alternative policy while all other firms use the estimated policies from $\hat{\sigma}$.

Once the inequalities are determined, calculate the objective function $Q(\alpha)$ for each trial value of $\alpha$ as follows:

1. Given the estimated first-stage parameters $\hat{\phi}$ and a vector of structural parameters $\alpha$, repeat the following steps for each of the initial market structures indexed by $b = 1, \ldots, B$:

   (a) Draw $S$ sample paths of length $T$, each starting at $(N^b_1, X^b_1, Z^b_1)$, using the laws of motion determined by the given parameters $\hat{\phi}$ and $\alpha$. Store the discounted profits for the chosen firm $i$ for the simulated path.

   Specifically, for each simulated path $s = 1, \ldots, S$, for each time $t = 1, \ldots, T$:

   i. Using the law of motion estimated in the first stage, draw shocks to simulate a new vector of firm-specific spillover levels, $Z_{t}^{b,s}$.
   
   ii. Using the fitted SUR model, draw shocks and simulate new values for each of the exogenous variables, $X_{t}^{b,s}$.

   iii. Using the new spillover levels and exogenous variables, draw structural shocks and evaluate the estimated policies $\hat{\sigma}$ in order to simulate each firm’s expansion or contraction decision, $n_{it}^{b,s}$.

   iv. Calculate the stock of outlets for each firm, $N_{it}^{b,s} = N_{i,t-1}^{b,s} + n_{it}^{b,s}$.

   v. Calculate the structural period profits for firm $i$ in period $t$ under $\tilde{\sigma}$, denoted $\hat{\pi}_{it}^{b,s}$:

      $$\hat{\pi}_{it}^{b,s} = \Pi(N_{it}^{b,s}, N_{i,t}^{b,s}, X_{t}^{b,s}, Z_{t}^{b,s}, \zeta_{t}, \alpha).$$

   (b) Calculate the discounted profits for firm $i$ from the perspective of the initial state for each simulated path, $V^{b,s}(\tilde{\sigma}, \alpha) = \sum_{t=1}^{T} \rho^{t-1} \hat{\pi}_{it}^{b,s}$.
(c) Estimate the ex-ante value of being in state \((N^b_1, X^b_1, Z^b_1)\) for firm \(i\) (when firms use the estimated policies in the profile \(\hat{\sigma}\)) by averaging the discounted profits over all \(S\) paths:
\[
\bar{V}^b(\hat{\sigma}, \alpha) = \frac{1}{S} \sum_{s=1}^{S} \hat{V}^{b,s}(\hat{\sigma}, \alpha).
\]

(d) Repeat steps in part 1a above to simulate \(S\) alternative paths of length \(T\), also starting at \((N^b_1, X^b_1, Z^b_1)\) and using the laws of motion determined by the given parameters \(\hat{\phi}\) and \(\alpha\), but using the alternative policy from \(\tilde{\sigma}\) for firm \(i\). Let \(\pi^{b,s}_{it}\) denote the profits earned by firm \(i\) in period \(t\) along path \(s\).

(e) Calculate the discounted profits for firm \(i\) from the perspective of the initial state for the alternative paths, \(V^{b,s}(\tilde{\sigma}, \alpha) = \sum_{t=1}^{T} \rho^{t-1} \pi^{b,s}_{it}\).

(f) Estimate the ex-ante value of being in state \((N^b_1, X^b_1, Z^b_1)\) for firm \(i\) (when using the alternative policy against the estimated policies of the rival firms) by averaging the discounted profits over all \(S\) paths: \(\bar{V}^b(\tilde{\sigma}, \alpha) = \frac{1}{S} \sum_{s=1}^{S} \tilde{V}^{b,s}(\tilde{\sigma}, \alpha)\).

(g) Calculate and store the difference in the ex-ante valuations, \(g^b(\hat{\sigma}, \alpha) = \bar{V}^b(\hat{\sigma}, \alpha) - \bar{V}^b(\tilde{\sigma}, \alpha)\).

2. Use the values \(g^b(\hat{\sigma}, \alpha)\) for each of the \(B\) initial states and alternative policies to calculate the value of the minimum distance function, \(Q(\alpha) = \frac{1}{B} \sum_{b=1}^{B} (\min\{g^b(\hat{\sigma}, \alpha), 0\})^2\).

A.3 Seemingly Unrelated Regressions (SUR) Model

To model the joint evolution of the exogenous state variables, we employ an SUR model. One justification for this approach is that all of these variables may be correlated at some level. For example, income and property value often move along similar trends. The SUR specification is as follows:

\[
\begin{bmatrix}
X_{1t} \\
X_{2t} \\
\vdots \\
X_{kt}
\end{bmatrix} =
\begin{bmatrix}
c_1 \\
c_2 \\
\vdots \\
c_k
\end{bmatrix} +
\begin{bmatrix}
A_{11} & A_{12} & \ldots & A_{1k} \\
A_{21} & A_{22} & \ldots & A_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
A_{k1} & A_{k2} & \ldots & A_{kk}
\end{bmatrix}
\begin{bmatrix}
X_{1t-1} \\
X_{2t-1} \\
\vdots \\
X_{kt-1}
\end{bmatrix}
+ \begin{bmatrix}
e_{1t-1} \\
e_{2t-1} \\
\vdots \\
e_{kt-1}
\end{bmatrix}
\]

where \(E[e_t e'_t] = \Omega\) and where \(c = (c_1, \ldots, c_k)\), \(A = (a_{ij})\), and \(\Omega\) are parameters to be estimated. Estimates of the intercepts \(c\) and the coefficients \(A\) are reported in Table 14 and estimates for the covariances are reported in Table 15.
Table 14: SUR Model Estimates

<table>
<thead>
<tr>
<th></th>
<th>( \text{Population}_{t-1} )</th>
<th>( \text{Income}_{t} )</th>
<th>( \text{Property Value}_{t} )</th>
<th>( \text{Minimum Wage}_{t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>1.0043</td>
<td>(0.0031)</td>
<td>0.0007</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>Income</td>
<td>0.0148</td>
<td>(0.1586)</td>
<td>0.8499</td>
<td>(0.0174)</td>
</tr>
<tr>
<td>Property Value</td>
<td>0.0118</td>
<td>(0.0147)</td>
<td>0.0018</td>
<td>(0.0016)</td>
</tr>
<tr>
<td>Minimum Wage</td>
<td>0.0010</td>
<td>(0.0008)</td>
<td>0.0092</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.0061</td>
<td>(0.0096)</td>
<td>0.0072</td>
<td>(0.0011)</td>
</tr>
</tbody>
</table>

Table 15: Estimated SUR Covariance Matrix

<table>
<thead>
<tr>
<th></th>
<th>( \text{Population}_{t-1} )</th>
<th>( \text{Income}_{t} )</th>
<th>( \text{Property Value}_{t} )</th>
<th>( \text{Minimum Wage}_{t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>2.2420</td>
<td>-0.0060</td>
<td>-0.0017</td>
<td>-0.3005</td>
</tr>
<tr>
<td>Income</td>
<td>-0.0060</td>
<td>0.0271</td>
<td>0.0011</td>
<td>-0.0102</td>
</tr>
<tr>
<td>Property Value</td>
<td>-0.0017</td>
<td>0.0011</td>
<td>0.9651</td>
<td>0.2625</td>
</tr>
<tr>
<td>Minimum Wage</td>
<td>-0.3005</td>
<td>-0.0102</td>
<td>0.2625</td>
<td>48.3756</td>
</tr>
</tbody>
</table>

Note: All entries have been multiplied by 10^3 for easier comparison.

A.4 Reduced Form Policy Estimates

Table 16 contains the reduced form policy estimates from the first stage for each specification as described in Section 4.1. As indicated in the table we include either city fixed effects or city-specific drift terms, denoted \( \eta_m \), and firm fixed effects or firm-specific drift terms in all specifications. For specifications which include the \( Z \) process the inclusion of drift terms is denoted as “\( Z \)” while in the specification with only i.i.d. unobservables, the inclusion of fixed effects is denoted as “Yes”. For both specifications including the \( Z \) process, we used 1000 particles to approximate the distribution of \( Z_{imt} \) for each firm \( i \), market \( m \), and time period \( t \).

A.5 Additional Simulation Analysis Details

To implement the model simulations and counterfactuals, we employ a similar forward simulation approach as in Benkard, Bodoh-Creed, and Lazarev (2010), which does not require one to solve a computationally intractable dynamic model. First, we estimate the different specifications (full model with \( Z \) process, model without \( Z \) process, model with \( Z \) process but no spillovers) and store the first and second stage estimates.

We then initialize the market characteristics (i.e., population, income, property value, and minimum wage) and market structure (i.e., the initial number of outlets for each chain) using data for the first year for each market. The unobserved and serially correlated \( Z \) process also needs to be initialized in our simulations. We draw the first period chain-market-specific \( Z_{i,m,1} \)’s from the corresponding steady-state distributions at time period \( t = 1 \) under the assumption of no size.
Table 16: Reduced Form Policy Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Z (Spillovers)</th>
<th>Z (No Spillovers)</th>
<th>No Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>-0.2379 (0.0019)</td>
<td>-0.1458 (0.0065)</td>
<td>-0.1416 (0.0948)</td>
</tr>
<tr>
<td>Income</td>
<td>0.1794 (0.0008)</td>
<td>0.2099 (0.0031)</td>
<td>0.2081 (0.0197)</td>
</tr>
<tr>
<td>Property Value</td>
<td>-0.1407 (0.0122)</td>
<td>-0.1447 (0.0073)</td>
<td>-0.1398 (0.0486)</td>
</tr>
<tr>
<td>Grey Cup Host</td>
<td>0.2053 (0.0024)</td>
<td>0.1435 (0.0181)</td>
<td>0.1479 (0.0755)</td>
</tr>
<tr>
<td>Smoking Regulation</td>
<td>-0.1608 (0.0019)</td>
<td>-0.0788 (0.0053)</td>
<td>-0.1395 (0.1196)</td>
</tr>
<tr>
<td>Minimum Wage</td>
<td>-0.1357 (0.0015)</td>
<td>-0.1304 (0.0049)</td>
<td>-0.1252 (0.0412)</td>
</tr>
<tr>
<td>Population²</td>
<td>0.0642 (0.0005)</td>
<td>0.0689 (0.0030)</td>
<td>0.0553 (0.0271)</td>
</tr>
<tr>
<td>Population × Income</td>
<td>0.0611 (0.0006)</td>
<td>0.0444 (0.0032)</td>
<td>0.0487 (0.0257)</td>
</tr>
<tr>
<td>Population × Prop. Value</td>
<td>-0.0695 (0.0006)</td>
<td>-0.0668 (0.0025)</td>
<td>-0.0646 (0.0325)</td>
</tr>
<tr>
<td>Population × Min. Wage</td>
<td>-0.0453 (0.0004)</td>
<td>-0.0535 (0.0015)</td>
<td>-0.0476 (0.0188)</td>
</tr>
<tr>
<td>Income²</td>
<td>-0.0174 (0.0001)</td>
<td>-0.0238 (0.0006)</td>
<td>-0.0233 (0.0035)</td>
</tr>
<tr>
<td>Income × Prop. Value</td>
<td>-0.1480 (0.0003)</td>
<td>-0.1375 (0.0011)</td>
<td>-0.1323 (0.0088)</td>
</tr>
<tr>
<td>Income × Min. Wage</td>
<td>0.0324 (0.0002)</td>
<td>0.0344 (0.0011)</td>
<td>0.0354 (0.0062)</td>
</tr>
<tr>
<td>Property Value²</td>
<td>0.1814 (0.0008)</td>
<td>0.1715 (0.0016)</td>
<td>0.1709 (0.0205)</td>
</tr>
<tr>
<td>Property Value × Min. Wage</td>
<td>0.0184 (0.0002)</td>
<td>0.0137 (0.0009)</td>
<td>0.0139 (0.0150)</td>
</tr>
<tr>
<td>Minimum Wage²</td>
<td>-0.0197 (0.0002)</td>
<td>-0.0139 (0.0008)</td>
<td>-0.0146 (0.0088)</td>
</tr>
<tr>
<td>Cutoff 1 (φ₁)</td>
<td>-4.6511 (0.0765)</td>
<td>-3.8976 (0.2678)</td>
<td>-3.4663 (0.8414)</td>
</tr>
<tr>
<td>Cutoff 2 (φ₂)</td>
<td>-3.8789 (0.0903)</td>
<td>-3.1656 (0.1352)</td>
<td>-2.8467 (0.1905)</td>
</tr>
<tr>
<td>Cutoff 3 (φ₃)</td>
<td>-2.8649 (0.1142)</td>
<td>-2.4011 (0.1219)</td>
<td>-2.1811 (0.1437)</td>
</tr>
<tr>
<td>Cutoff 4 (φ₄)</td>
<td>1.0366 (0.0182)</td>
<td>0.8544 (0.0281)</td>
<td>0.8186 (0.0776)</td>
</tr>
<tr>
<td>Cutoff 5 (φ₅)</td>
<td>1.9208 (0.0198)</td>
<td>1.6166 (0.0369)</td>
<td>1.5274 (0.0734)</td>
</tr>
<tr>
<td>Cutoff 6 (φ₆)</td>
<td>2.4517 (0.0314)</td>
<td>2.0893 (0.0561)</td>
<td>1.9487 (0.0805)</td>
</tr>
<tr>
<td>Cutoff 7 (φ₇)</td>
<td>3.3228 (0.0888)</td>
<td>2.8536 (0.1267)</td>
<td>2.6372 (0.1204)</td>
</tr>
</tbody>
</table>

City Fixed Effects (ηₘ)         | Z        | Z        | Yes                |
Firm Fixed Effects (µₙ)          | Z        | Z        | Yes                |
Observations                     | 5580     | 5580     | 5580               |
Particles                        | 1000     | 1000     | -                  |
Log Likelihood                   | -4009.45 | -4061.30 | -4059.40           |
spillovers from $t = 0$ as by definition $N_{i,m,0} = 0$ for all firms. Using the estimated $Z$ process, under these assumptions for each chain $i$ in market $m$ in the initial period $t = 0$, the stationary mean is $(\mu_i + \eta_m)/(1 - \delta_i)$, and the stationary variance is $\omega^2_i/(1 - \delta_i^2)$ (see e.g., Hamilton, 1994, p. 53).

Using the estimates, inferred policy functions from the first stage estimation, and SUR process for the exogenous market characteristics, along with the initializations, we then forward simulate the evolution of the number of stores and per-period profits across all markets $m$, for each of the specifications. In each market, we simulate 250 sample paths (with length of 36 years) given the initial market conditions, distribution of $Z$’s, and inferred policy functions.

**A.6 Additional Data Description and Reduced Form Analysis**

Figure 5 plots a fitted prediction, and demonstrates that not only is there heterogeneity in the degree of expansion or contraction and firm size, retailers differ in how expansion or contraction changes with lagged firm size. But in general, we generally see a positive correlation between the lagged firm size and subsequent expansion efforts, especially so for McDonald’s.

We have seen in the previous regressions that a chain’s future market share is positively related to its previous size. Such patterns may have implications on the competitiveness of the markets. To investigate further the implications of firm size on the competitiveness of markets, we run regressions of HHI on each retail chain’s size. Table 17 demonstrates a positive correlation between a firm’s lagged size, and the competitiveness of the market. The positive correlation is most pronounced for McDonald’s. Notice also that in the last specification in which we include each firm’s lagged size, it is only McDonald’s for which its size has a positive relationship with the HHI index. These results suggest that the success of McDonald’s comes at the cost of market competitiveness.
Figure 5: Yearly Expansion and Contraction vs. Chain Size
Table 17: HHI Regressions

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HHI</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged A&amp;W outlets</td>
<td>0.00100</td>
<td>-0.00287</td>
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<td>-0.0132***</td>
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<td>0.638***</td>
<td>0.662***</td>
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<td>0.662***</td>
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Standard errors in parentheses; ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.