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Preferences with frames: A new utility specification that allows for the framing of risks

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ABSTRACT

Experiments on decision-making show that, when people evaluate risk, they often engage in "narrow framing": that is, in contrast to the prediction of traditional utility functions defined over wealth or consumption, they often evaluate risks in isolation, separately from other risks they are already facing. While narrow framing has many potential real-world applications, there are almost no tractable preference specifications that incorporate it into the standard framework used by economists. In this paper, we propose such a specification and demonstrate its tractability in both portfolio choice and equilibrium settings.

1. Introduction

When economists model the behavior of an individual agent, they typically use utility functions defined over wealth or consumption. Such utility functions make a precise prediction as to how the agent evaluates a new gamble he is offered: he merges the new gamble with other risks he is already facing to determine its effect on the distribution of his future wealth or consumption, and then checks if the new distribution is an improvement.

Experiment-based research on decision-making under risk has uncovered many instances in which people do not appear to evaluate gambles in this way: instead of merging a new gamble with other risks they are already facing and checking if the combination is attractive, they often evaluate the new gamble in isolation, separately from their other risks. This is known as "narrow framing" (Kahneman and Lovallo, 1993; Kahneman, 2003). More formally, narrow framing means that, when an agent is deciding whether to accept a gamble, he uses a utility function that depends *directly* on the outcome of the gamble, not just indirectly via the gamble's contribution to his total wealth.

While narrow framing has been documented most clearly in experimental settings, it may also play a role outside the laboratory. In particular, there are numerous real-world situations in which people appear to neglect simple opportunities for diversification. Stock market non-participation—the fact that, historically, many U.S. households did not allocate any of their wealth to the stock market even though equity is relatively uncorrelated with other household risks—is one example

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(Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995; Heaton and Lucas, 2000). Another is home bias—the fact that many households do not diversify what holdings of domestic stock they do have with even a small position in foreign stock (French and Poterba, 1991). Yet another example is the large fraction of wealth some households allocate to just a few stocks (Curcuru et al., 2004).

At least in the absence of frictions, it is hard to explain such behavior based on utility functions defined only over wealth or consumption. Investors who care about the riskiness of their overall wealth are generally keen to take advantage of opportunities for diversification. An investor who evaluates risks in isolation, however, misses these opportunities and may therefore fail to exploit them. For example, an investor who evaluates an individual stock in isolation will fail to notice its diversification benefits; he will therefore be less willing to buy it, leaving him with a portfolio made up of too few stocks. Similarly, an investor who thinks about foreign stock market risk in isolation will fail to notice its diversification benefits and is therefore more likely to exhibit home bias.

These potential applications suggest that it may be useful to study the predictions of narrow framing more rigorously. It has not been easy to do this, however, because there are almost no tractable preference specifications that allow for narrow framing. The development of such preferences faces many hurdles. The specification should allow the agent to derive utility directly from the outcome of a specific gamble he is taking, but also, as in traditional models, to derive utility from total wealth or consumption. In other words, to be realistic, it must allow for both narrow and traditional "broad" framing. Moreover, it must be tractable even in the dynamic settings favored by economists.

In this paper, we present an intertemporal preference specification that meets these requirements. In our framework, the agent gets utility from a gamble's outcome both indirectly, via its contribution to his total wealth, but also directly. The specification is tractable, both in partial equilibrium, thereby allowing us to study portfolio and consumption choice, but also in equilibrium, thereby allowing us to study the effect of narrow framing on asset prices. We present examples of both kinds of analysis. We also show that, in our framework, the agent's indirect value function takes a simple form. This makes it easy to calibrate our preferences by checking the agent's attitude to timeless monetary gambles.

To explore applications of narrow framing, a researcher needs two things: a tractable preference specification that allows for narrow framing; and a theory of which risks agents will frame narrowly. In this paper, we focus on the first component, in other words, on developing a tractable preference specification. The second component is equally important, but is not the focus of our study. When necessary, we will rely on an existing theory of framing, namely Kahneman's (2003) "accessibility" theory. We describe this theory in Section 2.

One previous attempt to incorporate narrow framing into standard preferences is that of Barberis et al. (2001), who study the pricing of the aggregate stock market when investors derive utility directly from stock market fluctuations. Since the stock market is only one component of overall wealth, the authors are assuming that investors frame narrowly and their preference specification reflects this.

While Barberis et al.s' (2001) specification is tractable in equilibrium settings, it also has some limitations. First, it is intractable in partial equilibrium and so cannot be used to study the heterogeneity in household portfolios. Second, the agent's indirect value function cannot be computed explicitly, making it hard to calibrate the utility function by checking attitudes to timeless monetary gambles. Finally, to ensure stationarity in equilibrium, the narrow framing term in the preference specification has to be scaled by aggregate consumption in an ad hoc way.

The preference specification that we present in this paper improves on that of Barberis et al. (2001). Our preferences *are* tractable in partial equilibrium; they *do* admit an explicit value function; and they do *not* require any ad hoc scaling. Even in equilibrium settings, where Barberis et al.s' (2001) specification is tractable, our formulation offers an advantage: since it leads to an explicit value function, it allows the researcher to check whether the parameter values used in any particular application are *reasonable*, in terms of predicting sensible attitudes to timeless monetary gambles.

Barberis et al. (2006) apply the preferences that we develop here in an analysis of the stock market participation puzzle, while Barberis and Huang (2007) apply them in a study of the equity premium. The distinct contribution of this paper is the formal derivation of the equations that govern portfolio choice, asset pricing, and attitudes to timeless monetary gambles. For example, it is in this paper that we derive the first-order conditions for optimal consumption and portfolio choice; and it is in this paper that we show how our specification can be incorporated into a full equilibrium setting.

In Section 2, we review some experimental evidence on narrow framing. In Section 3, we present a preference specification that allows for narrow framing. Section 4 specifies a general portfolio problem, derives the first-order conditions of optimality, and presents an example. Section 5 explains how an agent with our preferences evaluates timeless monetary gambles while Section 6 shows how our specification can be used in an equilibrium setting. Section 7 concludes.

2. Narrow framing

Before presenting any formal analysis, we first review some of the experimental evidence on narrow framing. The classic demonstration is due to Tversky and Kahneman (1981) who ask 150 subjects the following question[1]:

[1] For more evidence and discussion of narrow framing, see Kahneman and Tversky (1983), Tversky and Kahneman (1986), Redelmeier and Tversky (1992), Read et al. (1999), and Rabin and Thaler (2001).

Imagine that you face the following pair of concurrent decisions. First examine both decisions, then indicate the options you prefer:

Choice (I) Choose between:

(A) a sure gain of \$240.
(B) 25% chance to gain \$1,000 and 75% chance to gain nothing.

Choice (II) Choose between:

(C) a sure loss of \$750.
(D) 75% chance to lose \$1,000 and 25% chance to lose nothing.

Tversky and Kahneman (1981) report that 84% of subjects chose (A), with only 16% choosing (B), and that 87% chose (D), with only 13% choosing (C). In particular, 73% of subjects chose the combination (A) and (D), namely

$$\text{25\% chance to win \$240}, \quad \text{75\% chance to lose \$760}, \tag{1}$$

which is surprising given that this choice is dominated by the combination (B) and (C), namely

$$\text{25\% chance to win \$250}, \quad \text{75\% chance to lose \$750}. \tag{2}$$

It appears that, instead of focusing on the *combined* outcome of decisions (I) and (II)—in other words, on the outcome that determines their final wealth—subjects are focusing on the outcome of each decision separately. Indeed, subjects who are asked *only* about decision (I) do overwhelmingly choose (A); and subjects asked *only* about decision (II) do overwhelmingly choose (D).

In more formal terms, we cannot model the typical subject as maximizing a utility function defined only over total wealth. Rather, his utility function appears to depend *directly* on the outcome of each of decisions (I) and (II), rather than just indirectly via the contribution of each decision to overall wealth. As such, this is an example of narrow framing.

More recently, Barberis et al. (2006) argue that the rejection of the gamble

$$(110, \tfrac{1}{2}; -100, \tfrac{1}{2}),$$

to be read as "gain \$110 with probability $\frac{1}{2}$, lose \$100 with probability $\frac{1}{2}$, independent of other risks," observed by Tversky and Kahneman (1992) in a majority of their subjects, may also be evidence of narrow framing. They reason that the subjects who are offered this gamble are typically already facing *other* kinds of risk—labor income risk, housing risk, or financial market risk, say. In the absence of narrow framing, they must therefore evaluate the 110/100 gamble by mixing it with these other risks and then checking if the combination is attractive. In general, the combination *is* attractive: since the 110/100 gamble is independent of other risks, it offers useful diversification benefits, which, even if "first-order" risk averse in the sense of Segal and Spivak (1990), people find appealing. The observed rejection of the 110/100 gamble therefore suggests that subjects are *not* fully merging the gamble with their other risks, but that, to some extent, they are evaluating it in isolation; in other words, that they are framing it narrowly.

As noted in the Introduction, our goal is not to provide a theory of when narrow framing occurs and when it does not, but rather to provide tools for exploring any specific hypothesis that posits the narrow framing of certain risks. Nevertheless, it may be helpful, as we go through the formal analysis, to keep at least one theory of framing in mind.

Kahneman (2003) presents one possible theory of framing. He argues that narrow framing occurs when decisions are made intuitively rather than through effortful reasoning. Since intuitive thoughts are by nature spontaneous, they are shaped by the features of the situation at hand that come to mind most easily; to use the technical term, by the features that are most "accessible." When an agent is offered a new gamble, the distribution of the gamble, considered separately, is often more accessible than the distribution of his overall wealth once the new gamble has been merged with his other risks. As a result, if the agent thinks about the gamble intuitively, the distribution of the gamble, taken alone, may play a more important role in his decision-making than would be predicted by traditional utility functions defined only over wealth or consumption.

In Tversky and Kahneman's (1981) example, the outcome of each of choices (A), (B), (C), or (D) is highly accessible. Less accessible, though, is the *overall* outcome once two choices—(A) and (D), say, or (B) and (C)—are combined: the distributions in (1) and (2) are less "obvious" than the distributions of (A), (B), (C), and (D). As a result, if subjects use their intuition when responding, the outcome of each of choices (I) and (II) may play a bigger role in their decision-making than predicted by traditional utility functions. Similar reasoning applies in the case of the 110/100 gamble.[2]

Narrow framing can also stem from fully rational considerations: for example, from the desire to take non-consumption utility such as regret into account. Regret is the pain we feel when we realize that we would be better off today if we had

[2] Rabin and Weizsacker (2009) show that, if the implications of each pair of choices in Tversky and Kahneman's (1981) experiment are pointed out, subjects no longer choose a dominated pair. When the implications of each pair of choices are pointed out, those implications are suddenly accessible, to use Kahneman's (2003) term. Subjects therefore now act as if they derive utility from the outcome of a pair of choices, rather than from the outcome of each choice separately. As a result, they no longer choose a dominated alternative.

taken a different action in the past. Even if a gamble that an agent accepts is just one of many risks that he faces, it is still linked to a specific decision, namely the decision to accept the gamble. As a result, it exposes the agent to possible future regret: if the gamble turns out badly, he may regret the decision to accept it. Consideration of non-consumption utility can therefore also lead to preferences that depend *directly* on the outcomes of specific gambles that the agent faces.

The fact that narrow framing occurs in experimental settings does not necessarily mean that it also occurs in real-world financial settings. Real-world decisions often involve stakes much larger than those in the laboratory. An actual investor therefore has a greater incentive than does a laboratory subject to figure out the right course of action and to avoid narrow framing, if narrow framing is indeed a mistake. For example, he might consult a financial planner who, presumably, will try to prevent him from framing decisions narrowly.

At the same time, there are reasons to think that narrow framing may occur even when stakes are high. As noted above, narrow framing is thought to stem, in part, from the use of intuition. An investor who engages in narrow framing, then, is doing what feels intuitively right. As such, he may see no reason to reconsider his actions or to seek help from a financial planner. Even if he does consult a financial planner and is advised to change his behavior, he may be reluctant to do so—again, what he is doing feels intuitively right, while the financial planner's advice may, to some extent, feel counterintuitive. As a result, he may stick to his original behavior.

Another reason why narrow framing may occur even when stakes are high is because, as noted earlier, at least some part of narrow framing probably stems from *rational* considerations such as the desire to take potential regret into account. If narrow framing has some rational underpinnings, we can expect it to play a role even when stakes are high.

Ultimately, the question of whether narrow framing occurs in the real world cannot be answered by introspection. It can only be settled by deriving the predictions of narrow framing and then testing those predictions. But in order to derive the predictions of narrow framing, we need a preference specification that incorporates it in a tractable way. We now present such a specification.

3. A preference specification that allows for narrow framing

We work in discrete time throughout. At time t, the agent, whose wealth is denoted W_t, chooses a consumption level C_t and allocates his post-consumption wealth, $W_t - C_t$, across n assets, one of which may be risk-free. His wealth therefore evolves according to

$$\widetilde{W}_{t+1} = (W_t - C_t)\left(\sum_{i=1}^{n} \theta_{i,t}\widetilde{R}_{i,t+1}\right) \equiv (W_t - C_t)\widetilde{R}_{W,t+1}, \tag{3}$$

where $\theta_{i,t}$ is the fraction of post-consumption wealth allocated to asset i, $\widetilde{R}_{i,t+1}$ is the gross return on asset i between time t and $t+1$, and $\widetilde{R}_{W,t+1}$ is the gross return on wealth over the same interval.

We can think of each of the n assets available to the agent as a "gamble." His gamble in asset i, for example, consists of putting down capital of $\theta_{i,t}(W_t - C_t)$ at time t and receiving an uncertain payoff of $\theta_{i,t}(W_t - C_t)\widetilde{R}_{i,t+1}$ at time $t+1$. We want to allow for the possibility that the agent frames one or more of these n gambles narrowly; in other words, that he gets utility from their outcomes directly, not just indirectly via their contribution to total wealth. How can this be modeled?

A useful starting point for developing preferences that allow for narrow framing is recursive utility in which the agent's time t utility, V_t, is given by

$$V_t = H(C_t, \mu(\widetilde{V}_{t+1}|I_t)), \tag{4}$$

where $\mu(\widetilde{V}_{t+1}|I_t)$ is the certainty equivalent of the distribution of future utility $\widetilde{V}_{t+1}$ conditional on time t information I_t, and $H(\cdot,\cdot)$ is an aggregator function which aggregates current consumption C_t with the certainty equivalent of future utility to give current utility (see Epstein and Zin, 1989, for a detailed discussion). Most implementations of recursive utility assign $H(\cdot,\cdot)$ the form

$$H(C,x) = ((1-\beta)C^{\rho} + \beta x^{\rho})^{1/\rho}, \quad 0<\beta<1, \ 0\neq\rho<1, \tag{5}$$

and assume that $\mu(\cdot)$ is homogeneous. If a certainty equivalent functional is homogeneous, it is necessarily homogeneous of degree one, so that

$$\mu(k\widetilde{x}) = k\mu(\widetilde{x}), \quad \forall k>0. \tag{6}$$

The specification in (4) does not allow for narrow framing: an agent with these preferences cares about the outcome of a gamble he is offered only to the extent that the outcome affects his overall wealth risk. These preferences can, however, be naturally extended to allow for narrow framing. Suppose that the agent frames $n-m$ of the n assets narrowly—specifically, assets $m+1$ through n. In terms of Kahneman's (2003) accessibility theory of framing, asset n, say, is framed narrowly because information about the distribution of its future returns is very accessible; in particular, more accessible than information about the distribution of the agent's overall wealth once a position in asset n is added to his holdings of other assets. The fact that the distribution of asset n's returns is so accessible means that it plays a larger role in the agent's decision-making than traditional utility functions would suggest.

We propose that narrow framing of this kind can be captured by the following preference specification:

$$V_t = H\left(C_t, \mu(\widetilde{V}_{t+1}|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\widetilde{G}_{i,t+1}))\right), \tag{7}$$

where b_0 is non-negative and where

$$H(C, x) = ((1-\beta)C^\rho + \beta x^\rho)^{1/\rho}, \quad 0<\beta<1, \ 0 \neq \rho < 1, \tag{8}$$

$$\mu(k\widetilde{x}) = k\mu(\widetilde{x}), \quad \forall k>0, \tag{9}$$

$$\widetilde{G}_{i,t+1} = \theta_{i,t}(W_t - C_t)(\widetilde{R}_{i,t+1} - R_{i,z}), \quad i = m+1, \ldots, n, \tag{10}$$

$$\overline{v}(x) = \begin{cases} x & \text{for } x \geqslant 0, \\ \lambda x & x<0, \end{cases} \quad \lambda > 1. \tag{11}$$

The aggregator function $H(\cdot,\cdot)$ and the certainty equivalent function $\mu(\cdot)$ are the same as in the usual recursive specification in (4)–(6). The difference is that we now add $n-m$ new terms to the second argument of $H(\cdot,\cdot)$, one for each of the $n-m$ assets that the agent is framing narrowly. For example, $\widetilde{G}_{n,t+1}$ is the specific gamble the agent is taking by investing in asset n. By adding the new term $b_0 E_t(\overline{v}(\widetilde{G}_{n,t+1}))$, we allow the agent to get utility directly from the outcome of this gamble, rather than just indirectly via its contribution to next period's wealth. In less formal terms, the agent now evaluates asset n in isolation, to some extent.

The simplest way to express the gamble the agent is taking by investing in asset n, say, is

$$\widetilde{G}_{n,t+1} = \theta_{n,t}(W_t - C_t)(\widetilde{R}_{n,t+1} - 1),$$

the amount invested in the asset, $\theta_{n,t}(W_t - C_t)$, multiplied by its net return, $\widetilde{R}_{n,t+1} - 1$. This corresponds to Eq. (10) with $R_{i,z} = 1$. In this case, so long as $\theta_{n,t} > 0$, a positive net return is considered a gain and, from (11), is assigned positive utility; a negative net return is considered a loss and is assigned negative utility.

By using the more general specification in Eq. (10), we allow for some flexibility as to what counts as a gain—in other words, as to what kind of gamble outcome is assigned positive utility. Treating any positive net return as a gain and any negative net return as a loss—in other words, setting $R_{i,z} = 1$—is one possibility, but another that we consider later sets $R_{i,z} = R_{f,t}$, where $R_{f,t}$ is the gross risk-free return between time t and $t+1$. In this case, an asset's return is only considered a gain and hence is only assigned positive utility if it exceeds the risk-free rate.[3]

The next task is to specify the utility $\overline{v}(\cdot)$ the agent receives from narrowly framed gains and losses. We propose the piecewise-linear specification in (11). There are at least two motivations for this. The first is tractability. One way to increase tractability is to impose homotheticity. Since $\mu(\cdot)$ is homogeneous of degree one, homotheticity obtains so long as $\overline{v}(\cdot)$ is *also* homogeneous of degree one. At the same time, to ensure that the first-order conditions for the agent's decision problem are both necessary and sufficient for optimality, we need $\overline{v}(\cdot)$ to be concave. The only function that is both homogeneous of degree one and concave is precisely the piecewise-linear function in (11).

A second motivation for Eq. (11) is that $\overline{v}(\cdot)$ should be modeled as closely as possible on Kahneman and Tversky's (1979) prospect theory—a descriptive theory, based on extensive experimental evidence, of decision-making under risk. The reason is that, just as narrow framing is associated with intuitive thinking, so prospect theory is also often associated with intuitive thinking (Kahneman, 2003). Narrow framing and prospect theory therefore form a natural pair.[4]

Eqs. (10) and (11) show that we have adopted two of the main features of prospect theory in our specification of $\overline{v}(\cdot)$: outcomes are described in terms of gains and losses relative to a reference return $R_{i,z}$ and the agent is more sensitive to losses than to gains. The other elements of prospect theory—the concavity (convexity) of the value function in the region of gains (losses) and the probability weighting function—are more difficult to incorporate because they can lead to risk-seeking. The first-order conditions for the agent's decision problem are then no longer sufficient for optimality.

Just as prospect theory and narrow framing form a natural pair, so expected utility and broad framing may also form a natural pair: they are both associated with effortful reasoning rather than intuitive thinking. The $\mu(\widetilde{V}_{t+1}|I_t)$ term in Eq. (7) does not involve any narrow framing. When we later specify a form for $\mu(\cdot)$, it may therefore be more natural to specify an expected utility form.

The parameter b_0 controls the degree of narrow framing. A b_0 of 0 means no narrow framing at all while a large b_0 means that the agent is evaluating each of assets $m+1$ through n almost entirely in isolation. For simplicity, we take the

[3] Since $R_{i,z}$ determines whether a particular outcome is treated as a gain or as a loss, it is what the literature on decision-making calls a "reference point." An ongoing research effort is trying to understand what determines the reference points that people use in practice (Koszegi and Rabin, 2007).

[4] Some evidence consistent with this is that, in those experimental settings where people appear to be evaluating a gamble in isolation, they also appear to be using prospect theory to decide whether to accept the gamble. For example, the experiment of Tversky and Kahneman (1981) discussed in Section 2 points not only to narrow framing but also, through the preference for (A) over (B) and for (D) over (C), to risk aversion over gains and risk-seeking over losses, mirroring the prospect theory value function's concavity (convexity) in the region of gains (losses). Similarly, the rejection of the 110/100 gamble suggests not only narrow framing but also a greater sensitivity to losses than to gains, in line with the kink in the prospect theory value function.

degree of narrow framing to be the same for all $n-m$ assets, but our analysis extends easily to the more general case where

$$V_t = H\left(C_t, \mu(\widetilde{V}_{t+1}|I_t) + \sum_{i=m+1}^{n} b_{i,0} E_t(\overline{v}(\widetilde{G}_{i,t+1}))\right).$$

Finally, we note that the preferences in (7)–(11) are dynamically consistent. Today, the agent knows how he will frame future gains and losses and what function $\overline{v}(\cdot)$ he will apply to those narrowly framed gains and losses. Moreover, he takes all of this into account when making today's decisions. Standard dynamic programming techniques can therefore be applied and dynamic consistency follows.

4. The consumption–portfolio problem

In this section, we derive the first-order conditions for optimal consumption and portfolio choice. We then demonstrate the tractability of our framework by solving a simple portfolio problem.

The Bellman equation that corresponds to (7) is

$$\begin{aligned} V_t = J(W_t, I_t) &= \max_{C_t,\theta_t} H\left(C_t, \mu(J(\widetilde{W}_{t+1}, I_{t+1})|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\widetilde{G}_{i,t+1}))\right) \\ &= \max_{C_t,\theta_t}\left[(1-\beta)C_t^{\rho} + \beta\left[\mu(J(\widetilde{W}_{t+1}, I_{t+1})|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\widetilde{G}_{i,t+1}))\right]^{\rho}\right]^{1/\rho}. \end{aligned}$$

Eqs. (8)–(11) show that the aggregator function $H(\cdot,\cdot)$, the certainty equivalent function $\mu(\cdot)$, the gambles $\{\widetilde{G}_{i,t+1}\}_{i=m+1}^{n}$, and the utility function for narrowly framed gains and losses $\overline{v}(\cdot)$ are all homogeneous of degree one. This implies[5]

$$J(W_t, I_t) = A(I_t)W_t \equiv A_t W_t, \tag{12}$$

so that

$$A_t W_t = \max_{C_t,\theta_t}\left[(1-\beta)C_t^{\rho} + \beta(W_t - C_t)^{\rho}\left[\mu(A_{t+1}\theta_t'\widetilde{R}_{t+1}|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1} - R_{i,z})))\right]^{\rho}\right]^{1/\rho}, \tag{13}$$

where

$$\theta_t = (\theta_{1,t}, \ldots, \theta_{n,t})', \quad \widetilde{R}_t = (\widetilde{R}_{1,t}, \ldots, \widetilde{R}_{n,t})'.$$

Eq. (13) shows that the consumption and portfolio decisions are separable. In particular, the portfolio problem is

$$B_t^* = \max_{\theta_t}\left[\mu(A_{t+1}\theta_t'\widetilde{R}_{t+1}|I_t) + b_0 \sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1} - R_{i,z})))\right], \tag{14}$$

and after defining

$$\alpha_t \equiv \frac{C_t}{W_t},$$

the consumption problem becomes

$$A_t = \max_{\alpha_t}[(1-\beta)\alpha_t^{\rho} + \beta(1-\alpha_t)^{\rho}(B_t^*)^{\rho}]^{1/\rho}. \tag{15}$$

The first-order condition for optimal consumption choice α_t^* is

$$(1-\beta)(\alpha_t^*)^{\rho-1} = \beta(1-\alpha_t^*)^{\rho-1}(B_t^*)^{\rho}, \tag{16}$$

and the second-order condition confirms that Eq. (16) is not only necessary but also sufficient for a maximum. Combining Eqs. (15) and (16) gives

$$A_t = (1-\beta)^{1/\rho}(\alpha_t^*)^{1-1/\rho}, \tag{17}$$

and similarly,

$$A_{t+1} = (1-\beta)^{1/\rho}(\alpha_{t+1}^*)^{1-1/\rho},$$

[5] To see this, note that, given any time t wealth level W_t and any consumption plan $\{\widetilde{C}_\tau\}_{\tau=t,t+1\ldots}$ financed by W_t, the homogeneity assumptions imply $V_t(W_t, \{\widetilde{C}_\tau\}_{\tau=t,t+1\ldots}) = W_t V_t(1, \{\widetilde{C}_\tau/W_t\}_{\tau=t,t+1,\ldots})$. Let $A(I_t) \equiv J(1, I_t)$ be the value function of an agent with time t wealth of 1 and let $\mathscr{C}_t$ be the corresponding optimal consumption plan. Since the consumption plan $W_t\mathscr{C}_t$ is feasible for an agent with time t wealth of W_t, $J(W_t, I_t) \geqslant W_t J(1, I_t)$. But if $J(W_t, I_t) > W_t J(1, I_t)$, then the optimal consumption plan for an agent with time t wealth of W_t, when scaled down by $1/W_t$, would offer strictly higher utility than $J(1, I_t)$ for an agent with time t wealth of 1. This is a contradiction. Therefore $J(W_t, I_t) = W_t J(1, I_t)$, as in Eq. (12).

which, when substituted into (14), allows us to rewrite the portfolio problem as

$$B_t^* = \max_{\theta_t}\left[\mu((1-\beta)^{1/\rho}\alpha_{t+1}^{1-1/\rho}\theta_t'\widetilde{R}_{t+1}|I_t) + b_0\sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1}-R_{i,z})))\right]. \tag{18}$$

In Section 4.1, we present a simple example of a portfolio problem in the presence of narrow framing. For that example, and for many others a researcher might be interested in, the portfolio problem can be solved using only Eqs. (16) and (18). For some applications, though, it can be useful to lay out in full the necessary and sufficient first-order conditions for consumption and portfolio choice. We do this in Proposition 1.

Since we are primarily interested in the narrow framing component of (7), Proposition 1 restricts the form of the certainty equivalent function $\mu(\cdot)$ to the simple case of

$$\mu(\widetilde{x}) = (E(\widetilde{x}^{\zeta}))^{1/\zeta}, \quad 0 \neq \zeta < 1. \tag{19}$$

However, the same method of proof used for Proposition 1 can also be applied to other explicitly defined forms of $\mu(\cdot)$, whether expected utility or not, that satisfy the homogeneity property (9).[6]

Proposition 1. *The necessary and sufficient first-order conditions for the decision problem that maximizes* (7) *subject to* (3), (8), (10), (11) *and* (19) *are, for each t, that*

$$\left(\frac{1-\alpha_t}{\alpha_t}\right)^{1-1/\rho}\left[\beta^{1/\rho}[E_t(\alpha_{t+1}^{\zeta(1-1/\rho)}(\theta_t'\widetilde{R}_{t+1})^{\zeta})]^{1/\zeta} + b_0\left(\frac{\beta}{1-\beta}\right)^{1/\rho}\sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1}-R_{i,z})))\right] = 1 \tag{20}$$

and that there exists ψ_t *such that, for* $i = 1, \ldots n$,

$$\psi_t - (1-\beta)^{1/\rho}[E_t(\alpha_{t+1}^{\zeta(1-1/\rho)}(\theta_t'\widetilde{R}_{t+1})^{\zeta})]^{1/\zeta-1}E_t[\alpha_{t+1}^{\zeta(1-1/\rho)}(\theta_t'\widetilde{R}_{t+1})^{\zeta-1}\widetilde{R}_{i,t+1}]$$
$$\begin{cases} = b_0 1_{\{i>m\}}\mathrm{sign}(\theta_{i,t})E_t[\overline{v}(\mathrm{sign}(\theta_{i,t})(\widetilde{R}_{i,t+1}-R_{i,z}))] & \textit{for} \;\; \theta_{i,t}\neq 0, \\ \in [b_0 1_{\{i>m\}}E_t(\overline{v}(\widetilde{R}_{i,t+1}-R_{i,z})), -b_0 1_{\{i>m\}}E_t(\overline{v}(R_{i,z}-\widetilde{R}_{i,t+1}))] & \;\; \theta_{i,t} = 0. \end{cases} \tag{21}$$

Proof of Proposition 1. See the Appendix.

Eq. (20) is simply a rearrangement of the first-order condition for consumption choice in Eq. (16). Eq. (21) is the first-order condition for the portfolio problem in (18); in the Appendix, we show it to be both necessary and sufficient for optimality. The right-hand side of Eq. (21) is non-zero only if asset i is framed narrowly, in other words, only if $i > m$. Since $\overline{v}(\cdot)$ is not smooth at zero, the first-order condition takes the form of an inequality when $\theta_{i,t} = 0$.

Applications of recursive utility often consider the special case of (19) in which $\zeta = \rho$, in other words, the case in which the exponent in the certainty equivalent function is the same as the exponent in the aggregator function in (8). The corollary below presents the simplified first-order conditions that apply in this case. We use $1-\gamma$ to denote the common value of ζ and ρ.

Corollary 1. *When* $\zeta = \rho = 1-\gamma$, *the necessary and sufficient first-order conditions for the decision problem that maximizes* (7) *subject to* (3), (8), (10), (11) *and* (19) *are, for each t, that*

$$\left(\frac{1-\alpha_t}{\alpha_t}\right)^{-\gamma/(1-\gamma)}\left[\beta^{1/(1-\gamma)}\left[E_t(\alpha_{t+1}^{-\gamma}(\theta_t'\widetilde{R}_{t+1})^{1-\gamma})\right]^{1/(1-\gamma)} + b_0\left(\frac{\beta}{1-\beta}\right)^{1/(1-\gamma)}\sum_{i=m+1}^{n} E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1}-R_{i,z})))\right] = 1 \tag{22}$$

and that there exists ψ_t *such that, for* $i = 1, \ldots, n$,

$$\psi_t - (1-\beta)^{1/(1-\gamma)}[E_t(\alpha_{t+1}^{-\gamma}(\theta_t'\widetilde{R}_{t+1})^{1-\gamma})]^{\gamma/(1-\gamma)}E_t[\alpha_{t+1}^{-\gamma}(\theta_t'\widetilde{R}_{t+1})^{-\gamma}\widetilde{R}_{i,t+1}]$$
$$\begin{cases} = b_0 1_{\{i>m\}}\mathrm{sign}(\theta_{i,t})E_t[\overline{v}(\mathrm{sign}(\theta_{i,t})(\widetilde{R}_{i,t+1}-R_{i,z}))] & \textit{for} \;\; \theta_{i,t}\neq 0, \\ \in [b_0 1_{\{i>m\}}E_t(\overline{v}(\widetilde{R}_{i,t+1}-R_{i,z})), -b_0 1_{\{i>m\}}E_t(\overline{v}(R_{i,z}-\widetilde{R}_{i,t+1}))] & \;\; \theta_{i,t} = 0. \end{cases} \tag{23}$$

Conditions (21) and (23) simplify slightly when $\theta_{i,t} > 0$. In this case,

$$\mathrm{sign}(\theta_{i,t})E_t[\overline{v}(\mathrm{sign}(\theta_{i,t})(\widetilde{R}_{i,t+1}-R_{i,z}))] = E_t(\overline{v}(\widetilde{R}_{i,t+1}-R_{i,z})).$$

[6] Since the $\mu(\cdot)$ term in Eq. (7) does not involve narrow framing, it may be more natural to give it an expected utility form. As noted earlier, expected utility and broad framing form a natural pair because they are both associated with effortful reasoning as opposed to intuitive thinking.

Table 1
Parameter values for the return processes in a portfolio choice problem.

Parameter	
R_f	1.02
g_2	0.04
σ_2	0.10
g_3	0.04
σ_3	0.10
ω	0.10
$\overline{\theta}_2$	0.50

Notes: Asset 1 is riskless and earns a gross risk-free return of R_f in each period. Assets 2 and 3 are risky: g_2 and σ_2 (g_3 and σ_3) are the mean and standard deviation of the log gross return on asset 2 (asset 3); ω is the correlation of the log returns on assets 2 and 3. Finally, $\overline{\theta}_2$ is the fixed fraction of wealth held in asset 2.

4.1. An example

We now use the preceding analysis to solve a simple portfolio problem in which an investor allocates his wealth across three assets. Asset 1 is riskless and earns a constant gross risk-free return of R_f in each period. Assets 2 and 3 are risky with gross returns between time t and $t+1$ of $\widetilde{R}_{2,t+1}$ and $\widetilde{R}_{3,t+1}$, respectively, where

$$\log \widetilde{R}_{i,t+1} = g_i + \sigma_i \widetilde{\varepsilon}_{i,t+1}, \quad i = 2,3$$

and

$$\begin{pmatrix} \widetilde{\varepsilon}_{2,t} \\ \widetilde{\varepsilon}_{3,t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix} \right) \quad \text{i.i.d. over time.}$$

The investor's wealth evolves according to

$$\widetilde{W}_{t+1} = (W_t - C_t)((1 - \theta_{2,t} - \theta_{3,t})R_f + \theta_{2,t}\widetilde{R}_{2,t+1} + \theta_{3,t}\widetilde{R}_{3,t+1}),$$

where $\theta_{2,t}$ ($\theta_{3,t}$) is the fraction of post-consumption wealth allocated to asset 2 (3).

We can simplify the portfolio problem further and still demonstrate the effects of narrow framing by making one additional assumption: that the fraction of the investor's wealth in asset 2 is fixed at $\theta_{2,t} = \overline{\theta}_2$, so that the investor simply has to split the remainder of his wealth between the riskless asset and risky asset 3. We can think of asset 2 as a non-financial asset such as housing wealth or human capital and asset 3 as the domestic stock market. In this case, given a fixed position in the non-financial asset, the investor is thinking about how to allocate the rest of his wealth between the risk-free asset and a risky stock market. Alternatively, asset 2 could be domestic stock and asset 3, foreign stock.

We investigate what happens if, in making this decision, the investor frames asset 3 narrowly, so that his preferences are given by

$$V_t = H(C_t, \mu(\widetilde{V}_{t+1}) + b_0 E_t(\overline{v}(\widetilde{G}_{3,t+1}))), \tag{24}$$

$$H(C,x) = ((1-\beta)C^{1-\gamma} + \beta x^{1-\gamma})^{1/(1-\gamma)}, \quad 0<\beta<1, \ 0<\gamma\neq 1,$$
$$\mu(\widetilde{x}) = (E(\widetilde{x}^{1-\gamma}))^{1/(1-\gamma)}, \quad 0<\gamma\neq 1,$$
$$\widetilde{G}_{3,t+1} = \theta_{3,t}(W_t - C_t)(\widetilde{R}_{3,t+1} - R_f),$$
$$\overline{v}(x) = \begin{cases} x & \text{for} \quad x \geqslant 0, \\ \lambda x & \phantom{\text{for}} \quad x<0, \end{cases} \quad \lambda>1.$$

Relative to the general specification in Eqs. (7)–(11), we have set $n = 3$ and $m = 2$—in words, there are three assets and just one of them is framed narrowly—and we have given $\mu(\cdot)$ the simplest possible form: power utility with an exponent equal to the exponent in the aggregator function $H(\cdot,\cdot)$. Finally, we have set the reference return $R_{i,z}$ equal to the risk-free rate R_f.

In terms of Kahneman's (2003) accessibility theory of framing, the narrow framing of asset 3 indicates that information about the distribution of that asset's returns is very accessible to the investor—more so than information about the distribution of his overall wealth once a position in asset 3 is merged with his fixed position in asset 2.[7]

We use the return process parameter values in Table 1. We set the mean returns on the two risky assets, and also their volatilities, to the same value. The correlation of the log returns on the risky assets is $\omega = 0.1$. A fixed fraction $\overline{\theta}_2 = 50\%$ of wealth is allocated to asset 2, the non-financial asset, and the gross risk-free rate is 1.02. Finally, we set the preference

[7] One could argue that the investor should also frame asset 2 narrowly, on the grounds that the distribution of that asset's returns may also be more accessible than the distribution of overall wealth once the two risky assets are combined. While it is clear, from Eq. (7), that we can easily accommodate this, doing so adds little to the intuition of this section. For simplicity, then, we assume that only asset 3 is framed narrowly.

Table 2
Asset allocation by an investor who frames narrowly.

	$\gamma = 1.5$ $\lambda = 2$	$\gamma = 1.5$ $\lambda = 3$	$\gamma = 5$ $\lambda = 2$	$\gamma = 5$ $\lambda = 3$
$b_0 = 0$	100	100	90	90
$b_0 = 0.005$	100	100	88	68
$b_0 = 0.01$	100	100	86	44
$b_0 = 0.02$	100	70	82	0
$b_0 = 0.03$	100	0	76	0
$b_0 = 0.04$	100	0	72	0
$b_0 = 0.05$	100	0	68	0
$b_0 = 0.1$	100	0	40	0
$b_0 = 0.2$	52	0	0	0
$b_0 = 0.3$	0	0	0	0
$b_0 = 0.4$	0	0	0	0
$b_0 = 0.5$	0	0	0	0

Notes: The table shows, for given aversion to consumption risk γ, sensitivity to narrowly framed losses λ, and degree of narrow framing b_0, the percentage of his remaining wealth that an investor with 50% of his wealth already invested in a risky asset would invest in another similar, weakly correlated risky asset that he frames narrowly, rather than in the risk-free asset.

parameter β, which has little direct influence on attitudes to risk, to 0.98 and consider a range of values for the remaining preference parameters: γ, λ, and b_0.

Before presenting the results, we outline a way of solving the portfolio problem. Given the i.i.d. investment opportunity set, we conjecture that

$$(\theta_{3,t}, \alpha_t, A_t) = (\theta_3, \alpha, A), \quad \forall t.$$

The problem in (18) then becomes

$$B^* = \max_{\theta_3}\Big[(1-\beta)^{1/(1-\gamma)}\alpha^{-\gamma/(1-\gamma)}[E((\theta'\widetilde{R}_{t+1})^{1-\gamma})]^{1/(1-\gamma)} + b_0 E(\overline{v}(\theta_3(\widetilde{R}_{3,t+1} - R_f)))\Big]. \tag{25}$$

The only difficulty here is that the portfolio problem depends on the consumption policy constant α. We address this in the following way. Given a candidate optimal consumption policy α, we solve (25) for that α. We then substitute the resulting B^* into Eq. (16) to generate a new candidate α and continue the iteration until convergence occurs.

Table 2 shows the portfolios chosen by an investor who maximizes the utility function in (24). Recall that a fixed 50% of the investor's wealth is held in risky asset 2. For four pairs of values of γ and λ and for a wide range of values of b_0, the table reports the percentage of the investor's *remaining* wealth that is allocated to risky asset 3 as opposed to the risk-free asset. For example, the number "100" in the table means that 100% of remaining wealth, or 50% of total wealth, is invested in asset 3. In solving the portfolio problem, we allow the investor to short asset 3 but not to buy it on margin.

Note first that, when $b_0 = 0$, in other words, when there is no narrow framing at all, the investor either puts all of his remaining wealth, or the vast majority of it, into asset 3. The intuition is simple. Asset 3 not only earns a premium over the risk-free rate but is also almost uncorrelated with asset 2, thereby offering substantial diversification. Since the investor does not frame narrowly, he cares about overall wealth risk and therefore finds the diversification very attractive.

At the other end of the table, when $b_0 = 0.5$, the investor puts *none* of his remaining wealth into risky asset 3. At this level of narrow framing, the investor evaluates asset 3 so much in isolation that he misses its diversification benefits. Since he focuses narrowly on the asset's potential gains and losses and since, through the parameter λ, he is more sensitive to losses than to gains, he rejects the asset completely.

The results in Table 2—specifically, the fact that, for a wide range of values of b_0, the investor allocates nothing to asset 3, thereby ignoring its diversification benefits—immediately suggest a number of possible applications for narrow framing. In particular, there are many situations in which investors do indeed appear to reject obvious diversification opportunities. In what has come to be known as the stock market participation puzzle, many U.S. households have historically refused to add even a small amount of stock market risk to their portfolios even though the stock market is relatively uncorrelated with other major household risks. Very similar is the home bias puzzle: the fact that many investors are reluctant to diversify what holdings of domestic stock they do have with a position in foreign stock in spite of the low correlation between the two asset classes. Yet another example is the fact that some households invest a large fraction of their wealth in just a few stocks.

In the absence of frictions, it is hard to explain such behavior based on utility functions defined only over wealth or consumption. Investors who care about the riskiness of their overall wealth are generally keen to take advantage of opportunities for diversification; this is true even for investors whose utility functions exhibit first-order risk aversion (Barberis et al., 2006). Table 2 suggests that narrow framing, on the other hand, *can* potentially explain the under-diversification we see in practice: an investor who evaluates risks in isolation misses diversification opportunities and may therefore fail to exploit them.

The above discussion raises the following question: If we invoke narrow framing to explain why an investor holds only a few stocks, might we be forced to conclude that the investor holds no stocks at all? Roughly speaking, if the investor narrowly frames the *n*'th stock he considers and turns it down, then, if he also narrowly frames the first stock he considers, would he not also turn that stock down?

We emphasize that this reasoning is not correct. Even if an investor frames *all* the stocks that he considers narrowly, he would typically still be willing to hold *some* of them. The reason is that the narrow framer's decision as to whether to buy a stock depends not only on his preferences—on how sensitive he is to losses or on his degree of narrow framing—but also on his *beliefs*, in particular, on his beliefs about the stock's expected return. If a stock has a very high expected return, the investor will buy it even if he frames it narrowly. But if its expected return is low, he will turn it down. The key point is that, for a wide range of preference parameterizations, the narrow framer is less likely to buy a given stock than is an investor who maximizes a standard utility function defined over wealth or consumption: the narrow framer misses the diversification benefits while the expected utility investor appreciates them.

5. Attitudes to timeless gambles

In this section, we show how an agent who engages in narrow framing would evaluate a timeless gamble. This analysis is useful because it allows us to calibrate the parameters in our preference specification. For example, if we want to draw quantitative implications out of our specification, we need to take a stand on what values of b_0 and λ are reasonable. One way to do this is to look at what specific values of b_0 and λ predict about attitudes to timeless gambles. The analysis in this section explains how these attitudes can be computed.

Earlier papers have already discussed how an agent with the recursive utility preferences in (4) would evaluate a timeless gamble (Epstein and Zin, 1989). The reason we need to do more analysis is that, if the agent frames some risks narrowly, as the agent with the preferences in (7)–(11) does, then he may, in particular, frame timeless gambles narrowly.

The narrow framing of a timeless gamble can be motivated, as before, by Kahneman's (2003) notion of accessibility. Suppose that, at time τ, the agent is offered a timeless gamble $\widetilde{g}$, a 50:50 bet to gain \$$x$ or lose \$$y$, independent of other risks, and whose outcome provides no information about future investment opportunities. The gamble payoffs, x and y, are highly accessible and, in particular, may be more accessible than the distribution of overall wealth once $\widetilde{g}$ is mixed with the agent's other risks. As a result, the distribution of the gamble, taken alone, may play a more important role in the agent's decision-making than would be predicted by traditional utility functions.

Even if the gamble $\widetilde{g}$ is framed narrowly, there is still some flexibility as to how it is evaluated. One approach, proposed in the earlier literature on recursive utility, is that, when evaluating a timeless gamble, the agent inserts an infinitesimal time step $\Delta\tau$ around the moment where the gamble's uncertainty is resolved and then applies the recursive calculation over this time step (Epstein and Zin, 1989). In this case, then, the agent waits for the outcome of the timeless gamble to be revealed and then, at time $\tau+\Delta\tau$, decides what fraction of his wealth to consume.

Under this approach, the agent's utility from accepting the gamble is[8]

$$V_\tau = H(0, \mu(\widetilde{V}_{\tau+\Delta\tau}|I_\tau) + b_0 E(\overline{v}(\widetilde{g}))). \tag{26}$$

Since

$$\mu(\widetilde{V}_{\tau+\Delta\tau}|I_\tau) = \mu(A_{\tau+\Delta\tau}(I_{\tau+\Delta\tau})\widetilde{W}_{\tau+\Delta\tau}|I_\tau) = A_\tau \mu(W_\tau + \widetilde{g}|I_\tau) = A_\tau \mu(W_\tau + \widetilde{g}),$$

where the second equality comes from the fact that $\widetilde{g}$ provides no information about future investment opportunities, the third from the fact that $\widetilde{g}$ is independent of time τ information, and A_τ is defined in Eq. (12), Eq. (26) becomes

$$V_\tau = H\left(0, A_\tau \mu(W_\tau + \widetilde{g}) + b_0\left(\frac{x-\lambda y}{2}\right)\right). \tag{27}$$

If the agent chooses not to take the gamble, this reduces to

$$V_\tau = H(0, A_\tau W_\tau).$$

The gamble is therefore accepted if

$$A_\tau \mu(W_\tau + \widetilde{g}) + b_0\left(\frac{x-\lambda y}{2}\right) > A_\tau W_\tau. \tag{28}$$

If the potential outcomes of the gamble $\widetilde{g}$ are small relative to the agent's wealth and if μ is "smooth"—in the sense of exhibiting second-order risk aversion, say, as in Segal and Spivak (1990)—then $\mu(W_\tau + \widetilde{g}) \approx W_\tau + E(\widetilde{g})$ and condition (28) becomes

$$\frac{x}{y} > \frac{A_\tau + b_0\lambda}{A_\tau + b_0}.$$

[8] We suppose that the agent frames only the timeless gamble narrowly. It is straightforward to extend the calculations to the case where the agent also frames some of his other risks narrowly.

Note that, if b_0 is large relative to A_τ, the gamble is accepted if the ratio of x to y exceeds λ. Intuitively, when b_0 is large, the agent evaluates the gamble largely in isolation and therefore accepts it only if its ratio of gain to loss exceeds his sensitivity to losses λ.

A second possibility is that the agent evaluates the timeless gamble $\widetilde{g}$ over the same time interval he uses to evaluate his other risks, which, from (7), is the time interval between τ and $\tau+1$. In this case, then, the agent makes the time τ consumption decision before seeing the outcome of the timeless gamble.

Under this approach, if the agent does not take the gamble, his utility, from (12), is

$$V_\tau = A_\tau W_\tau.$$

If he does take the gamble, his utility is

$$V_\tau = \widehat{A}_\tau W_\tau = H(\widehat{C}_\tau, \mu(\widetilde{V}_{\tau+1}|I_\tau) + b_0 E(\overline{v}(\widetilde{g}))), \tag{29}$$

where the hats over $\widehat{A}_\tau$ and $\widehat{C}_\tau$ are a reminder that, if the gamble is accepted, optimal consumption and portfolio policies are affected. Since

$$\mu(\widetilde{V}_{\tau+1}|I_\tau) = \mu(A_{\tau+1}\widetilde{W}_{\tau+1}|I_\tau) = \mu(A_{\tau+1}((W_\tau - \widehat{C}_\tau)\widetilde{R}_{W,\tau+1} + \widetilde{g})|I_\tau),$$

Eq. (29) becomes

$$V_\tau = H\left(\widehat{C}_\tau, (W_\tau - \widehat{C}_\tau)\mu\left(A_{\tau+1}\left(\widetilde{R}_{W,\tau+1} + \frac{\widetilde{g}}{W_\tau - \widehat{C}_\tau}\right)\middle| I_\tau\right) + b_0\left(\frac{x-\lambda y}{2}\right)\right). \tag{30}$$

The agent therefore takes the gamble if

$$H\left(\widehat{C}_\tau, (W_\tau - \widehat{C}_\tau)\mu\left(A_{\tau+1}\left(\widetilde{R}_{W,\tau+1} + \frac{\widetilde{g}}{W_\tau - \widehat{C}_\tau}\right)\middle| I_\tau\right) + b_0\left(\frac{x-\lambda y}{2}\right)\right) > A_\tau W_\tau. \tag{31}$$

We now present an example. Consider an agent who, at time τ, has wealth of \$500,000 invested in a risky asset with gross return $\widetilde{R}_{t+1}$ given by

$$\log \widetilde{R}_{t+1} \sim N(0.04, 0.03) \quad \text{i.i.d. over time.}$$

The agent is offered a timeless gamble $\widetilde{g}$, a 50:50 bet to gain \$200 or lose \$100, independent of other risks. Suppose that the agent engages in narrow framing, so that he evaluates the gamble according to either (28) or (31). We set

$$H(C,x) = ((1-\beta)C^{1-\gamma} + \beta x^{1-\gamma})^{1/(1-\gamma)}, \quad 0<\beta<1, \ 0<\gamma\neq 1, \tag{32}$$

$$\mu(\widetilde{x}) = (E(\widetilde{x}^{1-\gamma}))^{1/(1-\gamma)}, \quad 0<\gamma\neq 1, \tag{33}$$

so that, as in Section 4.1, $\mu(\cdot)$ has a power utility form with an exponent equal to the exponent in the aggregator function.

The top panel in Fig. 1 shows, for $\beta = 0.98$ and $\gamma = 1.5$, the range of values of b_0 and λ for which the agent rejects the 200/100 gamble under the first evaluation method laid out above in Eq. (28).[9] The figure shows that, for high values of b_0, the agent rejects the gamble when $\lambda > 2$ and accepts it otherwise. The intuition is simple. For high values of b_0, the agent evaluates the gamble largely in isolation: whether or not he takes it therefore depends on his sensitivity to narrowly framed losses λ. If he is more than twice as sensitive to losses as to gains, the 200/100 gamble, with its 2:1 ratio of gain to loss, becomes unattractive.

As b_0 falls, it takes higher values of λ for the agent to reject the gamble. To understand this, consider the extreme case where $b_0 = 0$. In this case, the preferences in (7), coupled with (32) and (33), collapse to standard power utility. For such preferences, the agent is almost risk-neutral to small gambles and is therefore delighted to accept a small, independent, actuarially favorable gamble like 200/100. As b_0 falls towards 0 then, the agent becomes more interested in the 200/100 gamble and higher values of λ are required to scare him away from it.

The bottom panel in Fig. 1 shows, again for $\beta = 0.98$ and $\gamma = 1.5$, the range of values of b_0 and λ for which the agent rejects 200/100 under the second evaluation method laid out above in Eq. (31).[10] The figure shows that, for this gamble, the alternative procedure produces identical results. The reason is that, since the 200/100 gamble is small relative to the agent's wealth, it makes little difference whether the time τ consumption decision is made before or after observing the outcome of the gamble.

Sometimes, we are interested not in whether an agent accepts or rejects a gamble x/y, but in what premium π the agent would pay to avoid a symmetric gamble $\widetilde{g}$: a 50:50 bet to gain or lose a fixed amount x, say. For an agent who engages in narrow framing, the premium can be computed using the analysis above.

[9] See the Appendix for computational details.
[10] See the Appendix for computational details.

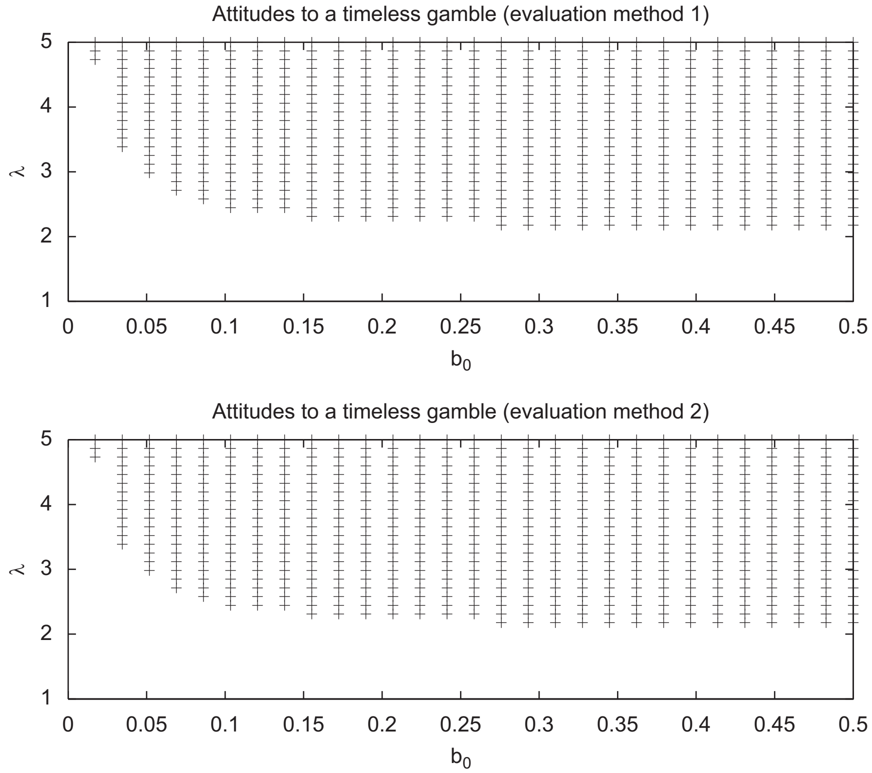

Fig. 1. The "+" signs in each panel mark the preference parameter values for which an agent with wealth of $500,000 rejects a timeless gamble offering a 50:50 chance to gain $200 or lose $100. The agent engages in narrow framing. The two panels correspond to different assumptions as to how he evaluates the gamble.

For example, following the first evaluation method in Eq. (27), the utility from taking the gamble is

$$V_\tau = H\left(0, A_\tau \mu(W_\tau + \widetilde{g}) + b_0 \frac{x}{2}(1-\lambda)\right),$$

while the utility after paying the premium is

$$V_\tau = H(0, A_\tau \mu(W_\tau - \pi) - b_0 \lambda \pi).$$

The premium is therefore given by

$$A_\tau \mu(W_\tau + \widetilde{g}) + b_0 \frac{x}{2}(1-\lambda) = A_\tau(W_\tau - \pi) - b_0 \lambda \pi,$$

so that

$$\pi = \frac{A_\tau(W_\tau - \mu(W_\tau + \widetilde{g})) + b_0 \frac{x}{2}(\lambda - 1)}{A_\tau + b_0 \lambda}. \tag{34}$$

Alternatively, the gamble can be evaluated according to the method assumed in Eq. (30), leading, in many cases, to similar results.

6. Equilibrium analysis

We now show that our preference specification is tractable not only in partial equilibrium but also in a full equilibrium setting, thereby allowing us to study the impact of narrow framing on asset prices.

Before going any further, we must first think about where, if anywhere, narrow framing is likely to have a significant impact on asset prices. Consider a heterogeneous agent model with two groups of investors. Investors in the first group are standard expected utility agents while investors in the second group are narrow framers. In such a model, the narrow framers are unlikely to have much of an effect on the prices of assets with close substitutes—if they did, that would present an attractive opportunity for the expected utility investors who would then trade aggressively against the narrow framers, reducing their impact on prices. Narrow framers will have a more significant impact, however, on the prices of assets *without* close substitutes—in this case, it is much riskier for the expected utility investors to trade against them.

We have not, as yet, found a tractable way of analyzing a heterogeneous agent model of this kind. As in other lines of asset pricing research, then, we start by studying a homogeneous agent model. In taking this approach, we are careful to pick an application where the prediction of the homogeneous agent model is likely to be qualitatively similar to the prediction of the more realistic heterogeneous agent model. That is why, in this section, we choose the equity premium as our application. If narrow framing affects the equity premium in a homogeneous agent model, it is likely to also affect the equity premium in a heterogeneous agent model: since the aggregate stock market does not have a close substitute, it would be too risky for expected utility investors to trade aggressively against the narrow framers. The narrow framers would therefore continue to have at least some impact on the equity premium.

Even if the implications of a homogeneous agent model for the equity premium are qualitatively similar to those of a heterogeneous agent model, they may nonetheless be *quantitatively* different. Even if expected utility investors cannot fully reverse the effect of narrow framers on the pricing of the aggregate stock market, they may *partially* reverse it. As such, the equity premium that we obtain in a homogeneous agent economy should be thought of as an upper bound on the equity premium that we would obtain in a more realistic heterogeneous agent economy.

In this section, then, we consider the simplest equilibrium implementation of narrow framing, one in which we assign our preferences to a representative agent. In this case, the first-order conditions (22) and (23) give the relationship that must hold between aggregate consumption and asset returns. The following lemma rewrites those first-order conditions in a way that brings the role of consumption out more clearly and that is therefore easier to apply in equilibrium settings.

Lemma. *Suppose that asset* 1 *is the risk-free asset and that the reference return is set to* $R_{i,z} = R_{f,t}$, $\forall i$. *If, moreover,* $\theta_{i,t} > 0, \forall i > 1$, *then the first-order conditions for consumption and portfolio choice in* Eqs. (22)–(23) *reduce to*

$$\left[\beta R_{f,t} E_t\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\right)\right]\left[\beta E_t\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\widetilde{R}_{W,t+1}\right)\right]^{\gamma/(1-\gamma)} = 1, \tag{35}$$

$$\frac{E_t\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}(\widetilde{R}_{i,t+1} - R_{f,t})\right)}{E_t\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\right)} + b_0 1_{\{i>m\}} R_{f,t}\left(\frac{\beta}{1-\beta}\right)^{1/(1-\gamma)}\left(\frac{1-\alpha_t}{\alpha_t}\right)^{-\gamma/(1-\gamma)} E_t(\overline{v}(\widetilde{R}_{i,t+1} - R_{f,t})) = 0, \quad i = 2, \ldots, n. \tag{36}$$

Proof of Lemma. See the Appendix.

One last equation that will prove useful is the weighted sum of the equations in (36), where the *i*'th equation is weighted by $\theta_{i,t}$:

$$\frac{E_t\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}(\widetilde{R}_{W,t+1} - R_{f,t})\right)}{E_t\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\right)} + b_0 R_{f,t}\left(\frac{\beta}{1-\beta}\right)^{1/(1-\gamma)}\left(\frac{1-\alpha_t}{\alpha_t}\right)^{-\gamma/(1-\gamma)} \sum_{i=m+1}^{n} E_t(\theta_{i,t}\overline{v}(\widetilde{R}_{i,t+1} - R_{f,t})) = 0. \tag{37}$$

We now show how Eqs. (35)–(37) can be applied in an equilibrium setting. Specifically, we use them to analyze the effect of narrow framing on the equity premium.

6.1. An example

Consider an economy with three assets. Asset 1, the risk-free asset, is in zero net supply and earns a gross return of $R_{f,t}$ between time t and $t+1$. Assets 2 and 3 are risky and are in positive net supply. We think of asset 2 as a non-financial asset such as housing wealth or human capital; it earns a gross return of $\widetilde{R}_{N,t+1}$ between time t and $t+1$. Asset 3 is the stock market and earns a gross return of $\widetilde{R}_{S,t+1}$ between time t and $t+1$.

We study the implications for the risk-free rate and for the stock market risk premium when the representative agent frames stock market risk narrowly, in other words, when he has the preferences

$$\begin{aligned}
&V_t = H(C_t, \mu(\widetilde{V}_{t+1}) + b_0 E_t(\overline{v}(\widetilde{G}_{S,t+1}))),\\
&H(C,x) = ((1-\beta)C^{1-\gamma} + \beta x^{1-\gamma})^{1/(1-\gamma)}, \quad 0<\beta<1, \; 0<\gamma\neq 1,\\
&\mu(\widetilde{x}) = (E(\widetilde{x}^{1-\gamma}))^{1/(1-\gamma)}, \quad 0<\gamma\neq 1,\\
&\widetilde{G}_{S,t+1} = \theta_{S,t}(W_t - C_t)(\widetilde{R}_{S,t+1} - R_{f,t}),\\
&\overline{v}(x) = \begin{cases} x & \text{for} \quad x \geqslant 0, \\ \lambda x & \quad x<0, \end{cases} \quad \lambda > 1,
\end{aligned}$$

where $\theta_{S,t}$ is the fraction of wealth allocated to the stock market at time t and where, relative to the general specification in (7)–(11), we have set $n=3$ and $m=2$, given $\mu(\cdot)$ a power utility form with exponent equal to the exponent in the aggregator function, and set the reference return $R_{i,z}$ equal to the risk-free rate $R_{f,t}$.

In terms of Kahneman's (2003) accessibility theory of framing, the narrow framing of the stock market indicates that information about the distribution of stock market returns is very accessible to the investor, perhaps because he is regularly exposed to such information in books, newspapers, and other media. In particular, this information is more accessible than information about the distribution of his overall wealth once the stock market is merged with his holdings of the non-financial asset.[11]

We consider an equilibrium in which: (i) the risk-free rate is a constant R_f; (ii) consumption growth and stock returns are distributed as

$$\log \frac{\widetilde{C}_{t+1}}{C_t} = g_C + \sigma_C \widetilde{\varepsilon}_{C,t+1}, \tag{38}$$

$$\log \widetilde{R}_{S,t+1} = g_S + \sigma_S \widetilde{\varepsilon}_{S,t+1}, \tag{39}$$

where

$$\begin{pmatrix} \widetilde{\varepsilon}_{C,t} \\ \widetilde{\varepsilon}_{S,t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_{CS} \\ \rho_{CS} & 1 \end{pmatrix} \right) \quad \text{i.i.d. over time;} \tag{40}$$

(iii) the consumption–wealth ratio α_t is a constant α, which, using

$$\widetilde{R}_{W,t+1} = \frac{\widetilde{W}_{t+1}}{W_t - C_t} = \frac{1}{1-\alpha} \frac{\widetilde{C}_{t+1}}{C_t},$$

implies

$$\log \widetilde{R}_{W,t+1} = g_W + \sigma_W \widetilde{\varepsilon}_{W,t+1}, \tag{41}$$

where

$$\begin{aligned} g_W &= g_C + \log \frac{1}{1-\alpha}, \\ \sigma_W &= \sigma_C, \\ \widetilde{\varepsilon}_{W,t+1} &= \widetilde{\varepsilon}_{C,t+1}; \end{aligned}$$

and (iv) the fraction of total wealth made up by the stock market, $\theta_{S,t}$, is a constant over time, θ_S, so that

$$\theta_{S,t} = \frac{S_t}{S_t + N_t} = \theta_S, \quad \forall t, \tag{42}$$

where S_t and N_t are the value of the stock market and of the non-financial asset, respectively.

Can the structure in conditions (i)–(iv) be embedded in a general equilibrium framework? Condition (iv), the condition that $\theta_{S,t}$ is constant over time, makes this a non-trivial challenge. For example, this condition cannot emerge from the simplest model of the production sector, the Lucas tree. In the Appendix, we show that a slightly richer model of production *can* be consistent with conditions (i)–(iv). While we place this analysis in the Appendix, it is nonetheless one of our more important contributions: it is this analysis that clears the way for our investigation of the equilibrium implications of narrow framing.

At the same time, we acknowledge that conditions (i)–(iv) are strong conditions. We impose them in order to make our analysis more tractable. We conjecture, however, that our results would be qualitatively similar even if we were to relax the conditions: the intuition for the results we obtain below does not depend on the conditions in a crucial way.

Under conditions (i)–(iv), Eqs. (35)–(37) simplify to

$$\beta^{1/(1-\gamma)}(1-\alpha)^{-\gamma/(1-\gamma)} R_f E\left(\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{-\gamma} \right) \left(E\left(\left(\frac{\widetilde{C}_{t+1}}{C_t} \right)^{1-\gamma} \right) \right)^{\gamma/(1-\gamma)} = 1, \tag{43}$$

[11] One could argue that the investor should also frame the non-financial asset narrowly on the grounds that the distribution of that asset's returns may also be more accessible than the distribution of overall wealth once the two risky assets are combined. While it is clear, from (7), that we can easily accommodate this, doing so leaves the equity premium largely unaffected. For simplicity, then, we assume that the investor only frames stock market risk narrowly.

Table 3
Parameter values for the return and consumption processes in a representative agent equilibrium model.

Parameter	
g_C	0.0184
σ_C	0.0379
σ_S	0.20
ρ_{CS}	0.10

Notes: g_C and σ_C are the mean and standard deviation of log consumption growth, respectively, σ_S is the standard deviation of the log stock market return, and ρ_{CS} is the correlation of log consumption growth and the log stock market return.

Table 4
The equity premium and risk-free rate in an equilibrium model with narrow framing.

γ	λ	b_0	$R_f - 1$ (%)	EP (%)	π_L ($)	π_S ($)
1.5	2	0	4.7	0.12	6371	0.63
1.5	2	0.01	4.2	1.39	6336	18
1.5	2	0.02	3.7	2.41	6312	31
1.5	2	0.03	3.4	3.15	6296	39
1.5	2	0.04	3.1	3.66	6286	44
1.5	3	0	4.7	0.12	6371	0.63
1.5	3	0.005	4.1	1.54	6836	20
1.5	3	0.010	3.4	2.99	7237	37
1.5	3	0.015	2.8	4.35	7552	50
1.5	3	0.020	2.3	5.45	7773	60

Notes: The table shows, for given aversion to consumption risk γ, sensitivity to narrowly framed losses λ, and degree of narrow framing b_0, the risk-free rate R_f and equity premium EP in an economy where the representative agent frames the stock market narrowly. π_L (π_S) is the premium the representative agent would pay, given his equilibrium holdings of risky assets and wealth of $75,000, to avoid a 50:50 bet to gain or lose $25,000 ($250).

$$\frac{E\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}(\widetilde{R}_{S,t+1}-R_f)\right)}{E\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\right)}+b_0R_f\left(\frac{\beta}{1-\beta}\right)^{1/(1-\gamma)}\left(\frac{1-\alpha}{\alpha}\right)^{-\gamma/(1-\gamma)}E(\overline{v}(\widetilde{R}_{S,t+1}-R_f))=0, \tag{44}$$

$$\frac{E\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}(\widetilde{R}_{W,t+1}-R_f)\right)}{E\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\right)}+b_0R_f\left(\frac{\beta}{1-\beta}\right)^{1/(1-\gamma)}\left(\frac{1-\alpha}{\alpha}\right)^{-\gamma/(1-\gamma)}\theta_S E(\overline{v}(\widetilde{R}_{S,t+1}-R_f))=0. \tag{45}$$

We now use these equations to compute the equilibrium equity premium. First, we set the return and consumption process parameters to the values in Table 3. These values are estimated from annual data spanning the 20th century and are standard in the literature. Then, for given preference parameters β, γ, λ, and b_0, and a given stock market fraction of total wealth θ_S, we solve Eqs. (43)–(45) for the consumption–wealth ratio α, the risk-free rate R_f, and the mean log stock return g_S, which, in turn, gives us the equity premium. The calculations are straightforward because there are analytical expressions for the expectation terms in Eqs. (43)–(45). These expressions can be found in the Appendix.[12]

Table 4 presents the results. We take $\beta = 0.98$, $\theta_S = 0.3$, $\gamma = 1.5$, and consider several pairs of values of λ and b_0; the parameter β has little effect on attitudes to risk and the results are quite similar over a range of values of θ_S. The table shows that narrow framing of the stock market can generate a substantial equity premium at the same time as a low risk-free rate. The parameter triple $(\gamma, \lambda, b_0) = (1.5, 3, 0.02)$, for example, produces an equity premium of 5.45% and a net risk-free rate of 2.3%. The intuition is simple: if the investor gets utility directly from changes in the value of the stock market

[12] The goal of this section is to provide a framework for an equilibrium analysis of narrow framing and, in particular, to derive the first-order conditions (43)–(45). In what follows, we present some brief computations based on these equations. This is not the focus of our paper, however. For more detailed numerical analysis, see Barberis and Huang (2007), who take Eqs. (43)–(45) as their starting point.

and, via the parameter λ, is more sensitive to losses than to gains, he finds the stock market risky and will only hold it if compensated by a high average return.[13]

The results in Table 4 are consistent with those of Barberis et al. (2001), who also incorporate narrow framing into standard preferences. These authors study the equity premium in an economy where investors derive utility directly from stock market fluctuations. Since the stock market is only one component of overall wealth, the authors are assuming that investors frame narrowly. They find, as in Table 4, that narrow framing of the stock market can generate large equity premia.

In the Introduction, we noted some ways in which our preference specification improves on that of Barberis et al. (2001). For example, our specification is tractable in partial equilibrium while theirs is not. And our specification leads to an explicit value function while theirs does not. This makes it difficult to calibrate their utility function by computing attitudes to timeless monetary gambles.

We can illustrate this last advantage of our specification in the context of our equity premium analysis. A natural question to ask about the results in Table 4 is whether the preference parameters we use to generate high equity premia are *reasonable*, in terms of making sensible predictions about attitudes to monetary gambles. Since our preferences admit an explicit value function, it is straightforward to answer this question.

Consider, for example, a thought experiment proposed by Epstein and Zin (1990) and Kandel and Stambaugh (1991). The experiment posits an agent with wealth of $75,000 and asks what premium he would pay to avoid a 50:50 bet to gain or lose $25,000; and also, what premium he would pay to avoid a 50:50 bet to gain or lose $250. By comparing the premia predicted by a particular parameterization of a utility function to our intuition as to what the answers should be, we can judge how reasonable that parameterization is.

The columns labeled π_L and π_S in Table 4 report, for each set of preference parameter values, the premia that the representative agent in our economy would pay, given his equilibrium holdings of risky assets and wealth of $75,000. The quantities π_L and π_S correspond to the large and small bets, respectively. We compute π_L and π_S from Eq. (34). The parameter A_τ in that equation can be computed from Eq. (17) using the consumption–wealth ratio α obtained from Eqs. (43)–(45).

The table shows that, while all the parameterizations produce reasonable values of π_L, the predicted values of π_S are more reasonable for $b_0 \leqslant 0.03$ in the case of $(\gamma, \lambda) = (1.5, 2)$ and for $b_0 \leqslant 0.01$ in the case of $(\gamma, \lambda) = (1.5, 3)$. Our model therefore provides an insight not available using Barberis et al.s' (2001) specification: that narrow framing of the stock market can easily produce large equity premia while also predicting reasonable attitudes to large-scale gambles; but that if we are also interested in making sensible predictions about attitudes to *small*-scale gambles, there is a limit to the size of the equity premium that narrow framing can generate.

7. Conclusion

Experiments on decision-making show that, when people evaluate risk, they often engage in narrow framing: that is, in contrast to the prediction of traditional utility functions defined over wealth or consumption, they often evaluate risks in isolation, separately from other risks they are already facing. While narrow framing has many potential applications outside the laboratory, there are almost no tractable preference specifications that incorporate it into the standard framework used by economists. In this paper, we propose such a specification and demonstrate its tractability in both portfolio choice and equilibrium settings.

We hope that the specification we present in this paper will make it easier for researchers to derive the predictions of narrow framing. The next step, of course, will be to test these predictions. A particularly clean way of testing hypotheses that involve narrow framing is through experiments—laboratory experiments, but also field experiments with larger stakes. Experiments are a good way to test for narrow framing because they allow the researcher to vary the accessibility of information, in Kahneman's (2003) sense.

To illustrate the testability of narrow framing, we briefly describe two recent tests. In a laboratory setting, Anagol and Gamble (2008) ask subjects to allocate money between cash and a small set of stocks over a number of trading periods. In one condition, they emphasize *stock*-level outcomes rather than portfolio outcomes when they give subjects feedback on their investing performance—for example, they might make the stock-level outcomes more visible on the feedback screen. This makes the return distributions of individual stocks more accessible and encourages stock-level framing. In another condition, they emphasize *portfolio*-level outcomes when giving feedback, thereby making the return distribution of the portfolio more accessible and encouraging portfolio-level framing. Their prediction is that subjects who are encouraged to use the narrower stock-level framing will take less overall risk: by focusing on the highly variable individual stock outcomes, they will find it less appealing to invest in the stocks. Anagol and Gamble (2008) find support for this prediction.

While experiments provide a clean way of testing for narrow framing, non-experimental tests are also possible. Kumar and Lim (2008) test the idea that narrow framing plays a role in the "disposition effect," the empirically observed tendency

[13] Chapman and Polkovnichenko (2009) study the equity premium in a heterogeneous agent economy in which the individual agents exhibit first-order risk aversion—as does our representative agent—but differ in their preference parameters. The authors show that, in such an economy, the equity premium, while still sizeable, can be lower than in a representative agent economy.

of individual investors to sell stocks in their portfolios that have risen in value since purchase, rather than fallen in value. Their prediction is that, under the narrow framing hypothesis, the disposition effect should be stronger for investors who trade at most once in any given day, rather than several times a day. The logic is that investors who trade at most once in a given day are more likely to think about stock-level outcomes than about portfolio-level outcomes; since are they so focused on stock-level outcomes, they find it particularly hard to sell a stock that is trading at a loss and therefore exhibit a particularly strong disposition effect. Kumar and Lim (2008) find that the data confirm their prediction.

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Appendix A

Proof of Proposition 1. Substituting the expression for B_t^* in Eq. (18) into the following rearrangement of Eq. (16),

$$\left(\frac{\beta}{1-\beta}\right)^{1/\rho}\left(\frac{1-\alpha_t}{\alpha_t}\right)^{1-1/\rho}B_t^* = 1$$

gives

$$\left(\frac{\beta}{1-\beta}\right)^{1/\rho}\left(\frac{1-\alpha_t}{\alpha_t}\right)^{1-1/\rho}\left[\mu((1-\beta)^{1/\rho}\alpha_{t+1}^{1-1/\rho}\theta_t'\widetilde{R}_{t+1}|I_t) + b_0\sum_{i=m+1}^{n}E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1}-R_{i,z})))\right] = 1,$$

which, after applying the form of $\mu(\cdot)$ in Eq. (19), gives Eq. (20).

To derive Eqs. (21), let $K(\theta_t)$ equal the argument being maximized in Eq. (18) and recall that the certainty equivalent functional is given by (19), so that

$$K(\theta_t) \equiv F(\theta_t) + G(\theta_t),$$

where

$$\begin{aligned} F(\theta_t) &\equiv (1-\beta)^{1/\rho}\mu(\alpha_{t+1}^{1-1/\rho}\theta_t'\widetilde{R}_{t+1}|I_t) \\ &= (1-\beta)^{1/\rho}[E_t(\alpha_{t+1}^{\zeta(1-1/\rho)}(\theta_t'\widetilde{R}_{t+1})^{\zeta})]^{1/\zeta} \\ G(\theta_t) &\equiv b_0\sum_{i=m+1}^{n}E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1}-R_{i,z}))). \end{aligned}$$

The optimal portfolio weights θ_t can be computed by solving

$$\max_{\theta_t}\left[F(\theta_t) + G(\theta_t) + \psi_t\left(1 - \sum_{i=1}^{n}\theta_{i,t}\right)\right], \tag{46}$$

where the Lagrange multiplier ψ_t satisfies

$$\sum_{i=1}^{n}\theta_{i,t} = 1. \tag{47}$$

Since $\mu(\cdot)$ is strictly concave and since α_{t+1} and $\{\widetilde{R}_{i,t+1}\}_{i=1}^{n}$ are all non-zero random variables, $F(\theta_t)$ is also strictly concave in θ_t. Moreover, since $\overline{v}(\cdot)$ is concave in θ_t, so is $G(\theta_t)$. The argument to be maximized in (46) is therefore strictly concave in θ_t and any local maximum is also a global maximum.

Since $F(\theta_t) + G(\theta_t)$ has well-defined first derivatives everywhere except at $\theta_{i,t} = 0$ for $i > m$, the necessary and sufficient conditions for optimality, other than the standard constraint (47), are

$$\frac{\partial F(\theta_t)}{\partial\theta_{i,t}} + \frac{\partial G(\theta_t)}{\partial\theta_{i,t}} - \psi_t = 0 \quad \text{for } \theta_{i,t} \neq 0$$

and

$$\begin{cases} \dfrac{\partial F(\theta_t)}{\partial\theta_{i,t}} + \dfrac{\partial G(\theta_t)}{\partial\theta_{i,t}^{+}} - \psi_t \leqslant 0 \\ \dfrac{\partial F(\theta_t)}{\partial\theta_{i,t}} + \dfrac{\partial G(\theta_t)}{\partial(-\theta_{i,t}^{-})} - \psi_t \geqslant 0 \end{cases} \quad \text{for } \theta_{i,t} = 0.$$

Writing out the partial derivatives in full gives conditions (21). □

A.1. Computing attitudes to timeless gambles

In order to use condition (28), we need to compute A_τ. Given that investment opportunities are i.i.d., it is straightforward to show that $A_t = A$, $\forall t$, and that $\alpha_t = \alpha$, $\forall t$. The quantity A can then be computed from (13), where, given our assumption that the investor does not frame any of his other risks narrowly, b_0 equals 0. Eq. (14) then becomes

$$B^* = A(E(\widetilde{R}_{t+1}^{1-\gamma}))^{1/(1-\gamma)},$$

which, when substituted into Eq. (16), gives

$$\alpha = 1 - \beta^{1/\gamma}(E(\widetilde{R}_{t+1}^{1-\gamma}))^{1/\gamma}.$$

A can then be computed from Eq. (17).

To implement condition (31), note that the left-hand side can be written

$$\max_{\alpha}\left\{(1-\beta)\alpha^{1-\gamma} + \beta(1-\alpha)^{1-\gamma}\left[A\left(E\left(\widetilde{R}_{\tau+1} + \frac{\widetilde{g}}{W_\tau(1-\alpha)}\right)^{1-\gamma}\right)^{1/(1-\gamma)} + b_0\frac{x-\lambda y}{2W_\tau(1-\alpha)}\right]^{1-\gamma}\right\}^{1/(1-\gamma)} W_\tau.$$

This maximization can be performed numerically.

Proof of Lemma. Note that

$$\alpha_{t+1}\theta_t'\widetilde{R}_{t+1} = \alpha_{t+1}\frac{\widetilde{W}_{t+1}}{W_t - C_t} = \frac{\alpha_t\widetilde{C}_{t+1}}{(1-\alpha_t)C_t}. \tag{48}$$

Substituting this into Eq. (22) gives

$$\beta^{1/(1-\gamma)}\left[E_t\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\theta_t'\widetilde{R}_{t+1}\right)\right]^{1/(1-\gamma)} + b_0\left(\frac{\beta}{1-\beta}\right)^{1/(1-\gamma)}\left(\frac{1-\alpha_t}{\alpha_t}\right)^{-\gamma/(1-\gamma)}\sum_{i=m+1}^{n}E_t(\overline{v}(\theta_{i,t}(\widetilde{R}_{i,t+1} - R_{f,t}))) = 1. \tag{49}$$

Substituting Eq. (48) into conditions (23), recalling that $\theta_{i,t} > 0$ for $i > 1$, and then taking the difference between Eq. (23) for asset $i > 1$ and Eq. (23) for asset 1, gives, for $i > 1$,

$$\beta^{1/(1-\gamma)}\left[E_t\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\theta_t'\widetilde{R}_{t+1}\right)\right]^{\gamma/(1-\gamma)} E_t\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}(\widetilde{R}_{i,t+1} - R_{f,t})\right)$$
$$+ b_0 1_{\{i>m\}}\left(\frac{\beta}{1-\beta}\right)^{1/(1-\gamma)}\left(\frac{1-\alpha_t}{\alpha_t}\right)^{-\gamma/(1-\gamma)} E_t(\overline{v}(\widetilde{R}_{i,t+1} - R_{f,t})) = 0. \tag{50}$$

Note that Eq. (50) also holds trivially for $i = 1$. Subtracting

$$\sum_{i=1}^{n}\theta_{i,t} \text{ (Eq. (50) for asset } i)$$

from Eq. (49) gives Eq. (35). Using Eq. (35) to rewrite the first term of Eq. (50), we further obtain Eq. (36). □

A.2. Simplifying the first-order conditions (43)–(45)

Using the results that, for any random variable $\widetilde{\varepsilon}$ with a standard Normal distribution,

$$E(e^{a\widetilde{\varepsilon}}) = e^{a^2/2},$$
$$E(1_{\widetilde{\varepsilon}<\widehat{\varepsilon}}) = N(\widehat{\varepsilon}),$$
$$E(1_{\widetilde{\varepsilon}<\widehat{\varepsilon}}e^{a\widetilde{\varepsilon}}) = e^{a^2/2}N(\widehat{\varepsilon} - a),$$

the first-order conditions (43)–(45) become

$$\alpha = 1 - \beta^{1/\gamma}R_f^{(1-\gamma)/\gamma}e^{(1/2)(1-\gamma)\sigma_C^2},$$

$$0 = b_0R_f\left(\frac{\beta}{1-\beta}\right)^{1/(1-\gamma)}\left(\frac{1-\alpha}{\alpha}\right)^{-\gamma/(1-\gamma)}[e^{g_S+(1/2)\sigma_S^2} - R_f + (\lambda-1)[e^{g_S+(1/2)\sigma_S^2}N(\widehat{\varepsilon}^S - \sigma_S) - R_fN(\widehat{\varepsilon}^S)]]$$
$$+ e^{g_S+(1/2)\sigma_S^2-\gamma\sigma_S\sigma_C\rho_{CS}} - R_f,$$

$$0 = b_0R_f\left(\frac{\beta}{1-\beta}\right)^{1/(1-\gamma)}\left(\frac{1-\alpha}{\alpha}\right)^{-\gamma/(1-\gamma)}\theta_S[e^{g_S+(1/2)\sigma_S^2} - R_f + (\lambda-1)[e^{g_S+(1/2)\sigma_S^2}N(\widehat{\varepsilon}^S - \sigma_S) - R_fN(\widehat{\varepsilon}^S)]]$$
$$+ \frac{1}{1-\alpha}e^{g_C+(1/2)\sigma_C^2-\gamma\sigma_C^2} - R_f,$$

where

$$\widehat{\varepsilon}^S = \frac{\log R_f - g_S}{\sigma_S}.$$

A.3. A general equilibrium model to support conditions (i)–(iv) of Section 6.1

We now show that the structure described in conditions (i)–(iv) of Section 6.1 can be embedded in a general equilibrium model. To repeat, the conditions are that: (i) the risk-free rate is a constant R_f; (ii) $\widetilde{R}_{S,t+1}$ and $\widetilde{C}_{t+1}/C_t$ follow the i.i.d. processes in (38)–(40); (iii) the consumption–wealth ratio is a constant α; and (iv) the fraction of wealth made up by the stock market is a constant θ_S. The last condition implies that the fraction of total wealth made up by the non-financial asset, $\theta_{N,t}$, is also constant over time and equal to $\theta_N = 1 - \theta_S$.

Consider an economy with two firms. Asset 3 in Section 6.1, the stock market, is a claim to the payout of one of the firms, the "stock" firm, say, while asset 2 in that section, the non-financial asset, is a claim to the payout of the other firm, the "non-stock" firm. Total output in the economy, Y_t, is the sum of the output of the stock firm, $Y_{S,t}$, and of the non-stock firm, $Y_{N,t}$,

$$Y_t = Y_{S,t} + Y_{N,t}.$$

Each firm divides its output between a consumption good payout and capital investment,[14]

$$Y_{S,t} = \widehat{C}_{S,t} + I_{S,t}, \quad Y_{N,t} = \widehat{C}_{N,t} + I_{N,t},$$

so that

$$Y_t = \widehat{C}_t + I_t,$$

where $\widehat{C}_t$ and I_t are the total consumption good payout and total investment in the economy,

$$\widehat{C}_t = \widehat{C}_{S,t} + \widehat{C}_{N,t}, \quad I_t = I_{S,t} + I_{N,t}.$$

The production technologies are

$$Y_{S,t+1} = f_S(I_{S,t}, I_{N,t}), \quad Y_{N,t+1} = f_N(I_{S,t}, I_{N,t}).$$

We do not model labor input explicitly. We also assume that capital investment at time t is fully depreciated after time $t+1$.

We use $\widehat{S}_{t-}$ and $\widehat{S}_t$ ($\widehat{N}_{t-}$ and $\widehat{N}_t$) to denote the total market value of all shares of the stock firm (non-stock firm) at time t, immediately before and after the consumption good payout, respectively, so that

$$\widehat{S}_{t-} = \widehat{C}_{S,t} + \widehat{S}_t, \quad \widehat{N}_{t-} = \widehat{C}_{N,t} + \widehat{N}_t.$$

We also define the total market value of both firms, before and after the consumption good payout, as

$$\widehat{W}_{t-} = \widehat{S}_{t-} + \widehat{N}_{t-}, \quad \widehat{W}_t = \widehat{S}_t + \widehat{N}_t.$$

The conditions for general equilibrium are the conditions for capital investment optimality

$$E_t\left(m\frac{\partial Y_{S,t+1}}{\partial I_{S,t}}\right) = 1, \quad E_t\left(m\frac{\partial Y_{N,t+1}}{\partial I_{N,t}}\right) = 1, \quad \forall t, \tag{51}$$

where m is the stochastic discount factor, and the market clearing conditions, both for the consumption good and for shares in the firms,

$$\widehat{C}_t = C_t, \quad \widehat{S}_t = S_t, \quad \widehat{N}_t = N_t, \quad \forall t.$$

The last equation implies

$$W_t - C_t = \widehat{W}_t, \quad W_t = \widehat{W}_{t-}.$$

We seek an equilibrium with the following properties:

$$\begin{aligned} &\widehat{C}_t = \xi Y_t, \quad I_t = (1-\xi)Y_t, \quad \forall t, \\ &I_{S,t} = \zeta I_t = \zeta(1-\xi)Y_t, \quad I_{N,t} = (1-\zeta)I_t = (1-\zeta)(1-\xi)Y_t, \quad \forall t, \end{aligned} \tag{52}$$

and where

$$\widehat{S}_t = A_S I_{S,t}, \quad \widehat{N}_t = A_N I_{N,t}, \quad \widehat{W}_{t-} = \Lambda Y_t. \tag{53}$$

[14] If the consumption equilibrium in conditions (i)–(iv) shares a variable with the production economy we consider here, we distinguish the latter with a hat sign.

Under these assumptions, the returns on the stock firm, on the non-stock firm, and on total wealth are

$$\widehat{R}_{S,t+1} = \frac{\widehat{S}_{(t+1)-}}{\widehat{S}_t} = \frac{(Y_{S,t+1} - I_{S,t+1}) + A_S I_{S,t+1}}{A_S I_{S,t}} = \frac{Y_{S,t+1} + (A_S - 1)\zeta(1-\xi)Y_{t+1}}{A_S I_{S,t}}, \tag{54}$$

$$\widehat{R}_{N,t+1} = \frac{\widehat{N}_{(t+1)-}}{\widehat{N}_t} = \frac{(Y_{N,t+1} - I_{N,t+1}) + A_N I_{N,t+1}}{A_N I_{N,t}} = \frac{Y_{N,t+1} + (A_N - 1)(1-\zeta)(1-\xi)Y_{t+1}}{A_N I_{N,t}}, \tag{55}$$

$$\widehat{R}_{W,t+1} = \frac{\widehat{W}_{(t+1)-}}{\widehat{W}_t} = (1-\xi)\frac{\Lambda}{\Lambda - \xi}\frac{Y_{t+1}}{I_t}. \tag{56}$$

Note also that since

$$\widehat{W}_{t-} = \widehat{C}_t + \widehat{S}_t + \widehat{N}_t, \tag{57}$$

we have

$$\Lambda = \xi + (1-\xi)(A_S\zeta + A_N(1-\zeta)). \tag{58}$$

We can now state:

Proposition 2. *There exists a consumption–production general equilibrium in which the consumption and return processes are given by* Eqs. (38), (39), (41) *and* (43)–(45), *and the production process is given by* (52) *and* (53), *with*

$$Y_{S,t+1} = (I_{S,t}I_{N,t})^{1/2}v_{S,t+1}, \quad Y_{N,t+1} = (I_{S,t}I_{N,t})^{1/2}v_{N,t+1}, \quad \forall t,$$

where

$$v_{S,t+1} = \frac{1+\alpha}{\alpha}\left(\frac{\theta_S}{\theta_N}\right)^{1/2}\left[e^{g_S+\sigma_S\varepsilon_{S,t+1}} - \frac{1}{1+\alpha}e^{g_C+\sigma_C\varepsilon_{C,t+1}}\right], \tag{59}$$

$$v_{S,t+1} + v_{N,t+1} = 2(\theta_S\theta_N)^{-1/2}\frac{1}{1-\alpha}e^{g_C+\sigma_C\varepsilon_{C,t+1}}, \tag{60}$$

and where the constant coefficients are given by

$$\xi = \frac{1+\alpha}{2}, \quad \Lambda = \frac{1+\alpha}{2\alpha}, \quad A_S = A_N = \frac{1+\alpha}{\alpha}, \quad \zeta = \theta_S.$$

Proof of Proposition 2. First note that conditions (i)–(iv) of Section 6.1 hold if and only if:

$$C_t = \widehat{C}_t \quad \text{for some } t, \text{ to set the scale}, \tag{61}$$

$$\frac{C_t}{W_t} = \frac{\widehat{C}_t}{\widehat{W}_t} = \frac{\xi Y_t}{\Lambda Y_t}, \quad \forall t \text{ which implies } \alpha = \frac{\xi}{\Lambda}, \tag{62}$$

$$\theta_S = \widehat{\theta}_S \equiv \frac{\widehat{S}_t}{\widehat{S}_t + \widehat{N}_t} = \frac{A_S\zeta}{A_S\zeta + A_N(1-\zeta)}, \quad \theta_N = \widehat{\theta}_N \equiv \frac{\widehat{N}_t}{\widehat{S}_t + \widehat{N}_t} = \frac{A_N(1-\zeta)}{A_S\zeta + A_N(1-\zeta)}, \tag{63}$$

$$R_{S,t} = \widehat{R}_{S,t}, \quad R_{N,t} = \widehat{R}_{N,t}, \quad \forall t. \tag{64}$$

The last condition implies $R_{W,t} = \widehat{R}_{W,t}$, $\forall t$.

We now prove the proposition by explicit construction. Suppose that

$$Y_{S,t+1} = (I_{S,t})^a(I_{N,t})^b v_{S,t+1}, \quad Y_{N,t+1} = (I_{S,t})^{a'}(I_{N,t})^{b'} v_{N,t+1}.$$

Then[15]

$$\begin{aligned}\frac{\partial Y_{S,t+1}}{\partial I_{S,t}} &= a\frac{Y_{S,t+1}}{I_{S,t}} = aA_S\widehat{R}_{S,t+1} - a(A_S-1)\zeta(1-\xi)\frac{Y_{t+1}}{I_{S,t}} \\ &= aA_S\widehat{R}_{S,t+1} - a(A_S-1)\frac{\Lambda-\xi}{\Lambda}\widehat{R}_{W,t+1},\end{aligned}$$

[15] Here, we are assuming that the "stock" firm is one of infinitely many identical "stock" firms, and likewise for the "non-stock" firm. This allows us to ignore strategic behavior. Even in an economy where firms do behave strategically, however, an equilibrium satisfying conditions (i)-(iv) of Section 6.1 can still be constructed.

and similarly

$$\frac{\partial Y_{N,t+1}}{\partial I_{N,t}} = b' A_N \widehat{R}_{N,t+1} - b'(A_N - 1)\frac{\Lambda - \xi}{\Lambda}\widehat{R}_{W,t+1}.$$

The conditions for capital investment optimality in (51) become

$$aA_S - a(A_S - 1)\frac{\Lambda - \xi}{\Lambda} = 1, \quad b'A_N - b'(A_N - 1)\frac{\Lambda - \xi}{\Lambda} = 1. \tag{65}$$

Our independent equations are therefore (58), (62)–(65), and the unknowns are ξ, Λ, A_S, A_N, ζ, a, b, a', b', $v_{S,t+1}$, and $v_{N,t+1}$. Since we have some degrees of freedom, we simplify by setting $A_S = A_N$. Then, from (65), $a = b'$. If we assume that $v_{S,t}$ and $v_{N,t}$ are i.i.d., the fact that $R_{S,t}$ and $R_{N,t}$ are i.i.d. implies, from (64), that $\widehat{R}_{S,t}$ and $\widehat{R}_{N,t}$ are also i.i.d. This means that

$$a + b = a' + b' = 1.$$

Further assuming that $a = b$, we have

$$a = b = a' = b' = \tfrac{1}{2}.$$

Finally, we obtain

$$\zeta = \theta_S, \quad A \equiv A_S = A_N = \frac{\Lambda - \xi}{1 - \xi}, \quad \frac{A}{2} - \frac{(A-1)}{2}\frac{\Lambda - \xi}{\Lambda} = 1.$$

Combining these, we obtain

$$\xi = \frac{1+\alpha}{2}, \quad \Lambda = \frac{1+\alpha}{2\alpha}, \quad A_S = A_N = \frac{1+\alpha}{\alpha},$$

as in the proposition. Putting these solutions into Eqs. (54)–(56), we obtain

$$\widehat{R}_{W,t+1} = \tfrac{1}{2}(\theta_S\theta_N)^{1/2}(v_{S,t+1} + v_{N,t+1}), \tag{66}$$

$$\widehat{R}_{S,t+1} = \frac{\alpha}{1+\alpha}\left(\frac{\theta_N}{\theta_S}\right)^{1/2} v_{S,t+1} + \frac{1-\alpha}{2(1+\alpha)}(\theta_S\theta_N)^{1/2}(v_{S,t+1} + v_{N,t+1}), \tag{67}$$

$$\widehat{R}_{N,t+1} = \frac{\alpha}{1+\alpha}\left(\frac{\theta_S}{\theta_N}\right)^{1/2} v_{N,t+1} + \frac{1-\alpha}{2(1+\alpha)}(\theta_S\theta_N)^{1/2}(v_{S,t+1} + v_{N,t+1}). \tag{68}$$

Setting Eq. (66) equal to Eq. (41), we obtain Eq. (60). Setting Eq. (67) equal to Eq. (39), we obtain Eq. (59). That $R_{N,t+1} = \widehat{R}_{N,t+1}$ follows from the portfolio identity. □

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