Prospect Theory Applications in Finance

Nicholas Barberis
Yale University

March 2010
Overview

• in behavioral finance, we work with models in which some agents are less than fully rational

• “rationality” is typically taken to mean two things:
  – rational beliefs: update beliefs using Bayes’ rule
  – rational preferences: make decisions according to EU, with a utility function defined over wealth or consumption
Overview, ctd.

• one source of inspiration on plausible departures from rationality is the psychology literature

• psychology of beliefs
  – deviations from Bayes’ rule
  – e.g. overconfidence, representativeness

• psychology of preferences
  – deviations from EU or a concern for non-consumption utility
  – e.g. prospect theory, narrow framing, ambiguity aversion

• today, look at implications of *prospect theory* and *narrow framing* for asset prices and portfolio choice
Prospect Theory

• almost all models of the stock market assume that investors evaluate risk according to EU

• however, extensive experimental evidence shows that people systematically violate EU

• there are now many non-EU models that try to capture the experimental evidence
  – Prospect Theory (Kahneman and Tversky, 1979) is the best-known
  – it does the best job capturing the evidence

• many financial phenomena are hard to explain within the EU framework
  – can we make progress by replacing EU with Prospect Theory?
Prospect Theory, ctd.

Consider the gamble \((x, p; y, q)\)

- under EU, it is assigned the value
  \[ pU(W + x) + qU(W + y) \]

- under Prospect Theory, it is assigned the value
  \[ \pi(p)v(x) + \pi(q)v(y) \]
Prospect Theory, ctd.

Four key features:

- the carriers of value are gains and losses, not final wealth levels
  - compare $v(x)$ vs. $U(W + x)$
  - inferred from experimental evidence
  - also consistent with the way we perceive other attributes

- $v(\cdot)$ has a kink at the origin
  - captures a greater sensitivity to losses (even small losses) than to gains of the same magnitude
  - “loss aversion”
  - inferred from aversion to $(110, \frac{1}{2}; -100, \frac{1}{2})$

- $v(\cdot)$ is concave over gains, convex over losses
  - inferred from $(500, 1) \succ (1000, \frac{1}{2})$ and $(-500, 1) \prec (-1000, \frac{1}{2})$
Prospect Theory, ctd.

• transform probabilities with a weighting function $\pi(\cdot)$ that overweights low probabilities
  
  – inferred from our simultaneous liking of lotteries and insurance, e.g. $(5, 1) \prec (5000, 0.001)$ and $(-5, 1) \succ (-5000, 0.001)$

Note:

• transformed probabilities should not be thought of as beliefs, but as decision weights
Cumulative Prospect Theory

- proposed by Tversky and Kahneman (1992)
- applies the probability weighting function to the cumulative distribution function:

\[(x_{-m}, p_{-m}; \ldots; x_{-1}, p_{-1}; x_0, p_0; x_1, p_1; \ldots; x_n, p_n),\]

where \(x_i < x_j\) for \(i < j\) and \(x_0 = 0\), is assigned the value

\[\sum_{i=-m}^{n} \pi_i v(x_i)\]

\[\pi_i = \begin{cases} 
\pi(p_i + \ldots + p_n) - \pi(p_{i+1} + \ldots + p_n) \\
\pi(p_{-m} + \ldots + p_i) - \pi(p_{-m} + \ldots + p_{i-1}) 
\end{cases} \quad \text{for } 0 \leq i \leq n \\
\pi_i = \begin{cases} 
\pi(p_{-m} + \ldots + p_{i-1}) - \pi(p_{-m} + \ldots + p_{i}) \\
\pi(p_{-m} + \ldots + p_0) - \pi(p_{-m} + \ldots + p_{-m}) 
\end{cases} \quad \text{for } -m \leq i < 0\]

- the agent now overweights the tails of a probability distribution
  - this preserves a preference for lottery-like gambles
Cumulative Prospect Theory, ctd.

- Tversky and Kahneman (1992) also suggest functional forms for $v(\cdot)$ and $\pi(\cdot)$ and calibrate them to experimental evidence:

$$v(x) = \begin{cases} 
  x^\alpha & \text{for } x \geq 0 \\
  -\lambda(-x)^\alpha & \text{for } x < 0 
\end{cases}$$

$$\pi(P) = \frac{P^\delta}{(P^\delta + (1 - P)^\delta)^{1/\delta}}$$

with

$$\alpha = 0.88, \lambda = 2.25, \delta = 0.65$$
FIGURE 2. THE TVERSKY-KAHNEMAN (1992) PROBABILITY WEIGHTING FUNCTION. Notes: The figure shows the form of the probability weighting function proposed by Tversky and Kahneman (1992), namely \( w(P) = \frac{P^\delta}{(P^\delta + (1 - P)^\delta)^{1/\delta}} \). The dashed line corresponds to \( \delta = 0.65 \), the dash-dot line to \( \delta = 0.4 \), and the solid line to \( \delta = 1 \).
Narrow framing

• in traditional models, an agent evaluates a new gamble by merging it with his pre-existing risks and checking if the combination is attractive

• but in experimental settings, people often seem to evaluate a new gamble *in isolation* (Tversky and Kahneman, 1981)
  – this is “narrow framing”
  – get utility *directly* from the outcome of the gamble, not just indirectly from its contribution to total wealth

• Barberis, Huang, and Thaler (2006) argue that the rejection of $(110, \frac{1}{2}; -100, \frac{1}{2})$ is not only evidence of loss aversion, but of narrow framing as well
  – an agent with pre-existing risk finds the above gamble attractive, even if loss averse
Narrow framing, ctd.

• how should narrow framing be interpreted?
• two possibilities:
  – it is related to non-consumption utility, e.g. regret, which is plausibly associated with a narrow frame
  – it stems from an intuitive attempt to maximize consumption utility
    * intuition uses “accessible” information, and the most accessible information may be about narrow components of wealth (Kahneman, 2003)

• in what follows, we sometimes define prospect theory’s “gains” and “losses” as gains and losses in specific components of wealth
  – e.g. gains and losses in stock market wealth
  – e.g. gains and losses in a specific stock
• this is narrow framing
Applications of prospect theory and narrow framing

*Probability weighting function*

I. Pricing of skewness

*Concavity/convexity of value function over gains/losses*

II. Disposition effect

*Loss aversion*

III. Equity premium

IV. Stock market non-participation
Applications, ctd.

In all cases:

- have to decide on a frame, narrow or broad
  - which asset do the gains and losses refer to?
- then decide on the precise definition of the gain/loss
  - e.g. what is the reference point?
Models and references

_Probability weighting function_


Concavity/convexity of value function over gains/losses


Models and references, ctd.

*Loss aversion*


I. Pricing of skewness (PW)

Barberis and Huang (2008), “Stocks as Lotteries...”

- single period model; a risk-free asset and $J$ Normally distributed risky assets
- agents have identical expectations about security payoffs
- agents have identical CPT preferences
  - defined over gains/losses in wealth (i.e. no narrow framing)
  - reference point is initial wealth scaled up by riskless rate, so utility defined over $\hat{W} = \hat{W}_1 - W_0R_f$
  - full specification is:
    \[ V(\hat{W}) = \int_{-\infty}^{0} v(W) d\pi(P(W)) - \int_{0}^{\infty} v(W) d\pi(1-P(W)) \]
    (continuous distribution version of Tversky and Kahneman, 1992)

Then:

- the CAPM holds
  - FOSD holds $\Rightarrow$ all investors are on the MVE frontier
I. Pricing of skewness (PW), ctd.

• now introduce a small, independent, positively skewed security into the economy
  – we obtain a novel prediction: the new security earns a negative excess return
  – skewness itself is priced, in contrast to concave EU model where only coskewness with market matters

• equilibrium involves heterogeneous holdings
  (assume short-sale constraints for now)
  – some investors hold a large, undiversified position in the new security
  – others hold no position in it at all
  – heterogeneous holdings arise from non-unique global optima, not from heterogeneous preferences

• since the new security contributes skewness to the portfolios of some investors, it is valuable, and so earns a low average return
FIGURE 3. A HETEROGENEOUS HOLDINGS EQUILIBRIUM. Notes: The figure shows the utility that an investor with cumulative prospect theory preferences derives from adding a position in a positively skewed security to his current holdings of a Normally distributed market portfolio. The skewed security is highly skewed. The variable $x$ is the fraction of wealth allocated to the skewed security relative to the fraction of wealth allocated to the market portfolio. The two lines correspond to different mean returns on the skewed security.
I. Pricing of skewness (PW), ctd.

• this only works if the new security is highly skewed
  – otherwise, would need too undiversified a position in order to add skewness to the portfolio

• results hold:
  – even if there are many skewed securities
  – even if short sales are allowed
  – qualitatively, even if EU agents are present
FIGURE 4. A HOMOGENEOUS HOLDINGS EQUILIBRIUM. Notes: The figure shows the utility that an investor with cumulative prospect theory preferences derives from adding a position in a positively skewed security to his current holdings of a Normally distributed market portfolio. The skewed security is only moderately skewed. The variable $x$ is the fraction of wealth allocated to the skewed security relative to the fraction of wealth allocated to the market portfolio. The three lines correspond to different mean returns on the skewed security.
FIGURE 5. SKEWNESS AND EXPECTED RETURN. Notes: The figure shows the expected return in excess of the risk-free rate earned by a small, independent, positively skewed security in an economy populated by cumulative prospect theory investors, plotted against a parameter of the security’s return distribution, $q$, which determines the security’s skewness. A low value of $q$ corresponds to a high degree of skewness.
Applications

• low average return on IPOs
  – IPO returns are highly positively skewed

• under-diversification
  – Mitton and Vorkink (2007) find that undiversified individuals hold stocks that are more positively skewed than the average stock

• diversification discount
  – Mitton and Vorkink (2008)

• low average return of volatile stocks (Ang et al., 2006)

• other:
  – low average return to “private equity”
  – low average return on distressed stocks
  – “over-pricing” of out-of-the-money options
I. Pricing of skewness (PW), ctd.

- a clear prediction of the model is that skewness should be priced in the cross-section of assets
- some recent papers test this idea, and find support for the prediction
  - Boyer, Mitton, Vorkink (2009)
  - Conrad, Dittmar, Ghysels (2009)
  - Zhang (2005)
  - Green and Hwang (2009)
  - Bali, Cakici, Whitelaw (2010)
II. Disposition effect (CC)

• Odean (1998) studies the trading activity, from 1987-1993, of 10,000 households with accounts at a large discount brokerage firm

• he finds that, when individual investors sell a stock, they have a greater propensity to sell one that has risen in value since purchase, rather than fallen in value

  – the “disposition effect”
II. Disposition effect (CC)

How exactly does Odean (1998) measure this?

• whenever an investor sells shares of a stock, classify each of the stocks in his portfolio on that day as one of:
  – “realized gain”, “realized loss”, “paper gain”, or “paper loss”

• add up total number of realized gains and losses and paper gains and losses over all accounts over the sample, and compute:

\[
PGR = \frac{\text{no. of realized gains}}{\text{no. of realized gains} + \text{no. of paper gains}}
\]

\[
PLR = \frac{\text{no. of realized losses}}{\text{no. of realized losses} + \text{no. of paper losses}}
\]

(e.g. PGR is “proportion of gains realized”)

• the disposition effect is the finding that \( PGR > PLR \)
  – specifically, \( 0.148 = PGR > PLR = 0.098 \)
II. Disposition effect (CC), ctd.

The most obvious potential explanations fail to capture important features of the data

- e.g. informed trading
  - the subsequent return of winners that people sell is *higher* than that of losers they hold on to
- e.g. taxes, rebalancing, transaction costs
  - an important piece of evidence is that the disposition effect is stronger among *less* sophisticated individual investors (Dhar and Zhu, 2006)
II. Disposition effect (CC), ctd.

Two non-standard hypotheses have gained prominence

- an irrational belief in mean-reversion
- an explanation based on prospect theory

- prospect theory, in combination with stock-level narrow framing, does appear, at first sight, to offer an explanation of the disposition effect
- but it turns out that we need to be careful how we implement prospect theory
  - prospect theory defined over *annual* stock-level trading profits does *not* generate a disposition effect
  - Barberis and Xiong (2009), “What Drives the Disposition Effect?...”
II. Disposition effect (CC), ctd.

• consider a simple portfolio choice setting

  – $T + 1$ dates: $t = 0, 1, \ldots, T$
  – a risk-free asset, gross return $R_f$ each period
  – a risky asset with an i.i.d binomial distribution across periods:

$$R_{t,t+1} = \begin{cases} R_u > R_f & \text{with probability } \frac{1}{2} \\
R_d < R_f & \text{with probability } \frac{1}{2} \end{cases}, \text{i.i.d.}$$

• the investor has prospect theory preferences defined over his “gain/loss”

  – simplest definition of gain/loss is trading profit between 0 and $T$, i.e. $W_T - W_0$
  – we use $W_T - W_0 R_f^T$
  – call $W_0 R_f^T$ the “reference” wealth level
II. Disposition effect (CC), ctd.

The investor therefore solves

$$\max_{x_0,x_1,\ldots,x_{T-1}} E[v(\Delta W_T)] = E[v(W_T - W_0 R^T_f)]$$

where

$$v(x) = \begin{cases} x^\alpha & \text{for } x \geq 0 \\ -\lambda (-x)^\alpha & \text{for } x < 0 \end{cases}$$

subject to

$$W_t = (W_{t-1} - x_{t-1} P_{t-1}) R_f + x_{t-1} P_{t-1} R_{t-1,t}$$

$$W_T \geq 0$$

(Note that we ignore probability weighting here)

- using the Cox-Huang (1989) methodology, can derive an analytical solution for any number of trading periods
II. Disposition effect (CC), ctd.

Results

• the investor usually exhibits the opposite of the disposition effect
  – only when $T$ is high and $\mu$ is low does he exhibit a disposition effect

• for $T = 2$ and for the Tversky and Kahneman (1992) parameterization, he always exhibits the opposite of the disposition effect
II. Disposition effect (CC), ctd.

Why does the disposition effect always fail in the two-period case for the TK parameters? \((t = 0, 1, 2)\)

- for the investor to buy the stock at time 0, in spite of his loss aversion, it must have a high expected return
  - this implies that the time 1 gain is larger than the time 1 loss in magnitude
  - it also implies that, after a time 1 gain, the investor gambles to the edge of the concave region (\(v(\cdot)\) is only mildly concave over gains)
  - after a time 1 loss, the investor gambles to the edge of the convex region

- but it takes a larger position to gamble to the edge of the concave region after the time 1 gain than it does to gamble to the edge of the convex region after the time 1 loss

\(\Rightarrow\) the investor takes more risk after a gain than after a loss, contrary to the disposition effect
Time 1 and time 2 gains/losses plotted on the value function

---

45
II. Disposition effect (CC), ctd.

So how *can* we explain the disposition effect?

- one idea is to apply prospect theory to *realized* gains and losses
  - i.e. to assume “realization utility” (Shefrin and Statman, 1985)
- e.g. if you buy a stock at $40 and sell it at $60
  - you get a jolt of positive utility *at the moment of sale*, based on the size of the realized gain
- prospect theory applied to realized gains and losses *does* predict a disposition effect more reliably (Barberis and Xiong, 2009)
- realization utility may be a consequence of “mental accounting”
  - people often think about their investing history as a series of investing episodes or stories
  - when an investor sells an asset at a gain, he feels good because he is creating a *positive* new investing episode
II. Disposition effect (CC), ctd.

• Weber and Camerer (1995) provide useful experimental support for the realization utility view of the disposition effect

• in a laboratory setting, they ask subjects to trade six stocks over a number of periods

  – each stock has some probability of going up in each period, ranging from 0.35 to 0.65
  – subjects are not told which stock is associated with which up-move probability

• subjects exhibit a disposition effect

• more interestingly, in one condition, the experimenter liquidates subjects’ holdings and then allows them to reallocate however they like

  – subjects do not re-establish their positions in prior losers
II. Disposition effect (CC), ctd.

- Barberis and Xiong (2010), “Realization Utility,” study *linear* realization utility, coupled with a positive time discount factor
  - an infinite horizon, continuous-time model with a risk-free asset and many risky stocks
  - at each moment of time, the investor must allocate all of his wealth either to the risk-free asset or to one of the stocks
  - if he sells a stock, he derives utility from the realized gain or loss

*Solution*

- if the expected return on stocks is too low, the agent invests in the risk-free asset forever

- if the expected return on stocks is high enough, he buys a stock at time 0
  - and sells it only if its value rises a certain percentage amount above purchase price
  - he then immediately reinvests in another stock, and so on
II. Disposition effect (CC), ctd.

• applications:
  – the disposition effect
  but also:
  – “excessive trading”
  – the underperformance of individual investors even before transaction costs
  – the greater turnover in bull markets
  – the greater selling propensity above historical highs
  – the negative premium to volatility in the cross-section
  – the fact that overpriced assets are also heavily traded

Note:

• although not a feature of this model, realization utility may also generate momentum in asset prices
  – Grinblatt and Han (2005), Frazzini (2006)
II. Disposition effect (CC), ctd.

Summary

• a model in which the investor derives prospect theory utility from annual trading profits does not deliver a disposition effect very reliably

• a model in which the investor derives prospect theory utility from realized gains and losses delivers a disposition effect more reliably
  – but the disposition effect follows even from linear realization utility, coupled with a positive time discount factor

Note:

• some authors have argued that we shouldn’t give up on the “annual trading profit” implementation of prospect theory
  – e.g. Meng (2009) considers an alternative reference point assumption
II. Disposition effect (CC), ctd.

• the trading models we have looked at ignore probability weighting
  – in dynamic settings, probability weighting leads to a time inconsistency
  – this may be relevant in some contexts, e.g. casinos
III. Equity premium (LA)

• a model in which investors are loss averse over annual changes in the value of their stock market holdings predicts a large equity premium
  – Benartzi and Thaler (1995)

• three elements:
  – loss aversion
  – annual evaluation
  – narrow framing

• Benartzi and Thaler (1995) emphasize the first two elements
  – “myopic loss aversion”
III. Equity premium (LA), ctd.

- to make the argument more rigorous, need to embed it in a dynamic setting
  - e.g. so that preferences include a “utility of consumption” term alongside the prospect theory term
- two ways of doing this:
  - Barberis, Huang, and Santos (2001a)
  - Barberis and Huang (2009)
- Barberis and Huang (2007) reviews both methods
III. Equity premium (LA), ctd.

Method I (Barberis, Huang, and Santos, 2001a)

• intertemporal model; three assets: risk-free ($R_{f,t}$), stock market ($R_{S,t+1}$), non-financial asset ($R_{N,t+1}$)

• representative agent maximizes:

$$E_0 \sum_{t=0}^{\infty} \left[ \rho^t \frac{C_t^{1-\gamma}}{1-\gamma} + b_0 \rho^{t+1} C_t^{-\gamma} v(G_{S,t+1}) \right]$$

$$G_{S,t+1} = \theta_{S,t}(W_t - C_t)(R_{S,t+1} - 1)$$

$$v(x) = \begin{cases} 
  x & \text{for } x \geq 0 \\
  \lambda x & \text{for } x < 0, \lambda > 1 
\end{cases}$$

- frame stock market narrowly
- $v(\cdot)$ captures only loss aversion

• for “reasonable” parameters, get substantial equity premium
III. Equity premium (LA), ctd.

- three elements in the Benartzi and Thaler (1995) hypothesis
  - loss aversion, annual evaluation, narrow framing
  - the term “myopic loss aversion” emphasizes the first two ingredients
- but narrow framing is just as critical
  - loss aversion over total wealth fluctuations doesn’t produce as large an equity premium
  - the “loss aversion / narrow framing” approach?
  - how do we justify the narrow framing?
III. Equity premium (LA), ctd.

Method II (Barberis and Huang, 2009)

- start from standard recursive specification
  \[ V_t = H(C_t, \mu(V_{t+1})) \]
  \[ W(C, x) = ((1 - \beta)C^\rho + \beta x^\rho)^\frac{1}{\rho}, \quad 0 < \beta < 1, \quad 0 \neq \rho < 1 \]
  \[ \mu(x) = (E(x^\zeta))^\frac{1}{\zeta} \]

- can adjust this to incorporate narrow framing
  \[ V_t = H\left(C_t, \mu(V_{t+1}) + b_{i,0} \sum_i E_t(v(G_{i,t+1})) \right) \]

- in 3-asset context from before:
  \[ V_t = H\left(C_t, \mu(V_{t+1}) + b_0 E_t(v(G_{S,t+1})) \right) \]
  \[ G_{S,t+1} = \theta_{S,t}(W_t - C_t)(R_{S,t+1} - 1) \]
  \[ v(x) = \begin{cases} x & \text{for } x \geq 0 \\ \lambda x & \text{for } x < 0, \quad \lambda > 1 \end{cases} \]
  \[ \zeta = \rho \]
III. Equity premium (LA), ctd.

- this specification is better than Method I
  - it is tractable in partial equilibrium
  - it admits an explicit value function $\Rightarrow$ easy to check attitudes to monetary gambles
  - it does not require aggregate consumption scaling $\bar{C}$

- can now show that for parameter values that predict reasonable attitudes to large and small-scale monetary gambles, get substantial equity premium
III. Equity premium (LA), ctd.

- Barberis, Huang, and Santos (2001a) also build in dynamic aspects of loss aversion
  - “house money effect”
  - generates high volatility and time-series predictability in addition to a high equity premium
- but dynamic risk-taking phenomena are still poorly understood
  - e.g. after a prior loss, do we see more or less risk-taking?
IV. Stock market non-participation (LA)

- the combination of loss aversion and narrow framing may shed light on other puzzling phenomena
  - in particular, on examples of under-diversification

- potential applications:
  - stock market non-participation (Barberis, Huang, and Thaler, 2006)
  - low number of stocks held directly
  - home bias

- narrow framing is crucial in all these applications
  - e.g. loss aversion over total wealth does not predict stock market non-participation

- even a loss averse agent would want some position in equities
  - since the stock market has a low correlation with other household risks, it offers useful diversification
Summary

• probability weighting $\Rightarrow$ pricing of skewness
  – no narrow framing needed
• concavity/convexity of value function $\Rightarrow$ disposition effect (sometimes!)
  – but probably need realization utility
• loss aversion $\Rightarrow$ equity premium, under-diversification
  – need narrow framing

Future work?

• test prospect theory hypotheses for various facts
• use theoretical analysis to develop new predictions