# A Model of Casino Gambling

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September 2011

# Overview

- casino gambling is a very popular activity worldwide
  - in 2007, 54 million people made 376 million trips to casinos in the U.S. alone
  - revenues for U.S. casinos that year totalled $59 billion
- however, we still have very few models of casino gambling
  - constructing a model is difficult
  - the standard economic model of risk attitudes – Expected Utility with a concave utility function – cannot explain gambling
- some progress has been made:
  - Expected Utility with non-concave segments in the utility function
  - utility of gambling
    - ∗ either indirectly related to the bets themselves
    - ∗ or directly related to them (Conlisk, 1993)
  - misperception of odds

## Overview

- we present a new model of casino gambling based on (cumulative) prospect theory
- why prospect theory?
  - because it already explains risk attitudes in many *other* settings
- it is initially surprising that prospect theory would be able to explain gambling
  - gambling seems inconsistent with loss aversion
- we show that, in fact, prospect theory can offer a rich theory of gambling
  - for a wide range of parameter values, a prospect theory agent *would* want to gamble in a casino, even if the bets on offer have no skewness, and zero or negative expected value
  - the model also predicts a plausible time inconsistency
  - this, in turn, predicts heterogeneity in casino behavior

# Overview

Other contributions:

- we draw attention to a source of time inconsistency – nonlinear probability weighting – that has not been studied very much
- and use our framework to suggest some links between casino gambling and stock market phenomena

# Prospect Theory

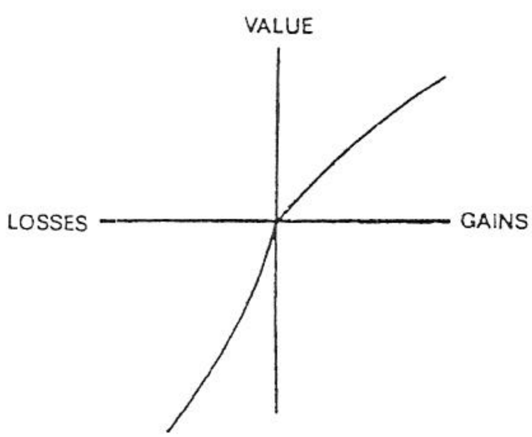
Consider the gamble $(x, p; y, q)$

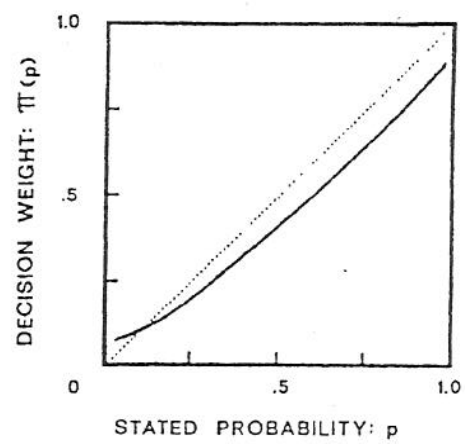
- under EU, it is assigned the value

$$pU(W + x) + qU(W + y)$$

- under Prospect Theory, it is assigned the value

$$\pi(p)v(x) + \pi(q)v(y)$$



# Prospect Theory

*Four* key features:

- the carriers of value are *gains* and *losses,* not final wealth levels
  - compare $v(x)$ to $U(W+x)$
- $v(\cdot)$ has a kink at the origin
  - captures a greater sensitivity to losses (even small losses) than to gains of the same magnitude
  - "loss aversion"
  - inferred from aversion to $(110, \frac{1}{2}; -100, \frac{1}{2})$
- $v(\cdot)$ is concave over gains, convex over losses
  - inferred from $(500, 1) \succ (1000, \frac{1}{2})$ and $(-500, 1) \prec (-1000, \frac{1}{2})$

# Prospect Theory

- transform probabilities with a weighting function $\pi(\cdot)$ that overweights *low* probabilities
  - inferred from our simultaneous liking of lotteries and insurance, e.g. $(5, 1) \prec (5000, 0.001)$ and $(-5, 1) \succ (-5000, 0.001)$

Note:

- transformed probabilities should not be thought of as beliefs, but as decision weights

# Cumulative Prospect Theory

- proposed by Tversky and Kahneman (1992)
- applies the probability weighting function to the *cumulative* distribution function:

$$(x_{-m}, p_{-m}; \ldots; x_{-1}, p_{-1}; x_0, p_0; x_1, p_1; \ldots; x_n, p_n),$$

where $x_i < x_j$ for $i < j$ and $x_0 = 0$, is assigned the value

$$\sum_{i=-m}^{n} \pi_i v(x_i)$$

$$\pi_i = \begin{cases} \pi(p_i + \ldots + p_n) - \pi(p_{i+1} + \ldots + p_n) \\ \pi(p_{-m} + \ldots + p_i) - \pi(p_{-m} + \ldots + p_{i-1}) \end{cases} \text{for} \begin{array}{l} 0 \leq i \leq n \\ -m \leq i < \end{array}$$

- the agent now overweights the *tails* of a probability distribution
  - this preserves a preference for lottery-like gambles

# Cumulative Prospect Theory

- Tversky and Kahneman (1992) also suggest functional forms for $v(\cdot)$ and $\pi(\cdot)$ and calibrate them to experimental evidence:

$$v(x) = \begin{cases} x^{\alpha} \\ -\lambda(-x)^{\alpha} \end{cases} \text{for} \begin{array}{c} x \geq 0 \\ x < 0 \end{array}$$

$$\pi(P) = \frac{P^{\delta}}{(P^{\delta} + (1-P)^{\delta})^{1/\delta}}$$

with

$$\alpha = 0.88, \lambda = 2.25, \delta = 0.65$$

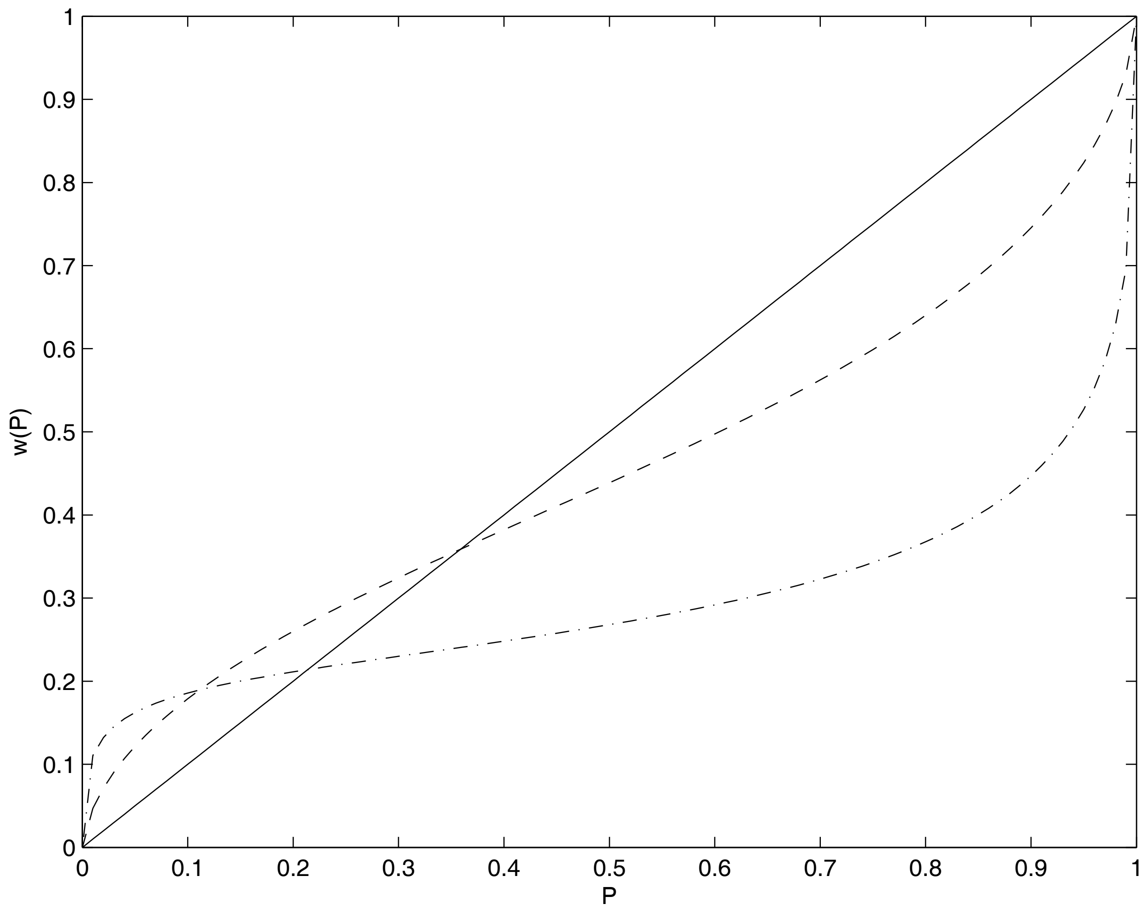



Figure 2. The figure shows the form of the probability weighting function proposed by Tversky and Kahneman (1992), namely $w(P) = P^\delta/(P^\delta + (1-P)^\delta)^{1/\delta}$. The dashed line corresponds to $\delta = 0.65$, the dash-dot line to $\delta = 0.4$, and the solid line to $\delta = 1$.

# Background

- in the U.S., the term "gambling" usually refers to one of four activities

  [1] casino gambling (slot machines and blackjack are the most popular games)

  [2] the buying of lottery tickets

  [3] pari-mutuel betting on horses at racetracks

  [4] fixed-odds betting, through bookmakers, on sports such as football or basketball

- here, we focus on casino gambling
  - it differs from the other types of gambling on certain dimensions
  - from the perspective of prospect theory, it is perhaps the most challenging to explain

# A Model

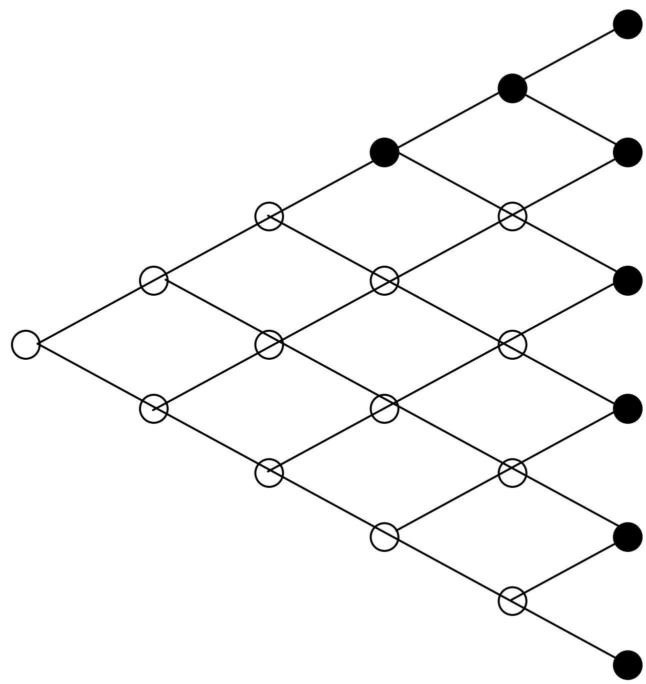
- $T + 1$ dates: 0, 1,. . . , T
- at time 0, the agent is offered a 50:50 bet to win or lose a fixed amount $\$h$
  - if he declines, the game is over
  - if he accepts, the gamble is played out and the outcome announced at time 1

The game continues as follows:

- if, at time $t \in [0, T - 2]$, the agent agreed to play a gamble, then, at time $t+1$, he is offered another 50:50 bet to win or lose $\$h$
  - if he declines, the game is over: the agent settles his account and leaves the casino
  - if he accepts, the gamble is played out and the outcome announced in the next period
- at time $T$, the agent *must* leave the casino if he hasn't already done so

# A Model

- the model corresponds most closely to blackjack
  - but will also have something to say about slot machines
- we represent the casino graphically as a binomial tree
  - refer to a node by a pair $(t, j)$, where $t$ is time and $j$ is the position in the column of nodes for that time period
  - use a black/white color scheme to indicate the agent's behavior
- the basic gamble involves equiprobable gains and losses
  - but our conclusions also hold for bets with somewhat negative expected values



# A Model

- we assume that the agent maximizes the cumulative prospect theory value of his accumulated winnings or losses at the moment he leaves the casino
- this immediately raises a key issue:
  - the probability weighting function introduces a *time inconsistency*

## Time Inconsistency

Suppose $T = 5$ and $h = \$10$

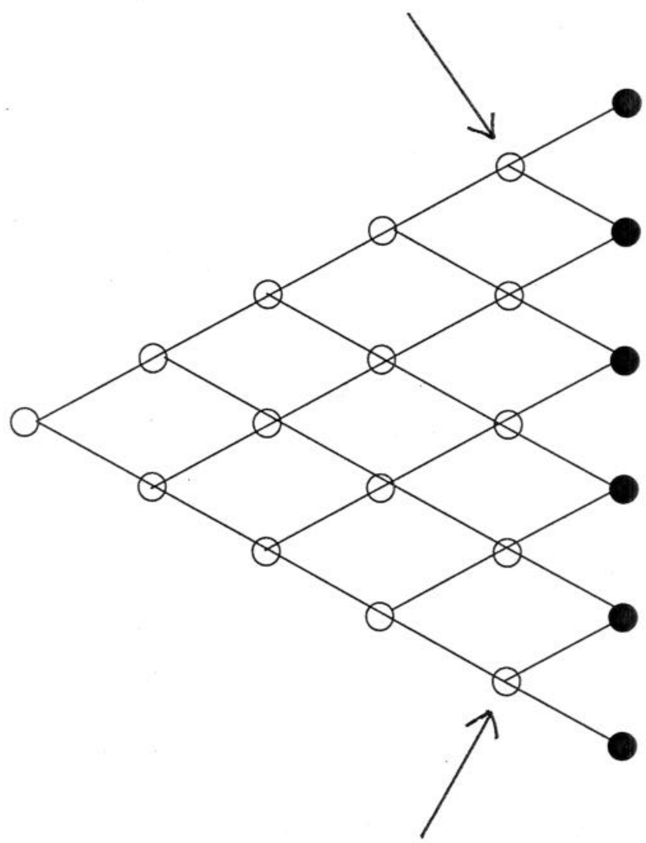
*Upper* part of the tree:

- from the perspective of time 0, the agent would like to keep gambling in node (4,1), should he later arrive in that node
  - but if he actually arrives in that node, i.e. from the perspective of time 4, he would prefer to stop gambling

*Lower* part of the tree:

- from the perspective of time 0, the agent would like to stop gambling in node (4,5), should he later arrive in that node
  - but if he actually arrives in that node, i.e. from the perspective of time 4, he would prefer to keep gambling

# Time Inconsistency



# Time Inconsistency

- given the time inconsistency, we consider three types of agents

*Naive agents*

- they are unaware that, at time $t > 0$, they will deviate from any plan they select at time 0

*No-commitment sophisticates*

- they are aware that they will deviate from any plan they select at time 0
  - and they are unable to find a way of committing to their inital plan

*Commitment-aided sophisticates*

- they are also aware that they will want to deviate from any plan they select at time 0
  - but they are able to find a way of committing to their initial plan

## The Naive Agent

- we divide our analysis of this agent into two parts:
  - the agent's *initial* decision, at time 0, as to whether to enter the casino
  - his *subsequent* gambling behavior, at time $t > 0$

# The Naive Agent

*The initial decision*

- the agent selects a "plan" at time 0
  - i.e. a mapping from all future nodes to one of two actions, "exit" or "continue"
  - as $T$ increases, the number of possible plans becomes very large
- denote the set of plans as $S_{(0,1)}$
  - each $s \in S_{(0,1)}$ corresponds to a random variable $\tilde{G}_s$ which represents the different possible winnings or losses if the agent leaves the casino in the manner specified by $s$
- the naive agent solves:

$$max_{s \in S_{(0,1)}} V(\tilde{G}_s)$$

- we solve this numerically, focusing on $T = 5$; but we cannot use dynamic programming
  - instead, step through each $s \in S_{(0,1)}$ in turn and compute $\tilde{G}_s$ and then $V(\tilde{G}_s)$
  - optimal plan is $s^*$ and has utility $V^*$
  - the agent enters the casino if $V^* > 0$

# The Naive Agent

- first, look at the range of preference parameter values for which the naive agent is willing to enter the casino
  - find that he *is* willing to enter for a wide range of parameter values
- to see the intuition, look at the optimal exit strategy, e.g. for $(\alpha, \delta, \lambda) = (0.95, 0.5, 1.5)$
  - the agent plans to keep gambling as long as possible when he is winning, but to stop gambling and leave the casino as soon as he starts accumulating losses
  - this gives his perceived *overall* casino experience a positively skewed distribution
  - with sufficient probability weighting, this distribution can be attractive

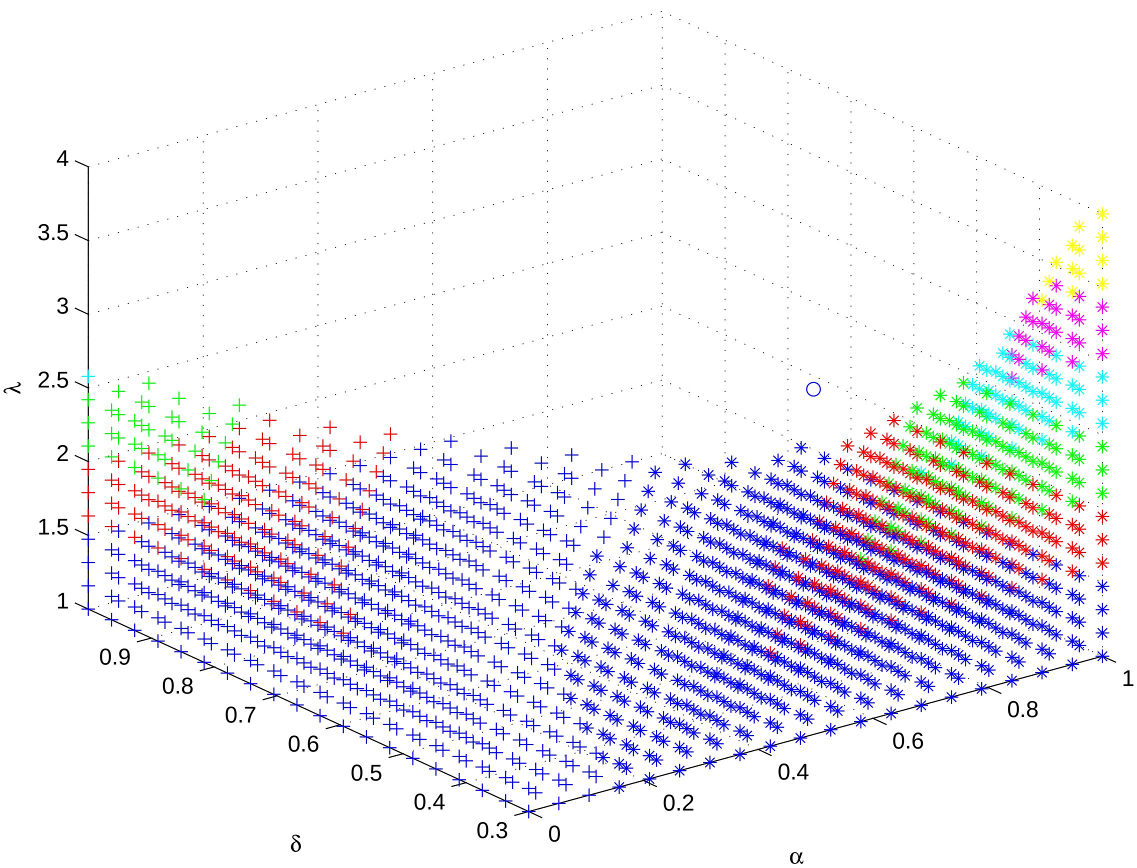
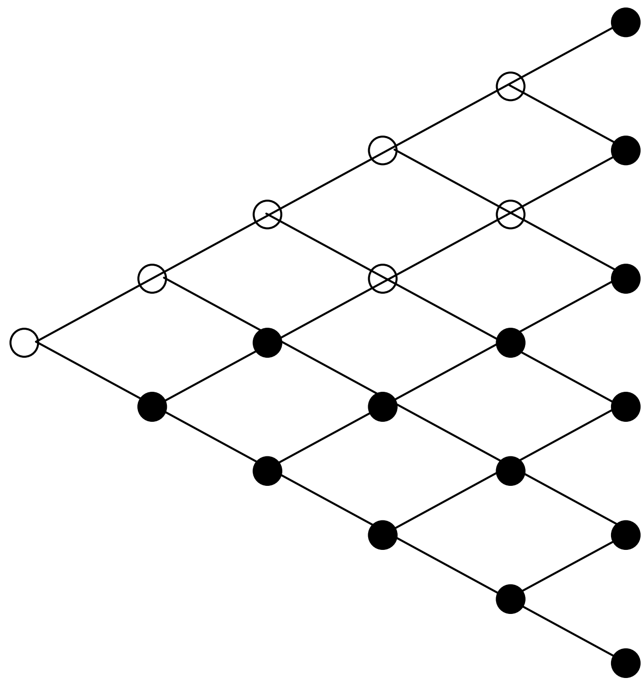



Figure 3. The "*" and "+" signs mark the preference parameter triples $(\alpha, \delta, \lambda)$ for which an agent with prospect theory preferences would be willing to enter a casino offering 50:50 bets to win or lose a fixed amount $\$h$. The agent is naive: he is not aware of the time inconsistency generated by probability weighting. The "*" signs mark parameter triples for which the agent's planned strategy is to leave early if he is losing but to stay longer if he is winning. The "+" signs mark parameter triples for which the agent's planned strategy is to leave early if he is winning but to stay longer if he is losing. The blue, red, green, cyan, magenta, and yellow colors correspond to parameter triples for which $\lambda$ lies in the intervals [1,1.5), [1.5,2), [2,2.5), [2.5,3), [3,3.5), and [3.5,4], respectively. The circle marks Tversky and Kahneman's (1992) median estimates of the parameters, namely $(\alpha, \delta, \lambda) = (0.88, 0.65, 2.25)$. A lower value of $\alpha$ means greater concavity (convexity) over gains (losses); a lower $\delta$ means more overweighting of tail probabilities; and a higher $\lambda$ means greater loss aversion.



# The Naive Agent

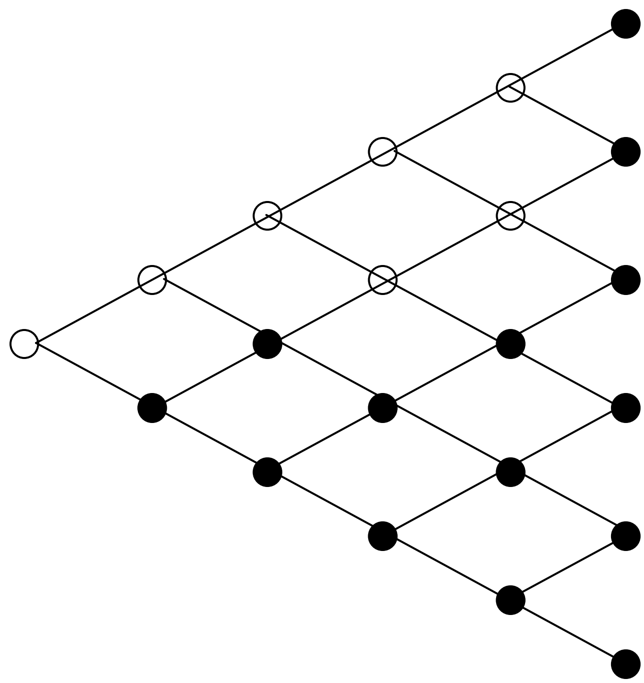
*Subsequent behavior ($t > 0$)*

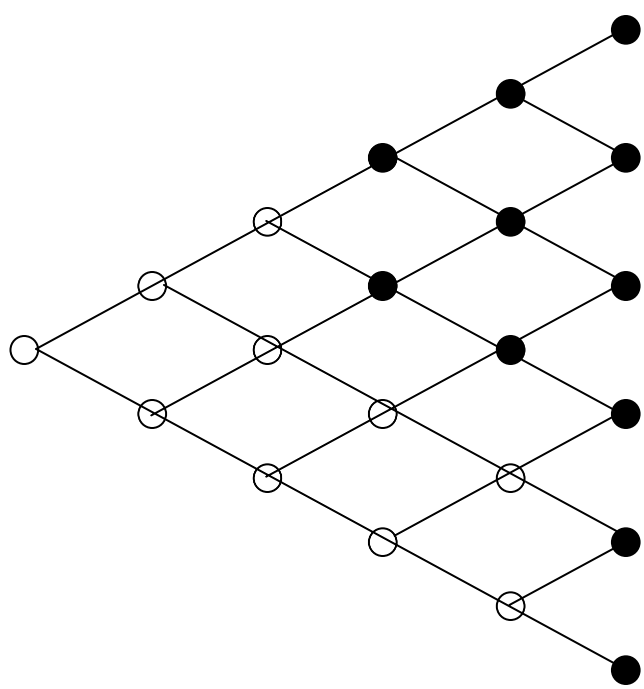
- in node $(t, j)$, where $t > 0$, the naive agent gambles if

$$max_{s \in S_{(t,j)}} V(\tilde{G}_s) > v(h(t + 2 - 2j))$$

  - $S_{(t,j)}$ is the set of possible plans available from that node

- now look, for $(\alpha, \delta, \lambda) = (0.95, 0.5, 1.5)$, at what this implies for his behavior after entering the casino
  - we find that, roughly speaking, he does the opposite of what he originally intended at time 0
  - he continues gambling as long as possible when he is losing, but stops gambling and leaves the casino once he accumulates a significant gain





# The No-commitment Sophisticate

- this agent is aware that he will deviate from his time 0 plan, but cannot find a way of committing to this initial plan
  - he uses dynamic programming to decide on a course of action
- the agent therefore gambles in node $(t, j)$ if

$$V(\widetilde{G}_{t,j}) > v(h(t+2-2j))$$

- we find that he almost always chooses *not* to enter the casino
  - he knows that he will keep gambling when he is losing and will stop gambling when he accumulates some gains
  - this makes his overall casino experience *negatively* skewed, which, under probability weighting, is unattractive

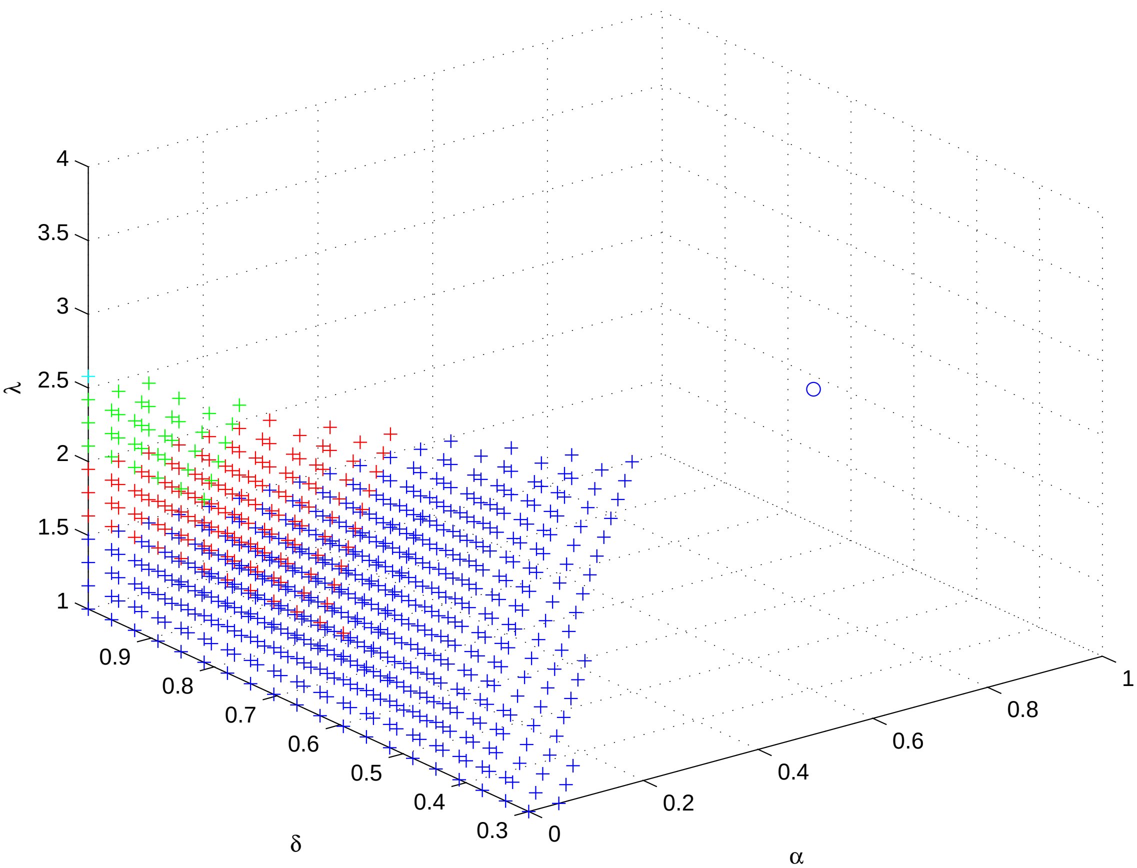


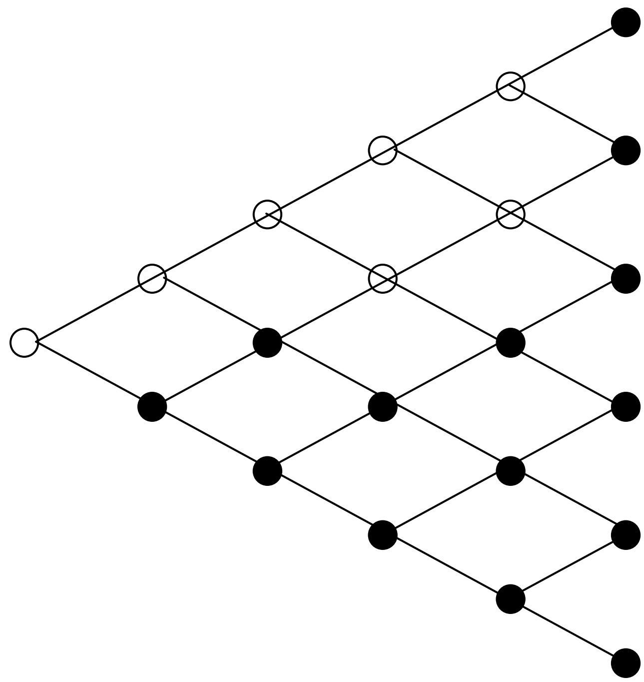

Figure 5. The "+" signs mark the preference parameter triples $(\alpha, \delta, \lambda)$ for which an agent with prospect theory preferences would be willing to enter a casino offering 50:50 bets to win or lose a fixed amount $\$h$. The agent is sophisticated: he is aware of the time inconsistency generated by probability weighting. The blue, red, green, and cyan colors correspond to parameter triples for which $\lambda$ lies in the intervals [1,1.5), [1.5,2), [2,2.5), and [2.5,4], respectively. The circle marks Tversky and Kahneman's (1992) median estimates of the parameters, namely $(\alpha, \delta, \lambda) = (0.88, 0.65, 2.25)$. A lower value of $\alpha$ means greater concavity (convexity) over gains (losses); a lower $\delta$ means more overweighting of tail probabilities; and a higher $\lambda$ means greater loss aversion.

## The Commitment-aided Sophisticate

- this agent is aware that he will want to deviate from his time 0 plan
  - and he is able to find a way of committing to this plan
- at time 0, he solves

$$max_{s \in S_{(0,1)}} V(\tilde{G}_s)$$

  - he is able to commit to the objective-maximizing plan $s^*$
  - he therefore enters the casino if $V(\tilde{G}_{s^*}) > 0$
- how can the agent commit?
  - in the region of losses: he brings a fixed amount of cash to the casino and leaves his ATM card at home
  - in the region of gains: the casino can help by offering vouchers for free accommodation and food



# Discussion

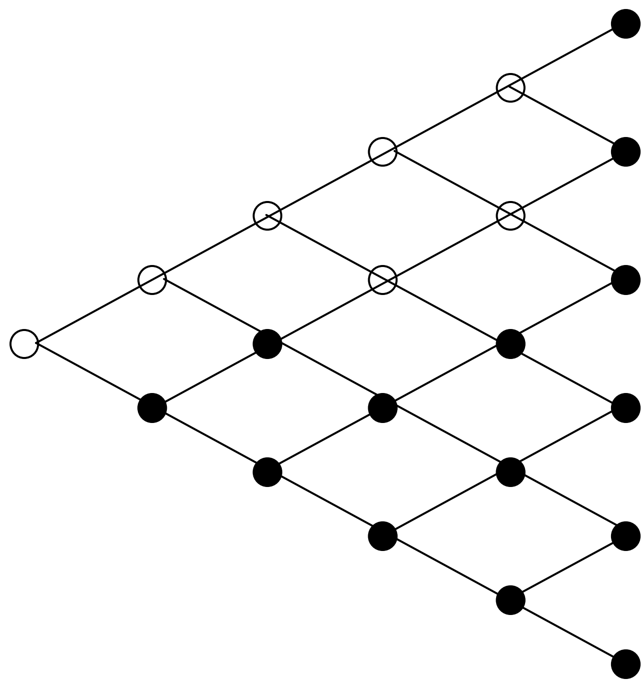
A few remarks on:

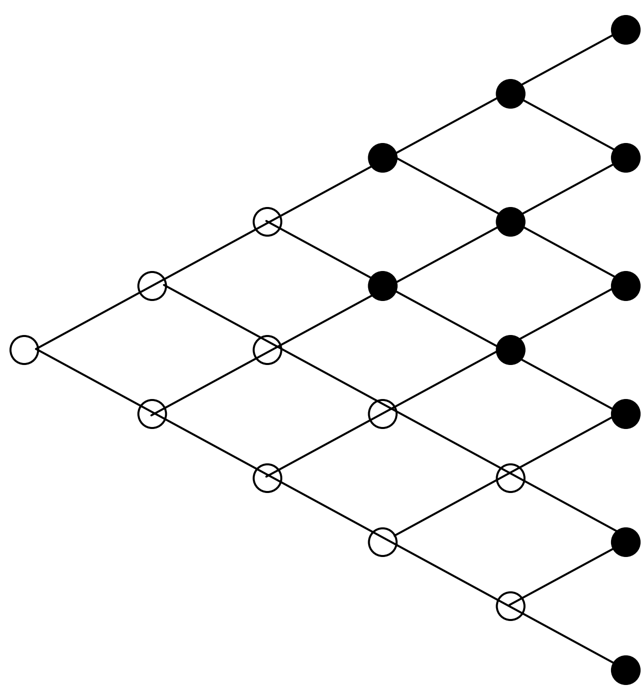
- average losses
- testable predictions
- competition with lotteries
- connections to the stock market

## Average Losses

- which of "naive agents" and "commitment-aided sophisticates" has larger average losses?
- when the basic casino bet is a 50:50 bet to win or lose \$$h$, both types have zero average losses
  - but if the basic bet has negative expected value, then the agent who gambles for longer has larger average losses
- we solve for the gambling behavior of the two types when the basic bet is a 0.46 chance to win \$$h$ and a 0.54 chance to lose \$$h$
- we find that the naive agent stays in the casino almost twice as long, on average

  $\Rightarrow$ "naivete" is costly





# Predictions and Other Evidence

- the model predicts that people will gamble *more* than they planned to after incurring some losses
  - and less than they planned to after making some gains
- it also predicts that the disparity between planned and actual behavior will be larger for less sophisticated gamblers

- Barkan and Busemeyer (1999) and Andrade and Iyer (2009) confirm some of these predictions in a laboratory setting
  - subjects gamble more in the region of losses than they planned to
  - and, in the first study, gamble less in the region of gains

# Competition with Lotteries

- can casinos survive competition from lottery providers?
  - lotteries may offer a more convenient source of positive skewness
- we use a simple equilibrium model with competitive provision of both casinos and lotteries to show that casinos *can* survive the competition
- in equilibrium, lottery providers attract the no-commitment sophisticates and earn zero average profits
- casinos compete by offering slightly better odds
  - they attract the naive agents and the commitment-aided sophisticates
  - and also manage to earn zero average profits
  - they lose money on the commitment-aided sophisticates, but make this up on the naive agents, who gamble longer than they planned to

# Competition with Lotteries

- the economy contains two kinds of firms: casinos $\{i\}$ and lottery providers $\{j\}$
- each casino has the form seen earlier, with fixed $T$ and $h$, but can choose its own basic bet win probability $p_i$

$$(\$h, p_i; -\$h, 1 - p_i)$$

- lottery provider $j$ offers a one-shot bet $\tilde{L}_j$
  - restrict attention to $\tilde{L}_j$ that can be dynamically constructed from a casino with same $T$ and $h$ but basic bet win probability $q_j$

$$(\$h, q_j; -\$h, 1 - q_j)$$

# Competition with Lotteries

- there are a continuum of consumers
  - fraction of each type is $\mu_N$, $\mu_{S,NC}$, and $\mu_{S,CA}$
  - each consumer either plays in a casino, plays a lottery, or does nothing
  - he chooses the option with the highest prospect theory value
  - each firm incurs a cost $C > 0$ per consumer served
- we look for a competitive equilibrium $\{p_i\}$ and $\{\tilde{L}_j\}$ (based on some $\{q_j\}$) such that:
  - once consumers have made their optimal choices, all firms break even
  - there is no profitable deviation for any firm

# Competition with Lotteries

*Results*

- set $(\alpha, \delta, \lambda) = (0.95, 0.5, 1.5)$ for all agents, $T = 5$, $h = \$10$, $(\mu_N, \mu_{S,NC}, \mu_{S,CA}) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, and $C = 2$
- then there is an equilibrium in which all lottery providers choose $q_j = 0.45$ and all casinos pick $p_i = 0.465$
- lottery providers attract the no-commitment sophisticates
  - the lottery's expected value is -\$2, so lottery providers earn zero average profits
- casinos attract the naive agents and the commitment-aided sophisticates
  - commitment-aided sophisticates lose \$1.43, on average
  - naive agents *think* they will lose \$1.43, on average, but actually lose \$2.57, on average
  - casinos therefore also earn zero average profits

# Link to Financial Markets I

- think of the binomial tree of casino winnings as the price path of a stock
- then the model suggests how a prospect theory investor would trade a stock over time
- e.g. it suggests that the trader may be time-inconsistent
  - he plans to sell a stock if it goes down and to hold it if it goes up, but actually does the opposite
  - i.e. he plans to exhibit the opposite of the disposition effect, but actually exhibits the disposition effect itself
- it also suggests the use of commitment devices

# Link to Financial Markets II

- our model of casino gambling is based on probability weighting
- in financial markets, probability weighting predicts that positively skewed assets will be overpriced, and will earn low average returns
  - Barberis and Huang (2008), "Stocks as Lotteries..."
- there is now significant empirical support for this prediction
  - Zhang (2006), Boyer, Mitton, Vorkink (2010), Conrad, Dittmar, Ghysels (2010)
- and it offers a way of understanding some puzzling financial phenomena
  - e.g. the average returns of IPOs, options, volatile stocks, and OTC stocks; the diversification discount; under-diversification
- according to our model, casino gambling is related to these financial phenomena
  - all of them are driven by the same psychological mechanism

# Conclusion

- we present a new model of casino gambling based on (cumulative) prospect theory
  - we find that, for a wide range of parameter values, a prospect theory agent would be willing to enter a casino even if it offers bets with no skewness and zero or negative expected value
  - the model also predicts a plausible time inconsistency
  - and traces heterogeneity in casino behavior to differences in sophistication and ability to commit

- according to the model, the popularity of casinos is driven by two aspects of our psychological make-up
  - probability weighting
  - naivete