Product Choice and Product Switching

Andrew B. Bernard†
*Tuck School of Business at Dartmouth
**National Bureau of Economic Research

Stephen Redding†
†London School of Economics
‡Centre for Economic Policy Research

Peter K. Schott§
§Yale School of Management
*National Bureau of Economic Research

February, 2003

Abstract

This paper explores product choice and product switching by firms. Product changes are frequent and important events at U.S. manufacturing firms. Over two thirds of continuing firms add and/or drop at least one product every five years. Added products account for more than a third of current firm output. The paper presents a theoretical model that integrates endogenous product choice into a dynamic analysis of industry evolution with entry and exit and heterogeneous firms. In equilibrium, firm productivity is correlated with product-level fixed costs, with the most productive firms endogenously choosing to make the products with the highest fixed costs. Changes in market structure result in systematic patterns of firm entry/exit and product switching.

Keywords: product switching, product differentiation, sunk costs, entry and exit

JEL classification: L11, D21, L60

*We thank Jonathan Haskel, Marc Melitz, Tony Venables, and seminar participants at CEPR, Dartmouth, Nottingham, and LSE for helpful comments. Bernard and Schott gratefully acknowledge research support by the National Science Foundation. Redding gratefully acknowledges financial support from the Leverhulme Trust. The research in this paper was conducted at the Center for Economic Studies. Research results and conclusions expressed are those of the authors and do not necessarily indicate concurrence by the Bureau of the Census or by the National Bureau of Economic Research. The paper has not undergone the review the Census Bureau gives its official publications. It has been screened to insure that no confidential data are revealed.

†100 Tuck Hall, Hanover, NH 03755, tel: (603) 646-0302, fax: (603) 646-0995, email: andrew.b.bernard@dartmouth.edu

‡Houghton Street, London, WC2A 2AE, tel: (44 20) 7955-7483, fax: (44 20) 7831-1840, email: s.j.redding@lse.ac.uk

§135 Prospect Street, New Haven, CT 06520, tel: (203) 436-4260, fax: (203) 432-6974, email: peter.schott@yale.edu
1. Introduction

This paper explores product choice and product switching by firms. Product changes are frequent and important events at U.S. manufacturing firms yet have received scant attention. The paper presents a model of endogenous product choice by heterogeneous firms. To motivate the need for such a model, we provide the first evidence on the extent of product switching by U.S. manufacturing firms.

The choice by firms about which product to make is a fundamental business decision. Product switching, i.e. the addition or deletion of a product from the firm’s output mix, is an important aspect of firm behavior. Roughly two thirds of continuing U.S. manufacturing firms alter their product mix over a five year period. For firms, adding (or dropping) a product is an important event, affecting more than one third of current products and output. Despite the extent and importance of product switching, it is completely absent from existing dynamic models of industry equilibrium and has been almost entirely neglected in empirical work using industry and plant-level datasets such as the Longitudinal Research Database of the United States Bureau of the Census.

Our theoretical model allows firms to choose which product within an industry to produce as well as whether to enter or exit the market. Product choice is determined by an interaction between heterogeneous firm characteristics and product characteristics. Motivated by existing work, we model firms as being heterogeneous in terms of their productivities and products as differing in terms of their fixed costs of production. In equilibrium, firm productivity is correlated with product choice, with the most productive firms endogenously choosing to make the products with the highest fixed costs. Changes in market structure, such as changes in industry sunk costs of entry, result in systematic patterns of firm entry and exit as well as product switching. We derive the implications of firm-level choices for relative product prices and aggregate industry productivity.

Our framework builds on Melitz’s (2002) dynamic industry model with heterogeneous firms. We extend that model by introducing multiple products defined by their fixed and variable costs of production. In the model, variation in product fixed costs are meant to capture differences in physical capital, skill intensity, or technological sophistication across products. Variable cost differences are modelled as affecting the marginal productivity
of labor but can equivalently be thought of as capturing differences in product quality.\footnote{This mirrors the equivalence, for many intents and purposes, between process innovation and quality improvements in endogenous growth models such as Aghion and Howitt (1992).} Our framework incorporates endogenous firm entry and exit decisions and variation in firm productivity (as in Hopenhayn 1992); horizontal product differentiation and monopolistic competition (as in Krugman 1980); and endogenous product choice based on vertical differentiation (as in Shaked and Sutton 1987 and Sutton 1998).\footnote{For other models with heterogeneous firm productivity (but no endogenous product choice), see Bernard et al. (2003) and Yeaple (2002).} The interaction of firm and product heterogeneity this framework yields implications for aggregate industry-wide productivity. For example, an increase in the sunk cost of entry will raise ex post industry profitability, inducing firms to switch products such that each product market contains firms with a larger range of productivities, and thereby reducing average productivity in each market.

A large empirical literature has presented evidence of substantial variation in productivity across firms within highly disaggregated industries (see for example Baily et al. 1992, Bernard and Jensen 1995, and the recent survey by Bartelsman and Doms 2000). Our focus is on a different form of within-industry heterogeneity, the variation in products across firms and over time. We present the first empirical evidence on the extent of product switching at manufacturing firms by documenting the breadth of such activity across firms as well as the importance of product switches in overall output within the firm.

Our emphasis on heterogeneity in product choice also finds resonance in recent work in industrial organization. Classic treatments of product choice include Hotelling (1929), Chamberlin (1951) and Lancaster (1966). More formal treatments of horizontal differentiation and vertical differentiation include Dixit and Stiglitz (1977), Shaked and Sutton (1982) and Spence (1976), synthesized in Tirole (1988). More recently, Sutton (1998, 2001) has emphasized the importance of 'firm capability' modeled in terms of the relationship between quality and productivity. Other research has sought to control for product quality by restricting attention to individual industries where highly disaggregated information on prices and quantities is available, and combining this disaggregate information with structural models of industry equilibrium (see for example Berry et al. 1995 and
The remainder of the paper is structured as follows. Section 2 provides evidence on product switching by U.S. manufacturing firms from 1972-1997. Section 3 presents a non-technical overview of the theoretical model. In section 4, we present the model while section 5 solves for general equilibrium. A technical appendix at the end of the paper contains important derivations and proofs of the propositions. Section 6 examines the effects of changes in the model’s parameters on firm-level choices and industry equilibrium. Section 7 concludes.


In this section we provide the first complete view of product switching by U.S. manufacturing firms. We look at the importance of adding and dropping products for the manufacturing sector a whole as well as for individual firms. We start by documenting the extent of this activity across firms in the U.S. manufacturing sector. Next, we consider the importance of product-switching at the firms themselves. Finally, we ask whether product switches are associated with significant changes in firm product mix.

Our data comes from the Censuses of Manufactures (CM) of the Longitudinal Research Database (LRD) of the U.S. Bureau of the Census starting in 1972 and conducted every fifth year through 1997. The sampling unit for the Census is a manufacturing establishment, or plant, and the sampling frame in each Census year includes detailed information on inputs, output, and products on all establishments. We aggregate the plant-level data up to the enterprise, or firm, for all the results reported in this paper. Examining the product mix of the firm, rather than the plant, has several advantages: it is both the level at which decisions are made about products and it avoids a potential problem of a firm shuffling its existing mix of products.

3Streitweiser (1992) uses U.S. Census data to describe the number of products (SIC5) produced by U.S. manufacturing plants during the period 1972-1982. Her focus is on the similarity of products produced at multiple product plants, not on product switching. Gollop and Monahan (1991) also use U.S. Census data to construct an index of firm diversification based on 5-digit products. They report increasing firm diversification and declining plant diversification within most 2-digit industries from 1963-1982 but do not address issues related to product switching.
products across plants. From the Census, we construct firm characteristics including the total value of shipments, the number of products produced, total value of each product produced, and information on the births and deaths of firms.

In constructing our sample, we make several modifications to the basic data. We use information on all manufacturing establishments in the six Censuses. While the LRD does contain basic information on small plants (so-called Administrative records), we do not include them in this study due to the lack of information on products or inputs. We aggregate the establishment level data in the Censuses up to the level of the firm. All our results are based on firm-level statistics. On average we are left with 141,561 continuing firms in each year.\(^4\)

We refer to a product as a unique five-digit category in the 1987 Standard Industrial Classification (SIC5).\(^5\) In Census years, plant output is recorded either at the five-digit or seven-digit SIC level of detail. Roughly 7000 of the 15,000 seven-digit categories are recorded directly in the LRD, the rest are recorded at a five-digit level of aggregation. We aggregate seven-digit categories up to their five-digit ‘product-class’ to obtain a complete set of SIC5 products for all manufacturing firms. Our terminology differs slightly from that of the Census Bureau. We [Census] define four-digit categories as industries [industries] and five-digit categories as products [product classes].

Product switching is a pervasive activity among U.S. manufacturing firms. During a typical five year period, two thirds of continuing firms add or drop a (SIC5) product from their output mix.\(^6\) This activity is

---

\(^4\)On average, over one third of manufacturing firms do not survive from one Census to the next. In these tables, we focus on product-switching at surviving, or continuing, firms. The subsequent theoretical model allows for firm failure as an outcome.

\(^5\)We characterize where firms are located in product space and the extent of product switching using Standard Industrial Classification codes. This has a number of important advantages: firms report several SIC5 codes and we can explicitly observe the addition and deletion of products; also, SIC5 codes are typically chosen on the basis of distinct product characteristics and are more aggregated than varieties of the same product. See US Census (1996), http://www.census.gov/prod/2/manmin/mc92-1.pdf, for a complete list of SIC5 categories.

\(^6\)A product addition in the data can represent one of two general activities. The first, related to product innovation, is the creation of an entirely new good which is classified in an existing SIC5 category. The second is the start of production of an existing good,
Table 1: Average Share of US Manufacturing Firm Activity During 5-Year Census Period Intervals, 1972 to 1997

<table>
<thead>
<tr>
<th>Type of Firm</th>
<th>Percent of Firms</th>
<th>Percent of Output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Firms</td>
<td>Single Product Firms</td>
</tr>
<tr>
<td>Firm takes no action</td>
<td>31</td>
<td>47</td>
</tr>
<tr>
<td>Firm drops products only</td>
<td>13</td>
<td>na</td>
</tr>
<tr>
<td>Firm adds products only</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Firm both adds and drops products</td>
<td>43</td>
<td>38</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type of Firm</th>
<th>Percent of Output</th>
<th>All Firms</th>
<th>Single Product Firms</th>
<th>Multiple Product Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm takes no action</td>
<td>7</td>
<td>51</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Firm drops products only</td>
<td>8</td>
<td>na</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Firm adds products only</td>
<td>5</td>
<td>21</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Firm both adds and drops products</td>
<td>81</td>
<td>29</td>
<td>86</td>
<td></td>
</tr>
</tbody>
</table>

Note: Table displays average share of continuing US manufacturing firms engaging in each activity across five year intervals from 1972 to 1997. Add and Drop refer to firms adding or dropping at least one five-digit SIC manufacturing 'product' during the five years between Censuses of Manufactures. The upper panel reports the distribution of firms while the lower panel reports the distribution of output. Results are reported for the full sample of firms as well as for single- and multiple-product firms separately. On average, there are roughly 140,000 firm observations in each Census year. "na" indicates not applicable: single product surviving firms cannot drop their only product and continue in business.

Summarized in the first panel of Table 1, which reports a breakdown of product switching for continuing U.S. manufacturing firms. The numbers in the table refer to the average fraction of firms undertaking the activity described in the first column across five year Census intervals from 1972 to 1997. The numbers reported in all the tables are averages from 1972-1997, there is little variation across Censuses. We divide all continuing firms into one of four types: (a) those that leave their product mix unchanged, i.e. neither add nor drop a SIC5 product; (b) those that drop at least one product and do not add any; (c) those that add at least one product but do not remove an existing product from their output mix; and (d) firms that both add and drop at least one product. The first four rows of the table summarize these four mutually exclusive activities.

New to the firm but not to the market, also in an existing SIC5 category. Innovation of a new good can occur but not be captured as a product addition if it occurs in a SIC5 category where the firm is currently active.
Every five years, 43% of continuing firms shuffle their product mix by both dropping and adding products, column 1 of Table 1. Dropping alone and adding alone are far less frequent events, occurring at 14% and 12% of firms respectively.

The second and third columns of Table 1 compare the activity of single versus multiple product firms. Single-product firms are likely to leave their product mix unchanged (47%) or change it completely by dropping their existing product and add one or more new ones (38%). Half of all multiple-product firms both add and drop products, while 31% of such firms narrow their product mix by dropping one or more products. On average 89% of continuing multiple product firms alter their product mix during a five year period. These numbers clearly demonstrate that product-switching is widespread among U.S. manufacturers and that simultaneous additions to and deletions from the existing product mix are frequent events.

While product-switching occurs at a large proportion of firms it is even more important in terms of output. The second panel of Table 1 shows the fraction of total output produced by the four firm types. More than 80% of total output is generated at firms that both add and drop products. Multi-product firms that are large in size are the most active in changing their product mix in both directions.

Table 2: Average Number of Products Produced, Added and Dropped Across 5-Year Intervals, 1972 to 1997

<table>
<thead>
<tr>
<th>Type of Firm</th>
<th>Number of Products</th>
<th>Number of Adds</th>
<th>Number of Drops</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Firms</td>
<td>2.3</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>Single Product Firms</td>
<td>1.0</td>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>Multiple Product Firms</td>
<td>4.0</td>
<td>1.4</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Note: Table displays mean number of products produced by continuing US manufacturing firms across Census of Manufacturing years 1972-1997. Adds and Drops refer to the number of five-digit SIC manufacturing products added to or dropped from a firm's product mix during the five years between censuses. Results are reported for the full sample of firms as well as for single- and multiple-product firms separately.

Product turnover among manufacturing firms is substantial and occurs at both single product and multiple product firms. Table 2 reveals that the average continuing U.S. manufacturing firm produces 2.3 products at the
beginning of a five year period, and subsequently adds and drops almost half of its product mix (1.1 products). On average, single-product firms add products while on average multiple-product firms drop half their existing products (2 dropped out of 4 produced) and add 1.4 new products.\textsuperscript{7}

Table 3: Average Share of Firm Output in Products Added and Dropped in Multi-Product Firms Across 5-Year Intervals, 1972 to 1997

<table>
<thead>
<tr>
<th>Activity</th>
<th>Number of Firms</th>
<th>Mean Share of Firm Output</th>
<th>Median\textsuperscript{*} Share of Firm Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Added Products</td>
<td>150,854</td>
<td>50</td>
<td>42</td>
</tr>
<tr>
<td>Dropped Products</td>
<td>142,178</td>
<td>46</td>
<td>34</td>
</tr>
</tbody>
</table>

Notes: Table displays mean and median value of continuing US manufacturing firm's added and dropped products as a percent of total firm output. Products are five-digit SIC aggregates. Figures are based on all adding and dropping behavior across five-year Census of Manufacturing years 1972 to 1997. Because of Census disclosure rules, which prohibit the reporting of an actual median, the reported median is the average of the 100 observations around the true median.

The products that are added and dropped by switching multi-product firms account for a large fraction of firm output.\textsuperscript{8} On average products that were added in the last five years account for 50\% of current firm output by value. Products that will be dropped over the next five years represent 46\% of current shipments, Table 3. For the median firm, recently added products are 42\% of shipments while products to be dropped are more than a third of output by value.

The previous facts demonstrate that product-switching is a widespread phenomenon in the U.S. manufacturing sector, occurring at more than two thirds of continuing firms that produce more than 90\% of total output. In addition, the results reveal that a sizable fraction of products are turned over every five years and that these products represent a large share of existing firm output. However, there remains the possibility that such activity involves switches among products that are quite similar. For example, switches between Canned Fruits and Canned Vegetables, both of which are five-digit products in SIC 2033, are unlikely to indicate important

\textsuperscript{7}Over time from 1972-1997, there is no systematic change in the number of products produced by continuing plants, nor in the fraction of products added or dropped.

\textsuperscript{8}Obviously, single-product firms that switch change all their output.
economic changes at a firm (see Appendix A for further discussion of the data).

Table 4: Share of Product Additions Outside the Existing Industry Mix of the Firm, 1972 to 1997

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of added products in:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>existing SIC4 industries</td>
<td>46</td>
<td>42</td>
</tr>
<tr>
<td>new SIC4 industries</td>
<td>54</td>
<td>58</td>
</tr>
<tr>
<td>new SIC3 industries</td>
<td>35</td>
<td>41</td>
</tr>
<tr>
<td>new SIC2 industries</td>
<td>17</td>
<td>22</td>
</tr>
</tbody>
</table>

Notes: Table reports the extent to which firms add products outside their existing mix of industries. The first column of the table reports the share of added five-digit SIC products that are outside of the firm’s existing stable of four-, three- and two-digit SIC manufacturing industries. The second column of the table reports the share of firms adding at least one five-digit SIC manufacturing product outside its existing mix of four-, three- and two-digit manufacturing industries.

To identify whether firms are merely adding products that are similar in nature, we examine the fraction of added products that extend the range of production of the firm across industries. We focus our attention on product additions on multi-product firms. The first column of Table 4 shows that 54% of all added products are outside existing 4-digit (SIC4) industries produced by the firm. Furthermore, more than a third of all new products are in a new three digit (SIC3) industry while 17% of added products involve a new two digit (SIC2) industry. The second column of the same table reports similar percentages for firms. Of firms that add one or more products, 58% also add a new 4-digit industry, while 41% (22%) add a new three (two) digit industry to the mix produced by the firm. These results confirm that product-switching involves substantial changes in industry composition and tends to move a firm well beyond its existing mix of industries.

This section has documented the pervasiveness and quantitative importance of product switching. The remainder of the paper develops a theoretical model that explicitly incorporates product switching, while also remaining consistent with existing empirical work emphasizing heterogeneous firm characteristics and continuing entry and exit. We develop a

---

9 Almost all empirical work on U.S. manufacturing identifies an “industry” by its 4-digit (SIC) classification.
3. Many Firms and Multiple Products

The theoretical model extends the heterogeneous firm framework of Melitz (2002) and Hopenhayn (1992) to allow firms to choose endogenously between products that vary in terms of their production technologies. At the heart of the model lies an interaction between both firm and product heterogeneity that shapes product choice and product switching. The model portrays an industry with a number of distinct products each containing a continuum of horizontally differentiated varieties. The products are distinguished by their production technologies, specifically the fixed and variable costs, and each firm is allowed to produce only one product at a time. In addition to product choice, the model incorporates firm births and deaths, imperfect competition, and firm heterogeneity.

The model allows us to examine how changes in market conditions affect product choice through the interaction of firm characteristics and product attributes. Endogenous product switching by firms affects industry productivity and firm profitability. For example, changes in the sunk cost of starting a firm (entry cost) affect the distribution of firm productivity and, at the same, induce some firms switch products.

The demand structure of the model is quite simple. Representative consumers demand positive quantities of both products and have a taste for variety within each product. Each variety is produced with a single input, labor, and is subject to a fixed cost (repeated each period) and a constant variable cost. The key feature of the model is that the period-by-period fixed costs vary across products (but not across varieties within

---

10 The framework can be extended to include an arbitrary number of products, each containing a continuum of varieties. While Melitz (2002) develops an open-economy model with symmetric countries, we focus on a multiple-product, closed economy.

11 While we focus on the interaction of firm and product characteristics, we recognize that there are a variety of additional reasons that firms may add or drop products. These include, but are not limited to, the industrial organization literature cited earlier as well as issues related to corporate diversification, mergers and divestitures, and evolving firm capabilities and factor accumulation. See, among others, Amihud and Baruch (1981), Bolton and Farrell (1990), Chandler (1990), and Milgrom and Roberts (1990).
products). Variable costs are also identical for all varieties but may or may not differ across products.\textsuperscript{12} This provides a particularly tractable way of formalizing the more general idea that there are differences in production technologies across products.

One interpretation of the difference in fixed costs across products is that it captures a difference in physical capital intensity. Although labor is the sole factor of production in the model, the labor used in paying the product fixed cost can be thought of as labor set aside to build and maintain a machine. Another interpretation is that the difference in fixed costs captures ongoing expenditures that allow one product to be a higher quality than another. In this case, since the variable cost of production can be thought of as an inverse measure of product quality, the high fixed cost product must be characterized by a lower variable cost of production (higher product quality). More generally, as noted above, we make no assumptions about the relative value of the variable cost of production for the two products.

To keep the modelling as clean as possible, the theoretical work focuses on the case of a single industry whose two products enter directly into final consumption. It is relatively straightforward to introduce many industries. The framework can also be extended to allow for any number of products within the industry. Both extensions merely complicate the analysis without changing the model’s key insights.\textsuperscript{13}

There is a large pool of ex-ante identical potential entrants into the industry. To enter the industry (and hence either product market within the industry), firms must pay a sunk cost of entry. Firms then observe their productivity, drawn from a common distribution. At this point each firm decides whether to produce or exit the market. Firms with low productivity draws immediately exit. If the firm decides to produce it chooses to manufacture either Product 1 or Product 2 depending on the relative profitability of each product for that firm given its observed productivity.

\textsuperscript{12}Yeaple (2002) allows technologies for producing varieties to vary in terms of both fixed and variable costs, but the model is static with a single product and cannot address product choice and product switching.

\textsuperscript{13}Our model assumes that all products already exist (although new varieties are created when the number of producing firms increases). The equally interesting issue of the creation of new products within a framework with firm heterogeneity and industry dynamics remains an area for further research.
All firms, regardless of the product they produce, face a common exogenous probability of death in every period which, for simplicity, is assumed not to depend on their productivity.\textsuperscript{14}

The solution of the model depends on three key equilibrium conditions: the zero profit cutoff condition which identifies the lowest productivity firm that chooses to produce; the product indifference cutoff condition which identifies the productivity level at which a firm is exactly indifferent between producing either of the two products\textsuperscript{15}; and the free entry condition which forces expected profits before entry to equal the sunk cost of entry and is driven by the existence of a competitive fringe.

Our heterogeneous firm model permits examination of the factors which prompt firms to switch products in response to changes in the competitive environment. Bernard et al. (2002) provide evidence of U.S. manufacturing firms switching between industries in a systematic manner over the last three decades. In the previous section, we provided evidence that within industry product switching by manufacturing firms is pervasive and affects a significant proportion of firm output.

In the next three sections, we introduce the theoretical model in further detail, characterize general equilibrium, and explore how firms’ product choice and the distribution of productivity across firms responds to changes in the underlying parameters of the model.

\textsuperscript{14}This assumption supports a particularly tractable model of continuing endogenous entry/exit, product choice and firm heterogeneity. See Hopenhayn (1992) for an analysis of industry dynamics where productivity affects the probability of firm death, but that abstracts from product choice and imperfect competition.

\textsuperscript{15}A firm with this productivity would be either the highest productivity firm in the low fixed cost good or the lowest productivity firm in the high fixed cost good.
4. A Theory of Endogenous Product Choice

4.1. Demand

The preferences of a representative consumer are given by a Cobb-Douglas utility function over two product markets,\(^{16}\)

\[ U = C_1^{\alpha_1} C_2^{\alpha_2}. \]  

Each market contains a continuum of horizontally differentiated varieties indexed by \(\omega\),

\[ C_i = \left[ \int_{\omega \in \Omega_i} q_i(\omega)^\rho d\omega \right]^{1/\rho} \]  

where \(\{\Omega_i\}\) is the mass (number) of available varieties in sector \(i\). Within each market, varieties are substitutes, implying \(0 < \rho < 1\) and an elasticity of substitution between any two varieties of \(\sigma = \frac{1}{1-\rho} > 1\).

The varieties in each market have an aggregate price index

\[ P_i = \left[ \int_{\omega \in \Omega_i} p_i(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}. \]  

Expenditures over varieties of each product are given by

\[ r_i(\omega) = R_i \left( \frac{p_i(\omega)}{P_i} \right)^{1-\sigma} = \alpha_i R \left( \frac{p_i(\omega)}{P_i} \right)^{1-\sigma} \]  

where \(R = P_1 C_1 + P_2 C_2 = \int_{\omega \in \Omega_1} r_1(\omega) d\omega + \int_{\omega \in \Omega_2} r_2(\omega) d\omega\) is aggregate expenditure.

\(^{16}\)We assume homogeneity of degree 1, i.e. \(\alpha_1 = \alpha\) and \(\alpha_2 = 1 - \alpha\). While we adopt the Cobb-Douglas functional form for tractability, all we require for the analysis that follows is that consumers demand both products in positive quantity in equilibrium. If that is the case, relative prices will adjust to ensure that varieties of products are produced. Prices will adjust for any utility function satisfying the Inada conditions or, alternatively, one can assume the existence of different types of consumers, one of which demands product 1 and the other of which demands product 2.
4.2. Production

The presence of fixed costs of production means that in equilibrium each of a continuum of firms chooses to produce a unique variety $\omega$ in one of the two product markets. Production requires one factor, labor, which is inelastically supplied at its aggregate level $L$, which also indexes the size of the economy.

As well as differing in terms of price $\{P_1, P_2\}$, the two products have different production technologies. The fixed and (constant) variable costs for product 1 are given by $\{f_1, b_1\}$ and for product 2 by $\{f_2, b_2\}$, where $f_2 > f_1$ and $b_i$ can be any non-negative number. The key distinction between products is that the second good is subject to a higher fixed cost of production.

Given firm productivity $\varphi$, labor used is a linear function of output $q$: $l_i = f_i + \frac{b_i}{\varphi}q_i$. Profit maximization yields the standard result that equilibrium prices are a constant mark-up over marginal cost,

$$p_i(\varphi) = \frac{\sigma}{\sigma - 1} \frac{wb_i}{\varphi}. \quad (5)$$

We choose the wage as numeraire so that $w = 1$. Equilibrium profits for the two products are thus,

$$\pi_i(\varphi) = \left(\frac{\sigma}{\sigma - 1}\right) \left(\frac{P_i \varphi}{b_i}\right)^{\sigma - 1} - f_i \quad (6)$$

$$\pi_i(\varphi) = r_i(\varphi) \frac{\varphi}{\sigma} - f_i.$$

From firms’ revenue functions, the ratio of revenues for any two firms producing the same product depends only on their relative productivities,

$$\frac{r_i(\varphi')} {r_i(\varphi'')} = \left(\frac{\varphi'}{\varphi''}\right)^{\sigma - 1}. \quad (7)$$

Comparing revenues for firms producing different products is comparably

\footnote{Note that the variable cost for product 2 can be greater or less than the variable cost of producing product 1. However, the fixed cost of product 2 is greater than the fixed cost of producing product 1.}
straightforward; the ratio of revenues is a function of their relative productivities and relative prices of the two products,

\[
\frac{r_1(\varphi)}{r_2(\varphi')} = \left( \frac{\alpha}{1 - \alpha} \right) \left[ \left( \frac{\varphi'}{\varphi''} \right) \left( \frac{P_1}{P_2} \right) \right]^{\sigma-1}.
\]

These relationships are useful because they allow us to express the revenue earned by a firm with average productivity in each market in terms of the revenue earned by a firm with one of the cutoff levels of productivity.

4.3. Market Entry and Exit

To engage in production, firms must pay a common fixed entry cost, \( f_e > 0 \), which is thereafter sunk. Firms draw their initial productivity parameter, \( \varphi \), after entry from a common distribution \( g(\varphi) \). Upon entering, there is an exogenous probability of death each period, \( \delta \), which we interpret as due to force majeure events beyond managers’ control. This probability is the same for firms in both product markets and does not depend upon firm productivity.

The value of a firm with productivity \( \varphi \) is therefore,

\[
v(\varphi) = \max \left\{ 0, \sum_{t=0}^{\infty} (1 - \delta)^t \pi_1(\varphi), \sum_{t=0}^{\infty} (1 - \delta)^t \pi_2(\varphi) \right\}
\]

\[
= \max \left\{ 0, \frac{1}{\delta} \pi_1(\varphi), \frac{1}{\delta} \pi_2(\varphi) \right\}
\]

4.4. Endogenous Product Selection

Firms decide which good to produce based upon their productivity, taking as given the aggregate price indices \( \{P_1, P_2\} \). Firms with zero productivity have negative post-entry profits in both industries, with the loss greatest for the high fixed cost product 2:

\[
\pi_1(0) = -f_1 > \pi_2(0) = -f_2.
\]

A firm with productivity \( \varphi \) will produce good \( i \) if it brings positive profit and is more profitable than producing good \( j \). For both goods to be produced in equilibrium, it is sufficient for \( \pi_1(\varphi) > \pi_2(\varphi) > 0 \) for some values of \( \varphi \) and \( \pi_2(\varphi) > \pi_1(\varphi) > 0 \) for other values of \( \varphi \). Via (10),
these conditions require product 1 profits to increase with productivity 
\( \frac{d\pi_1(\phi)}{d\phi} > 0 \) and for product 2 profits to increase faster than product 1 
profits \( \frac{d\pi_2(\phi)}{d\phi} > \frac{d\pi_1(\phi)}{d\phi} \) for at least a range of values for \( \phi \). The second of 
these conditions is met if and only if
\[
\frac{d\pi_2/d\phi}{d\pi_1/d\phi} = \frac{(1 - \alpha)}{\alpha} \left( \frac{P_2}{P_1} \right)^{\sigma-1} > 1,
\] (11)
where we have imposed, \( \alpha = \alpha_1, \alpha_2 = 1 - \alpha, b_1 = 1, \text{and } b_2 = b. \) Whether 
this inequality is satisfied is independent of \( \phi \).

Two profit curves meeting these conditions are displayed in Figure 1. 
Firms with productivity below \( \phi^* \) exit immediately upon entering the 
market because their productivity is too low to earn positive profit in either 
product market. Firms with productivity between \( \phi^* \) and \( \phi^{**} \) enter the low
fixed cost product market while firms with productivity greater than \( \phi^{**} \) 
enter the high fixed cost product market. With both products demanded 
by consumers in equilibrium, it follows that relative prices will adjust to 
ensure that the condition in equation (11) is satisfied and \( \pi_2 \) intersects \( \pi_1 \) 
from below in the region where positive profits are made in Figure 1.18

Figure 1 provides intuition for two key equilibrium conditions that we 
use to solve the model. First, the **zero profit cutoff condition** determines the lowest level of productivity where product 1 is produced and is 
given by,
\[
(ZP) \quad \pi_1 (\phi^*) = 0.
\] (12)
Second the **product indifference cutoff condition** determines the lowest 
level of productivity where product 2 is produced and is given by,
\[
(PI) \quad \pi_2 (\phi^{**}) = \pi_1 (\phi^{**}).
\] (13)

4.5. **Free Entry**

A third equilibrium condition, the **free entry condition**, equates expected 
firm value and entry costs and is driven by the assumed existence

---

18 Of course, in order for both goods to be produced, we also require that the fixed 
costs of production are not large enough to exhaust the economy’s entire supply of labor. 
This condition must be satisfied as \( f_1 \to 0 \) and \( f_2 \to f_1 \).
of a competitive fringe of potential entrants. The expected value of a firm before entry, $v_e$, consists of two components. The first is the ex ante probability of producing product 1 times the expected profitability of producing this good until death. The second is the ex ante probability of producing product 2 times the expected profitability of producing this good until death. The cost of entry is equal to the sunk cost, $f_e$.

Using the analysis of endogenous production selection in the previous section (as captured in the zero profit and product indifference cutoff conditions), the free entry condition is given by:

\[ (FE) \quad v_e = \frac{G(\varphi^{**}) - G(\varphi^*)}{\delta} \pi_1 + \frac{1 - G(\varphi^{**})}{\delta} \pi_2 = f_e, \quad (14) \]

where $\pi_1$ and $\pi_2$ are the expected or average profits from producing products 1 and 2, respectively. In the Appendix we show that the average profit for each product is a function of the average productivity of firms in each product market. In addition, average firm productivity for each product is a function of the two cutoff productivity levels, $\varphi^*$ and $\varphi^{**}$. 

Figure 1: Profit Versus Productivity for the Two Products
5. Equilibrium

An equilibrium is referenced by the sextuple, \( \{ \varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2 \} \): all other endogenous variables may be written as functions of these variables. Equilibrium is fully characterized by the zero profit cutoff condition, the product indifference cutoff condition, the free entry condition, clearing in the labor market, a steady-state stability condition (constant aggregate variables) and the first order conditions from consumer maximization.

5.1. Equilibrium \( \varphi^* \) and \( \varphi^{**} \) for Fixed Relative Prices

We begin by solving for equilibrium \( \varphi^* \) and \( \varphi^{**} \) holding relative prices, \( P_2/P_1 \), fixed. To do so, we combine the three conditions \( \text{ZP}, \text{PI}, \text{FE} \) to solve for the two unknowns \( \{ \varphi^*, \varphi^{**} \} \). Using these three conditions, we demonstrate the existence and uniqueness of \( \{ \varphi^*, \varphi^{**} \} \) for any value for the relative price that satisfies \( \text{ZP} \) and \( \text{PI} \). Later, we endogenize relative prices and show that there exists a unique equilibrium relative price that satisfies \( \text{ZP}, \text{PI}, \text{and the first order conditions for consumer maximization.} \)

**Lemma 1** For a value of the relative price \( P_2/P_1 \) that satisfies the zero profit condition and the product indifference condition, the equilibrium product indifference cutoff productivity \( \varphi^{**} \) is proportional to the zero profit cutoff productivity \( \varphi^* \). The factor of proportionality is greater than 1 and depends on relative prices: \( \varphi^{**} = \Lambda(P_2/P_1)\varphi^* \) where \( \Lambda(P_2/P_1) > 1 \).

**Proof.** See Appendix. \( \blacksquare \)

For any value of the relative price that satisfies both \( \text{ZP} \) and \( \text{PI} \), \( \pi_2 \) intersects \( \pi_1 \) from below in the region where positive profits are made in Figure 1. The product indifference cutoff productivity \( \varphi^{**} \) is greater than the zero profit cutoff productivity \( \varphi^* \) and, in fact, \( \varphi^{**} \) is proportional to \( \varphi^* \) with the factor of proportionality depending only on relative prices. It follows that \( \varphi^{**} \) can be written as a function of \( \varphi^* \) and relative prices alone, and our system of three equations in two unknowns can be reduced to a single equation \( \text{FE} \) in one unknown \( \varphi^* \).

**Proposition 1 (Existence - \( P_2/P_1 \) Fixed)** For a value of the relative price \( P_2/P_1 \) that satisfies the zero profit condition and the product
**Proposition 2 (Uniqueness - \(P_2/P_1\) Fixed)** For a value of relative price \(P_2/P_1\) that satisfies the zero profit condition and the product indifference condition, a sufficient condition for the equilibrium value of the zero-profit cutoff productivity \(\varphi^*\) to be unique is \(\frac{d\varphi}{d\varphi^*} < 0\) for all \(\varphi\).

**Proof.** See Appendix.

Having solved for the equilibrium value of \(\varphi^*\), equilibrium \(\varphi^{**}\) is obtained immediately from \(ZP\) and \(PI\) at the fixed value of relative prices \((\varphi^{**} = \Lambda(P_2/P_1)\varphi^*)\).

To provide some intuition for the existence of a unique \(\varphi^*\) that satisfies the free entry condition, note that increases in \(\varphi^*\) have two effects on \(FE\). First, an increase in \(\varphi^*\) reduces the ex ante probability that a firm’s realization of productivity is high enough for it to profitably produce one of the two products, thereby reducing the expected value of entry. Second, an increase in \(\varphi^*\) will influence the average profitability of producing each product.

We show in the technical appendix that the average profitability of producing each product may be written in terms of the ratio of average productivity in each product to the zero-profit cutoff productivity \(\varphi^*\). Note that \(\varphi^*\) is the lower threshold for productivity above which production is profitable and, under weak regularity conditions on the distribution \(g(\varphi)\), increases in \(\varphi^*\) will reduce the value of these ratios, reducing average profitability for each product, and reducing the expected value of entry as shown graphically in Figure 2.

The sufficient condition for the expected value of entry to be monotonically decreasing in the zero-profit cutoff productivity is satisfied for a wide class of productivity distributions, including the Pareto, Exponential, and Lognormal (see the technical appendix for further discussion).
5.2. Equilibrium $\phi^*, \phi^{**}$, and Endogenous Relative Prices

We now endogenize relative prices and solve simultaneously for \{\phi^*, \phi^{**}, P_1/P_2\}. To do so, we combine the three conditions ZP, PI, and FE used in the previous section with an expression for relative prices derived from the first order conditions for consumer maximization.

**Proposition 3 (Existence and Uniqueness)**
There exist unique equilibrium values of $\phi^*$, $\phi^{**}$, and $P_2/P_1$ that satisfy the zero profit condition, the product indifference condition, the free entry condition, and the first order conditions for consumer maximization.

**Proof.** See Appendix

The existence of a unique $P_2/P_1$ is best understood in the context of relative supply and demand of the two products. Starting from equilibrium, consider the case of a higher relative price, $P_2/P_1$, for the same values of $\phi^*$ and $\phi^{**}$. On the supply side, an increase in the relative price of product 2 will raise the relative profitability of producing product 2 for every firm.
Firms will want to switch from product 1 to product 2, increasing the relative supply of product 2. On the demand side, a higher relative price for product 2 will reduce the relative quantity of good 2 demanded, as expenditure shares are constant across the products by assumption. Since relative supply has increased and relative demand has fallen, this cannot be an equilibrium and the relative price $P_2/P_1$ must fall in order to re-equate relative supply and demand.\(^{19}\)

Given the equilibrium values $\{\varphi^*, \varphi^{**}, P_2/P_1\}$, we may determine the absolute levels of the price indices $\{P_1, P_2\}$; equilibrium average productivity in each product $\bar{\varphi}(\varphi^*, \varphi^{**})$ and $\tilde{\varphi}(\varphi^*, \varphi^{**})$; and equilibrium average profit in each product $\pi_1$ and $\pi_2$. The next section completes our characterization of the equilibrium sextuple $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$ by solving for revenue in each market $\{R_1, R_2\}$.

5.3. Entry, Exit, and Labor Market Clearing

Two equilibrium conditions not yet used are the steady-state stability (constant aggregate variables) and labor market clearing conditions. The steady-state equilibrium is characterized by a constant mass of firms entering each period $M_e$, a constant mass $M_1$ producing product 1, and a constant mass $M_2$ producing product 2. Each period, a fraction $\delta$ of firms die. A condition for steady-state equilibrium is therefore that the mass of firms who enter and decide to produce a product equals the mass of current producers who die.

Combining the analysis of endogenous product selection above with the constant probability of firm death, these \textbf{steady-state stability conditions} (SC) are,

\begin{equation}
(15) \quad [1 - G(\varphi^{**})]M_e = \delta M_2 \quad \text{ (SC)}
\end{equation}

\begin{equation}
(16) \quad [G(\varphi^{**}) - G(\varphi^*)]M_e = \delta M_1
\end{equation}

The \textbf{labor market clearing condition} (LM), expressed in value terms, requires that total payments to labor equal payments to labor used in production and used in paying the sunk costs of entry,

\begin{equation}
(17) \quad L = L_p + L_e
\end{equation}

\(^{19}\)See the proof of Proposition 3 in the technical appendix for a formal analysis.
where, since labor has been chosen for the numeraire, \( w = 1 \).

Total payments to labor used in production equal the difference between total revenue, \( R \), and total firm profits, \( \Pi \), in the two markets together:
\[ L_p = R - \Pi. \]
Combining FE and SC, total payments to labor used in entry are exactly equal to total profits:
\[ L_e = M_e f_e = \Pi. \]
This result reflects the existence of a competitive fringe of firms entering until the expected value of entry exactly equals the sunk cost of entry.

Combining these expressions for \( L_p \) and \( L_e \) with LM, it follows that total revenue, \( R \), equals total payments to labor, \( L \). Utility maximization implies that consumers devote constant shares of expenditure to the two products, and revenue in the two markets is, therefore, \( R_1 = \alpha L \) and \( R_2 = (1 - \alpha)L \). Thus, we have fully characterized the equilibrium sextuple \( \{\psi^*, \psi^{**}, P_1, P_2, R_1, R_2\} \).

**Proposition 4 (Aggregate Variables)**
The equilibrium sextuple \( \{\psi^*, \psi^{**}, P_1, P_2, R_1, R_2\} \) defines unique equilibrium values of all the other endogenous variables of the model \( \{M_1, M_2, M_e, L_{P_1}, L_{P_2}, L_e, C_1, C_2\} \).

**Proof.** See Appendix □

6. Market Structure and Product Choice

In this section, we examine how changes in market structure alter firms’ product choice and result in product switching. Important features of market structure in the model are: the sunk costs of entry \( (f_e) \), the parameter capturing relative demand for the two products \( (\alpha) \), the fixed costs of production \( (f_1, f_2) \), and the variable cost of production for product 2 relative to product 1 \( (b) \). Changes in each of these parameters result in product switching, with the pattern depending on the interaction between firm and product characteristics that lies at the heart of the model.

For tractability, we focus on the results assuming that the productivity distribution \( g(\varphi) \) is Pareto with parameters \( a \) and \( k \): \( g(\varphi) = ak^a \varphi^{-(a+1)} \)
where $k > 0$, $a > 0$, $\varphi \geq k$; $G(\varphi) = 1 - \left(\frac{k}{\varphi}\right)^a$. \(^{20}\) The technical appendix contains formal derivations of the results presented in this section.

In the interests of brevity, we provide a complete analysis of one aspect of market structure - the sunk costs of entry ($f_e$). These may be thought of as capturing barriers to entry in the industry, and may be of particular interest in so far as they can be directly influenced by policy. The impact of changes in other aspects of market structure are analogous, and we end the section with a more general discussion of how changes in the other parameters of the model influence product switching.

Market structure directly determines the productivity thresholds at which different products are produced ($\varphi^*$, $\varphi^{**}$), the relative price of the two products ($P_2/P_1$), equilibrium average productivity in each market ($\bar{\varphi}(\cdot)$, $\bar{\varphi}(\cdot)$), equilibrium profitability in each market ($\bar{\pi}_1(\cdot)$, $\bar{\pi}_2(\cdot)$), and the equilibrium expected value of entry ($v_e$).

An increase in the sunk costs of entry in the industry ($f_e$) has the following effects on these endogenous outcomes of interest:

\[
\begin{align*}
\frac{\partial \varphi^*}{\partial f_e} &< 0 \\
\frac{\partial (\varphi^{**}/\varphi^*)}{\partial f_e} &= 0 \\
\frac{\partial \bar{\varphi}(\varphi^*,\varphi^{**})}{\partial f_e} &< 0 \\
\frac{\partial \bar{\varphi}^*(\varphi^*,\varphi^{**})}{\partial f_e} &< 0 \\
\frac{\partial \bar{\varphi}^*(\varphi^*,\varphi^{**})}{\partial f_e} &> 0 \\
\frac{\partial \bar{\pi}_1}{\partial f_e} &= 0, \quad \frac{\partial \bar{\pi}_2}{\partial f_e} = 0
\end{align*}
\]

Intuitively, as the sunk costs of entry rise above the expected value of entry, a smaller mass of firms, $M_e$, will enter the industry. For given values of $\varphi^*$ and $\varphi^{**}$, a smaller mass of entrants implies a smaller mass of firms with productivity realizations high enough to produce in each market. This fall in the mass of firms producing in each market increases \textit{ex post} profitability for incumbent firms who have already incurred sunk entry costs and for those firms that continue to enter.\(^{21}\)

\(^{20}\) The distribution of within industry (SIC4) cross-plant labor productivity in the US is well-approximated by a \textit{Pareto} distribution (formally a Pareto-1 distribution). Comparative statics without assuming a particular distribution for firm productivity are available upon request from the authors.

\(^{21}\) One interpretation of a rise in sunk entry costs is a decrease in the extent of product market competition. See Aghion \textit{et al.} (2002) and Nickell (1996) for analyses of the relationship between product market competition and innovation.
The increase in *ex post* profitability means that firms with lower realizations of productivity than before are able to cover the fixed costs of producing product 1. Hence, in equilibrium the zero profit cutoff productivity $\varphi^*$ falls. As $\varphi^*$ falls for a given value of $\varphi^{**}$, this increases the mass of firms in product 1 relative to the mass of firms in product 2, thereby reducing product 1’s relative profitability. Hence, some previously high productivity manufacturers of product 1 now find it more profitable to produce the high fixed cost product 2 and $\varphi^{**}$ also falls. With a *Pareto* productivity distribution, the equilibrium ratio of the two productivity cutoffs, $\varphi^{**}/\varphi^*$, is independent of the sunk costs of entry, and hence $\varphi^{**}$ falls by the same proportion as $\varphi^*$.

The fall in both $\varphi^*$ and $\varphi^{**}$ means that some low productivity firms who previously exited now produce product 1, while some previously high productivity manufacturers of product 1 now produce product 2. For both reasons, (weighted) average productivity in product 1, $\bar{\varphi}(\cdot)$, will fall. Similarly, the fall in $\varphi^{**}$ means that product 2 now includes some lower productivity firms who previously manufactured product 1. Hence, (weighted) average productivity in product 2, $\tilde{\varphi}(\cdot)$, will also fall.

The fall in $\varphi^*$ and $\varphi^{**}$ increases the mass of firms with productivity realizations high enough to produce in each market for a given mass of firms, $M_e$, that enter. With a *Pareto* distribution, this effect exactly offsets the impact of a smaller mass of firms entering the industry on *ex post* profitability, so that expected or average *ex post* profitability in each market $\{\bar{\pi}_1, \bar{\pi}_2\}$ is unchanged. The expected value of entry, $v_e$, rises to equal the new higher sunk costs of entry, $f_e$, because the fall in $\varphi^*$ and $\varphi^{**}$ increases the expected probability of a firm having a productivity realization high enough to be able to profitably manufacture either product 1 or product 2.

The effects of changes in other aspects of market structure on product choice can be seen most clearly from equation 47, which determines the relative value of the two productivity thresholds ($\varphi^{**}/\varphi^*$) with a *Pareto* distribution. Increases in the share of consumer expenditure spent on the low fixed cost product 1 ($\alpha$) raise the relative demand for this product, increasing the relative mass of firms that make product 1, and leading to a rise in $\varphi^{**}/\varphi^*$.

Increases in the fixed production cost for product 2 ($f_2$) reduce relative profitability in this product market, again increasing the relative mass of
firms that make product 1, and leading to a rise in $\varphi^{**}/\varphi^*$. Increases in the fixed production cost for product 1 ($f_1$) have exactly the opposite effect, increasing the relative mass of firms that make product 2, and reducing $\varphi^{**}/\varphi^*$. Changes in the variable production cost for product 2 ($b$) have no effect on the relative value of the two productivity thresholds ($\varphi^{**}/\varphi^*$), with changes in relative variable costs exactly offset by changes in the relative price of the two products (equation 46).

In each case, changes in market structure alter firms incentives to make the two products and result in systematic patterns of product switching. The interaction between firm heterogeneity and product characteristics shapes these patterns of product switching. Thus, when profitability rises in the market for the high fixed cost product 2, it is the more productive firms within product market 1 who switch towards this product. Similarly, when profitability rises in the market for the low fixed cost product 1, it is the less productive firms within product market 2 who switch.

7. Conclusions

This paper documents the importance of product switching by manufacturing firms and develops a model with endogenous product selection. Two thirds of continuing firms change their product mix in any five year period. Multiple-product firms that both drop and add products account for over three quarters of total output. On average, product switches affect more than 40% of firm output and almost half of existing products. Product additions also involve important changes in the industry composition of the firm, extending the range of industries in more than half of all cases.

Motivated by these stylized facts, the paper develops a theoretical model that integrates endogenous product choice into a dynamic analysis of industry evolution with entry and exit and heterogeneous firms. In equilibrium, firm productivity is correlated with product-level fixed costs, with the most productive firms endogenously choosing to make the products with the highest fixed costs. Changes in market structure result in systematic patterns of firm entry/exit and product switching. For example, increases in an industry’s sunk costs of entry make it profitable for less productive firms to survive in the market; result in product switching as the more productive producers of the low fixed cost product 1 ‘switch up’ to the
high fixed cost product 2; and induce a fall in average productivity within product markets.

There are a number of theoretical predictions of the current model that are amenable to empirical testing including the response of product switching to changes in market structure such as the sunk cost of entry. A particularly interesting area for further theoretical research is the introduction of international trade. Existing research suggests a number of ways in which firms in developed countries respond to increased globalization: the death of less productive firms; entry into exporting by high productivity firms; and changes in industry composition as more productive firms expand. More recent empirical research has provided evidence of firms switching industry in response to increased competition from low wage countries. This paper suggests that another important margin along which firms may adjust to increased globalization is through product choice and/or changes in the nature of the production process.
References


A Data Appendix

The data comes from the Censuses of Manufactures (CM) of the Longitudinal Research Database (LRD) of the U.S. Bureau of the Census starting in 1972 and conducted every fifth year through 1997. The sampling unit for the Census is a manufacturing establishment, or plant. Establishments that are not mailed a Census form, so-called 'administrative records (AR)', do not report product-level data. Any firm-year observation that includes an AR establishment is excluded from the sample. Product additions and deletions are recorded only for firms that are present in the sample in neighboring censuses, i.e. we exclude firm births and deaths as well as changes from AR status to regular Census status. Plant acquisitions and sales by firms will potentially result in changes in product mix.

In Census years, plant output is recorded either at the five-digit or seven-digit SIC level of detail. Roughly 7000 of the 15,000 seven-digit categories are recorded directly in the LRD, the rest are recorded at a five-digit level of aggregation. We aggregate seven-digit categories up to their five-digit 'product-class. A small fraction of firm-product-year observations report negative values for output. We recode these values to zero but leave the products in the firm mix (none of the results are sensitive to this choice).

For the most part, five-digit 1987 SIC categories correspond to products that are likely made by different production processes. For example, “Nonferrous Wiredrawing and Insulating” (SIC 3357) includes “Copper Wire”, “Power Wire and Cable”, and “Fiber Optic Cables”. Similarly, “Motor Vehicles” (SIC 3711) contains seven five-digit categories including “Passenger Cars”, “Buses”, “Combat Vehicles”, and four categories of “Trucks” distinguished by weight. Adding or dropping one of these products is likely to entail a substantial change in the production process. The SIC system does not always categorize products into five-digit codes by differences in their production technology. "Canned Fruits and Vegetables" (SIC 2033) contains two five-digit products, “Canned Fruits” and “Canned Vegetables, Except Mushrooms”. Switches between these products are unlikely to involve changes in production technique. Additional work is needed to determine the degree of differences in production technique associated with the product-switching in the data.
**B Technical Appendix**

This technical appendix contains detailed derivations of key results used in the text together with proofs of Lemmas and Propositions.

**B1. Ex Ante and Ex Post Productivity Distributions**

The *ex ante* distribution of firm productivity is \( g(\varphi) \). The *ex ante* probability of successful entry for every firm is \( [1 - G(\varphi^*)] \), where \( G(\cdot) \) is the cumulative distribution of \( g(\cdot) \). The *ex ante* probability of producing the low quality good is \( [G(\varphi^{**}) - G(\varphi^*)] \). The *ex ante* probability of producing the high quality good is \([1 - G(\varphi^{**})]\).

The *ex post* distribution of firm productivity for product 1, \( \mu_1(\varphi) \), is given by the conditional distribution of \( g(\varphi) \) on \([\varphi^*, \varphi^{**})\),

\[
\mu_1(\varphi) = \begin{cases} 
\frac{g(\varphi)}{G(\varphi^{**}) - G(\varphi^*)} & \text{if } \varphi^* \leq \varphi < \varphi^{**} \\
0 & \text{otherwise} 
\end{cases}
\]

while the *ex post* distribution of firm productivity for product 2, \( \mu_2(\varphi) \), is given by the conditional distribution of \( g(\varphi) \) on \([\varphi^{**}, \infty)\),

\[
\mu_2(\varphi) = \begin{cases} 
\frac{g(\varphi)}{1-G(\varphi^{**})} & \text{if } \varphi^{**} \leq \varphi < \infty \\
0 & \text{otherwise} 
\end{cases}
\]

**B2. Aggregate Price Indices**

This section of the appendix demonstrates that the aggregate price indices \( \{P_1, P_2\} \) may be written as functions of the cutoff productivities \( \{\varphi^*, \varphi^{**}\} \) alone.

Equilibrium is characterized by constant masses of firms, \( M_1 \) and \( M_2 \), producing the two products (and hence \( M_1 \) and \( M_2 \) varieties of the products). Aggregate price indices may be written as:

\[
P_1 = \left[ \int_0^\infty p_1(\varphi)^{1-\sigma} M_1 \mu_1(\varphi) \, d\varphi \right]^{1/1-\sigma}
\]

\[
P_2 = \left[ \int_0^\infty p_2(\varphi)^{1-\sigma} M_2 \mu_2(\varphi) \, d\varphi \right]^{1/1-\sigma}
\]
Using the equilibrium pricing rule (5), the price index for each product may be written as proportional to the price charged by a firm with the (weighted) average productivity of all firms producing that product,

\[ P_1 = M_1^{1/1-\sigma} p_1(\tilde{\varphi}), \quad P_2 = M_2^{1/1-\sigma} p_2(\tilde{\varphi}) \] (22)

Weighted average productivity is defined as

\[ \tilde{\varphi}(\varphi^*, \varphi^{**}) = \left[ \frac{1}{G(\varphi^{**}) - G(\varphi^*)} \int_{\varphi^*}^{\varphi^{**}} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{1/(\sigma-1)} \] (23)

\[ \tilde{\varphi}(\varphi^*, \varphi^{**}) = \left[ \frac{1}{1 - G(\varphi^{**})} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{1/(\sigma-1)} \] (24)

where \( \tilde{\varphi}(\cdot) \) and \( \tilde{\varphi}(\cdot) \) are weighted harmonic means of the \( \varphi \)'s. The weights reflect relative output shares of firms with different levels of productivities. Note that these averages depend solely on the two cutoff productivities \{\varphi^*, \varphi^{**}\} and the ex ante distribution \( g(\varphi) \).

As expressed in equation (22), the price indices depend upon the masses of firms producing the two products \{M_1, M_2\} as well as the productivity cutoffs \{\varphi^*, \varphi^{**}\}. In the proof of Proposition 3 below, we show that the relative price \( P_2/P_1 \) can be written as a function of the productivity cutoffs alone (equation 41). The proof of Proposition 4 demonstrates that the equilibrium masses of firms producing the two products \{M_1, M_2\} and the equilibrium mass of firms entering \{M_e\} are ultimately functions of \{\varphi^*, \varphi^{**}, P_2/P_1\} alone.

\section{Average Profitability}

This section of the appendix establishes two related results. First, expected or average profits in each market \{\pi_1, \pi_2\} are a function of the ratio of weighted average productivity (as defined above) to the zero profit cutoff level of productivity. Second, expected or average profits in each market may be written as functions of the zero profit cutoff productivity, the product indifference cutoff productivity, and the relative price, \{\varphi^*, \varphi^{**}, P_2/P_1\}. 

Taking averages in the profit function (6) and using the definitions of weighted average productivity \( \{ \bar{\varphi}(\cdot), \tilde{\varphi}(\cdot) \} \), average profit in each market is equal to the profit of a firm with weighted average productivity:

\[
\pi_1 = \pi_1(\bar{\varphi}), \quad \pi_2 = \pi_2(\tilde{\varphi}).
\]

The relationships between the revenues received by firms with different levels of productivity (equations 7 and 8) imply that the revenue of a firm with weighted average productivity in each market can be expressed relative to the revenue of a firm with the zero profit cutoff level of productivity \( \varphi^* \).

From the zero profit condition (ZP), the revenue of a firm with the zero profit cutoff level of productivity is proportional to the fixed cost:

\[ r_1(\varphi^*) = \sigma f_1 \]

Average profit in the two markets is thus equal to:

\[
\pi_1(\varphi^*, \varphi^{**}) = \left[ \left( \frac{\bar{\varphi}(\cdot)}{\varphi^*} \right)^{\sigma-1} - 1 \right] f_1 \tag{25}
\]

\[
\pi_2(\varphi^*, \varphi^{**}, P_2/P_1) = \left[ \frac{(1 - \alpha)}{\alpha} \left( \frac{P_2}{P_1} \right) \right]^{\sigma-1} \left( \frac{\tilde{\varphi}(\cdot)}{\varphi^*} \right)^{\sigma-1} \frac{f_2}{f_1} \tag{26}
\]

where these expressions depend solely on the zero profit cutoff productivity, the product indifference cutoff productivity, and relative prices \( \{ \varphi^*, \varphi^{**}, P_2/P_1 \} \).

\section*{B4. Proof of Lemma 1}

\textbf{Proof.} The conditions ZP and PI imply,

\[ r_1(\varphi^*) = \sigma f_1 \tag{27} \]

\[ r_2(\varphi^{**}) = r_1(\varphi^{**}) + \sigma (f_2 - f_1) \tag{28} \]

The relationship between the revenues received by firms of different productivity in product 1 (equation 7) implies,

\[ r_1(\varphi^{**}) = \left( \frac{\varphi^{**}}{\varphi^*} \right)^{\sigma-1} r_1(\varphi^*) \tag{29} \]

\[ r_2(\varphi^{**}) = \left( \frac{1 - \alpha}{\alpha} \right) \left[ \left( \frac{P_2}{P_1} \right) \frac{1}{b} \right]^{\sigma-1} r_1(\varphi^{**}) \tag{30} \]
Combining these four equations, we obtain,

$$\varphi^{**} = \Lambda(P_2/P_1)\varphi^*$$  \hspace{1cm} (31)

where

$$\Lambda = \left[ \frac{(\frac{P_2}{P_1} - 1)}{(1-\alpha) \left( \frac{P_2}{P_1} \right)^{\sigma - 1} - 1} \right]^{1/(\sigma - 1)}$$

In order for both conditions $ZP$ and $PI$ to be satisfied, equation (31) must hold. If conditions $ZP$ and $PI$ are both satisfied, both products are produced. Since productivity is a subset of $(0, \infty)$, it follows that equation (31) (and hence both conditions $ZP$ and $PI$) can only be satisfied for values of the relative price such that:

$$\left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{P_2}{P_1} \right)^{\sigma - 1} > 1$$  \hspace{1cm} (32)

Values of the relative price that satisfy this condition imply:

$$\frac{d\pi_2}{d\varphi} = \frac{d\pi_1}{d\varphi} = \frac{(1 - \alpha)}{\alpha} \left( \frac{P_2}{P_1} \right)^{\sigma - 1} > 1$$  \hspace{1cm} (33)

Firms with zero productivity have negative post-entry profits in both industries, with the loss greatest for the high fixed cost product 2:

$$\pi_1 (0) = -f_1 > \pi_2 (0) = -f_2.$$  \hspace{1cm} (34)

Combining the results in (33) and (34), it follows that $\varphi^{**} > \varphi^*$. Hence, a value of the relative price that satisfies both $ZP$ and $PI$ implies $\varphi^{**} = \Lambda(P_2/P_1)\varphi^*$ where $\Lambda(P_2/P_1) > 1$.

B5. Proof of Proposition 1

**Proof.** Substitute the expressions for average profit in equations (25) and (26) into the *free entry condition* $FE$. Combine $ZP$ and $PI$ to obtain the relationship between the cutoff productivity levels in equation (31). Substitute for $\varphi^{**}$ in $FE$ using this relationship. Use the definitions of...
average productivity \( \{ \tilde{\varphi}(\cdot), \bar{\varphi}(\cdot) \} \) in equations (23) and (24) to simplify FE so that it may be written in terms of \( \varphi^* \) alone:

\[
v_e = \frac{f_1}{\delta} \int_{\varphi^*}^{\Lambda \varphi^*} \left[ \left( \frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] g(\varphi) d\varphi \\
+ \frac{f_1}{\delta} \int_{\Lambda \varphi^*}^{\infty} \left[ \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{P_2}{P_1 b} \right)^{\sigma-1} \left( \frac{\varphi}{\varphi^*} \right)^{\sigma-1} - \frac{f_2}{f_1} \right] g(\varphi) d\varphi \\
= f_e.
\] (35)

where \( \Lambda = \Lambda(P_2/P_1) > 1 \) is a positive constant for a fixed value of the relative price \( P_2/P_1 \).

As \( \varphi^* \to 0, v_e \to \infty \). As \( \varphi^* \to \infty, v_e \to 0 \). Since \( v_e \) is a continuous function of \( \varphi^* \), it follows immediately that there is at least one value of \( \varphi^* \) such that \( v_e \) equals the positive constant \( f_e \).

**B6. Proof of Proposition 2**

**Proof.** Using equation (35), we have already shown that as \( \varphi^* \to 0, v_e \to \infty \), and as \( \varphi^* \to \infty, v_e \to 0 \). It follows immediately that a sufficient condition for the equilibrium value of \( \varphi^* \) to be unique is \( \frac{dv_e}{d\varphi^*} < 0 \) for all \( \varphi^* \).
Differentiating in equation (35), we have,

$$\frac{dv_e}{d\phi^*} < 0$$ \hspace{1cm} (36)

$$\Leftrightarrow f_1 \int_{\phi^*}^{\Lambda \phi^*} \left[ \phi^{\sigma-1}(1-\sigma)(\phi^*)^{-\sigma} g(\phi) + \left( \frac{\phi}{\phi^*} \right)^{\sigma-1} - 1 \right] \frac{dg(\phi)}{d\phi} d\phi$$

+ $$f_1 \Lambda [\Lambda^{\sigma-1} - 1] g(\Lambda \phi^*)$$ \hspace{1cm} Term A

$$+ f_1 \int_{\phi^*}^{\infty} \left[ \left( 1 - \frac{\alpha}{\sigma} \right) \left( \frac{P_2}{P_1 b} \right)^{\sigma-1} \phi^{\sigma-1}(1-\sigma)(\phi^*)^{-\sigma} g(\phi) 
+ \left[ \left( 1 - \frac{\alpha}{\sigma} \right) \left( \frac{P_2}{P_1 b} \right)^{\sigma-1} \left( \frac{\phi}{\phi^*} \right)^{\sigma-1} - \frac{f_2}{f_1} \right] \frac{dg(\phi)}{d\phi} \right] d\phi$$ \hspace{1cm} Term B

$$- f_1 \Lambda \left[ \left( 1 - \frac{\alpha}{\sigma} \right) \left( \frac{P_2}{P_1 b} \right)^{\sigma-1} \Lambda^{\sigma-1} - \frac{f_2}{f_1} \right] g(\Lambda \phi^*) < 0$$ \hspace{1cm} Term C

where \((1 - \sigma) < 0\). Note that the sum of Terms B and D may be written as,

$$f_1 \Lambda g(\Lambda \phi^*) \left[ \left( \frac{f_2}{f_1} - 1 \right) - \Lambda^{\sigma-1} \left( \left( 1 - \frac{\alpha}{\sigma} \right) \left( \frac{P_2}{P_1 b} \right)^{\sigma-1} - 1 \right) \right]$$

From the definition of \(\Lambda\) in equation (31), the term in square parentheses is exactly equal to zero. Hence, the sign of \(\frac{dv_e}{d\phi^*} < 0\) depends upon the sign of Term A + Term C.

For a wide class of distributions including the Pareto, Exponential, and Lognormal, the sufficient condition \(\frac{dv_e}{d\phi^*} < 0\) for all \(\phi\) is satisfied.

For example, if productivity is distributed Pareto-1 with parameters \(a > 0\) and \(k > 0\): \(g(\phi) = ak^a \phi^{-(a+1)}\) where \(\phi \geq k\). In this case, substituting \(dg(\phi)/d\phi = -(a+1)ak^a \phi^{-(a+2)}\) into equation (36), Term A + Term C < 0 and \(\frac{dv_e}{d\phi^*} < 0\) for all \(\phi\).
B7. Proof of Proposition 3

**Proof.** We will begin by deriving two relationships between $\varphi^*$, $\varphi^{**}$, and relative prices, the first is a demand-side relationship derived from consumer maximization, while the second is a supply-side relationship derived from ZP and PI.

**Relative Demand:** the first order conditions for consumer maximization imply:

$$\frac{P_2}{P_1} = 1 - \frac{\alpha C_1}{\alpha C_2}$$

(37)

Substituting for the consumption indices (equation 2) and using the CES demand functions for varieties of each product implied by equation (4), this condition for consumer maximization may be re-written as:

$$\frac{P_2}{P_1} = \frac{M_2^{1/1-\sigma}p_2(\varphi)}{M_1^{1/1-\sigma}p_1(\varphi)}$$

(38)

which can also be obtained by taking the ratio of the price indices in equation (22).

The steady-state equilibrium will be characterized by a constant mass of firms entering each period $M_e$, a constant mass $M_1$ producing product 1, and a constant mass $M_2$ producing product 2 yielding the following two steady-state stability conditions:

$$[1 - G(\varphi^{**})]M_e = \delta M_2$$

(39)

$$[G(\varphi^{**}) - G(\varphi^*)]M_e = \delta M_1$$

(40)

Combining equation (38), the expressions for $\varphi'(\cdot)$ and $\varphi'(\cdot)$ from equations (23) and (24) with the steady-state stability conditions from equations (39) and (40), we arrive at the following relative demand condition linking $\varphi^*$, $\varphi^{**}$, and relative prices:

$$\frac{P_2}{P_1} = b \left[ \frac{\int_{\varphi^*}^{\varphi^{**}} \varphi^{\sigma - 1}g(\varphi) d\varphi}{\int_{\varphi^{**}}^{\varphi^*} \varphi^{\sigma - 1}g(\varphi) d\varphi} \right]^{\frac{1}{\sigma - 1}}.$$  

(41)

**Relative Supply:** the zero profit and product indifference cutoff conditions ZP and PI also imply a supply-side relationship between $\varphi^*$, $\varphi^{**}$,
and relative prices. Rearranging equation (31), we obtain the following relative supply condition:

\[
(RS) \quad \frac{P_2}{P_1} = b \left[ \frac{\alpha}{1 - \alpha} \left( \phi^{**} \right)^{1-\sigma} \left( \frac{f_2}{f_1} - 1 \right) + 1 \right]^{1/(\sigma-1)}
\]  

(42)

Combining Relative Demand and Supply: the equilibrium value for the relative price must simultaneously satisfy RD and RS. Figure 3 graphs these two relationships with relative price on the vertical axis, \(P_2/P_1\), and the ratio of the two productivity cutoffs on the horizontal axis, \(\phi^{**}/\phi^*\).

Equation RD implies that relative prices begin from a value of zero at \(\phi^{**}/\phi^* = 1\) and increase monotonically with \(\phi^{**}/\phi^*\). When \(\phi^{**}/\phi^* = 1\):

\[
\frac{P_2}{P_1} = b \left[ \int_{\phi^*}^{\phi^{**}} \phi^{\sigma-1} g(\phi) d\phi \right]^{1/(\sigma-1)} = 0
\]  

(43)

As \(\phi^{**} \to \infty\) for finite \(\phi^*\):

\[
\frac{P_2}{P_1} = b \left[ \int_{\phi^*}^{\phi^{**}} \phi^{\sigma-1} g(\phi) d\phi \right]^{1/(\sigma-1)} \to \infty
\]  

(44)

Differentiating with respect to \(\phi^*\) and \(\phi^{**}\) in equation RD: \(d(P_2/P_1)/d(\phi^*) < 0\) and \(d(P_2/P_1)/d(\phi^{**}) > 0\). Noting that,

\[
d(\phi^{**}/\phi^*) = \frac{\phi^* d\phi^{**} - \phi^{**} d\phi^*}{(\phi^*)^2}
\]

It follows immediately that: \(d(P_2/P_1)/d(\phi^{**}/\phi^*) > 0\).

Equation RS implies that relative prices begin from a positive value \(b \left[ \frac{\alpha}{1 - \alpha} \left( \frac{f_2}{f_1} - 1 \right) + 1 \right]^{1/(\sigma-1)}\) at \(\phi^{**}/\phi^* = 1\) and decline monotonically with \(\phi^{**}/\phi^*\). Combining these results concerning RD and RS, it follows that there is a unique value of \(\phi^{**}/\phi^* > 1\) where both relative demand and supply are satisfied. Figure 3 shows this graphically. The analysis here is the formal proof that, because consumers demand both products in equilibrium, relative prices will adjust until both are produced. Rearranging RS (which combines ZP and PI), \(\phi^{**} = \Lambda(P_2/P_1)\phi^*\), and we have therefore shown that, at the equilibrium value for endogenous relative prices, \(\Lambda(P_2/P_1) > 1\).
It is clear from equation RD that, although \( d \left( \frac{P_2}{P_1} \right) / d \left( \frac{\varphi^{**}}{\varphi^*} \right) > 0 \), the value of this derivative will, in general, depend on the absolute values of \( \varphi^* \) and \( \varphi^{**} \) (an exception is the Pareto distribution considered further below). Nonetheless, given any positive finite value of \( \varphi^* \), we can always evaluate the derivative \( \frac{d(P_2/P_1)}{d(\varphi^{**}/\varphi^*)} > 0 \) for all \( \varphi^{**} \) and construct the relative demand curve in Figure 3.

The equilibrium triplet \((\varphi^*, \varphi^{**}, \hat{P}_2/\hat{P}_1)\) is jointly determined by the free entry condition FE, the zero profit condition ZP, the product indifference cutoff condition PI, and the first order-conditions for consumer maximization as captured in relative demand RD.

Lemma 1 showed how ZP and PI could be combined to yield a single equilibrium condition: \( \varphi^{**} = \Lambda(P_2/P_1)\varphi^* \) (equation (31)). This equilibrium condition is relative supply RS.

Determining the equilibrium triplet \((\varphi^*, \varphi^{**}, \hat{P}_2/\hat{P}_1)\) reduces to determining three unknowns using three equations: FE, RS \( (\varphi^{**} = \Lambda(P_2/P_1)\varphi^*) \) and RD.
Propositions 1 and 2 used FE to determine the equilibrium value of \( \varphi^* \) for any value of the relative price, \( P_2/P_1 \), that satisfies RS. We have now combined RS with RD to determine the equilibrium value for \( P_2/P_1 \) at which RS is satisfied and \( \Lambda(P_2/P_1) > 1 \).

Together, RS and RD determine equilibrium \( P_2/P_1 \) and \( \Lambda(P_2/P_1) = \varphi^{**}/\varphi^* \), while equation FE pins down equilibrium \( \varphi^* \) given \( P_2/P_1 \) and \( \Lambda(P_2/P_1) = \varphi^{**}/\varphi^* \).

**B8. Pareto Distribution**

In this section of the appendix, we consider the special case where productivity has a Pareto distribution. This simplifies the determination of equilibrium in the model. The analysis of comparative statics in the main text exploits some of the results derived here.

Suppose that the productivity distribution \( g(\varphi) \) is Pareto-1 with parameters \( a \) and \( k \): \( g(\varphi) = ak^a\varphi^{-(a+1)} \) where \( k > 0, a > 0, \) and \( \varphi \geq k \). The cumulative distribution function for productivity becomes \( G(\varphi) = 1 - \left( \frac{k}{\varphi} \right)^a \).

Assuming \( a > \sigma - 1 \), the term \( \varphi^{\sigma-1}g(\varphi) \) also has a Pareto distribution with parameters \( \gamma \equiv a - \sigma + 1 \) and \( k \),

\[
\begin{align*}
    h(\varphi) &\equiv \varphi^{\sigma-1}g(\varphi) = \xi\gamma k^\gamma \varphi^{-(\gamma+1)}, \quad k > 0, \gamma > 0, \varphi \geq k \\
    H(\varphi) &\equiv \int_0^\varphi \varphi^{\sigma-1}g(\varphi)d\varphi = \xi \left[ 1 - \left( \frac{k}{\varphi} \right)^\gamma \right]
\end{align*}
\]

where \( \xi \equiv ak^{a-\gamma}/\gamma \) is a positive constant. This property substantially simplifies the expressions for average productivity, average profitability, and the aggregate price indices derived above, all of which contain the term \( \varphi^{\sigma-1}g(\varphi) \).

Using this property in equation (41), we find that, with a Pareto distribution, the relative price \( P_2/P_1 \) depends solely on the relative value of the productivity cutoffs:

\[
\boxed{\text{(RD)} \quad \frac{P_2}{P_1} = b \left[ (\varphi^{**}/\varphi^*)^\gamma - 1 \right]^{1/\sigma-1}.}
\]

Combining this expression for relative demand with equation RS, we are able to derive an explicit expression for the equilibrium value of \( \varphi^{**}/\varphi^* \) as
a function of the parameters of the model:

$$\left(\frac{\varphi^{**}}{\varphi^*}\right)^\gamma - 1 = \frac{\alpha}{1 - \alpha} \left(\frac{\varphi^{**}}{\varphi^*}\right)^{1 - \sigma} \left(\frac{f_2}{f_1} - 1\right) + 1$$

(47)

**Proposition 5** If productivity is distributed Pareto-1 with parameters $a > \sigma - 1 > 0$ and $k > 0$:

(a) There exists a unique equilibrium value for the relative productivity cutoffs $\varphi^{**}/\varphi^* = \Lambda > 1$ that solves equation (47).

(b) The ratio of the product indifference productivity cutoff $\varphi^{**}$ to the zero-profit productivity cutoff $\varphi^*$ is monotonically increasing in
- the share of consumer expenditure on product 1 ($\alpha$)
- the size of the fixed cost of producing product 2 ($f_2$)
and monotonically decreasing in
- the size of the fixed cost of producing product 1 ($f_1$)

(c) The ratio of the product indifference productivity cutoff $\varphi^{**}$ to the zero-profit productivity cutoff $\varphi^*$ is independent of the sunk costs of entry ($f_e$)

**Proof.** The left-hand side of (47) is monotonically increasing in $\varphi^{**}/\varphi^*$ (since $\gamma > 0$), while the right-hand side is monotonically decreasing in $\varphi^{**}/\varphi^*$ (since $\sigma > 1$). Combining this with the value of the left and right-hand sides as one takes the limits $\varphi^{**}/\varphi^* \rightarrow 1$ and $\varphi^{**}/\varphi^* \rightarrow \infty$, yields Proposition 5. ■

The equilibrium condition (47) embodies both RD and RS. Substituting the equilibrium value for $\varphi^{**}/\varphi^* = \Lambda$ into RD, one obtains the equilibrium relative price, $P_2/P_1$. Substituting equilibrium $\varphi^{**}/\varphi^* = \Lambda$ into FE, one obtains equilibrium $\varphi^*$. Thus, we have fully characterized the equilibrium triplet $(\varphi^*, \varphi^{**}, P_2/P_1)$.

**B9. Comparative Statics: Sunk Cost of Entry $f_e$**

This section provides a more formal analysis of the model’s comparative statics, which supplements the intuitive discussion in the main text.

From FE, an increase in the sunk costs of entry $f_e$ requires a corresponding increase in the expected value of entry $v_e$. For a given value of $P_2/P_1$ and for a Pareto distribution which satisfies the conditions specified in Proposition 2, the expected value of entry is monotonically decreasing.
in the zero-profit cutoff productivity $\varphi^*$. From Proposition 5 and equation (46), $\varphi^{**}/\varphi^*$ and $P_2/P_1$ are independent of the sunk costs of entry $f_e$ when productivity is Pareto distributed. Hence, as the sunk costs of entry rise, relative prices will remain unchanged, and to restore equilibrium the zero-profit cutoff productivity $\varphi^*$ must fall so as to increase the expected value of entry equal to the new higher sunk cost.

With relative prices unchanged, $\varphi^{**} = \Lambda(P_2/P_1)\varphi^*$ will also fall by the same proportion, and we have established: $\partial \varphi^*/\partial f_e < 0$, $\partial \varphi^{**}/\partial f_e < 0$, $\partial(\varphi^{**}/\varphi^*)/\partial f_e = 0$, and $\partial(P_2/P_1)/\partial f_e = 0$.

From the definition of (weighted) average productivities, $\bar{\varphi}(\cdot)$ and $\bar{\varphi}(\cdot)$, in equations (23) and (24), and from the analysis of the Pareto distribution in section B8:

$$[\bar{\varphi}(\varphi^*, \varphi^{**})]^{\sigma-1} = \frac{H(\varphi^{**}) - H(\varphi^*)}{G(\varphi^{**}) - G(\varphi^*)}$$

$$= (\varphi^*)^{\sigma-1} \xi \left[ \frac{k^\gamma - (\frac{k}{\varphi^*})^\gamma}{k^\alpha - (\frac{k}{\varphi^*})^\alpha} \right]$$

$$[\bar{\varphi}(\varphi^*, \varphi^{**})]^{\sigma-1} = \frac{1 - H(\varphi^{**})}{1 - G(\varphi^{**})}$$

Since $\sigma > 1$, it follows that $d\bar{\varphi}(\cdot)/d\varphi^* > 0$ and hence $d\bar{\varphi}(\cdot)/df_e = (d\bar{\varphi}(\cdot)/d\varphi^*)(d\varphi^*/df_e) < 0$. Similarly, $d\bar{\varphi}(\cdot)/d\varphi^{**} > 0$ and hence $d\bar{\varphi}(\cdot)/df_e = (d\bar{\varphi}(\cdot)/d\varphi^{**})(d\varphi^{**}/df_e) < 0$.

When productivity is Pareto distributed, relative prices are unchanged following the increase in the sunk costs of entry. Hence, the change in average profitability in equations (25) and (26) depends upon the change in the ratios of (weighted) average productivity, $\bar{\varphi}(\cdot)$ and $\bar{\varphi}(\cdot)$, to the zero profit cutoff level of productivity, $\varphi^*$:

$$\left( \frac{\bar{\varphi}(\varphi^*, \varphi^{**})}{\varphi^*} \right)^{\sigma-1} = \frac{1}{(\varphi^*)^{\sigma-1}} \left[ \frac{H(\varphi^{**}) - H(\varphi^*)}{G(\varphi^{**}) - G(\varphi^*)} \right]$$

$$= \xi \left[ \frac{k^\gamma - (\frac{k}{\varphi^*})^\gamma}{k^\alpha - (\frac{k}{\varphi^*})^\alpha} \right]$$
\[
\left( \frac{\varphi(\varphi^*, \varphi^{**})}{\varphi^*} \right)^{\sigma-1} = \frac{1}{(\varphi^*)^{\sigma-1}} \left[ \frac{1 - H(\varphi^{**})}{1 - G(\varphi^{**})} \right]
\]

(51)

From the above, it is clear that \( \partial \bar{\pi}_1 / \partial \varphi^* = \partial \bar{\pi}_2 / \partial \varphi^{**} = 0 \) and \( \partial \bar{\pi}_1 / \partial \varphi^* = \partial \bar{\pi}_2 / \partial \varphi^{**} = 0 \). Hence, \( \partial \bar{\pi}_1 / \partial f_e = 0 \) and \( \partial \bar{\pi}_2 / \partial f_e = 0 \).

**B10. Proof of Proposition 4**

**Proof. (a)**

\[
M_1 = \frac{G(\varphi^{**}) - G(\varphi^*)}{\delta M_e}
\]

(52)

\[
M_2 = \frac{1 - G(\varphi^{**})}{\delta M_e}
\]

(53)

\[
M_e = \frac{\Pi}{f_e} = \frac{M_1 \bar{\pi}_1 + M_2 \bar{\pi}_2}{f_e}
\]

(54)

Equations (52) to (54) provide three equations which can be solved for the three unknowns \( \{M_1, M_2, M_e\} \).

(b) Total payments to labor used in production in each market \( i \) equal the difference between revenue, \( R_i \), and total firm profits, \( \Pi_i \). Labor is the numeraire, so

\[
L_{p1} = R_1 - \Pi_1 = \alpha \bar{L} - (M_1 \bar{\pi}_1)
\]

\[
L_{p2} = R_2 - \Pi_2 = (1 - \alpha) \bar{L} - (M_2 \bar{\pi}_2)
\]

where \( \bar{\pi}_1 \) and \( \bar{\pi}_2 \) can be written as functions of elements of the equilibrium sextuple \( \{\varphi^*, \varphi^{**}, P_2/P_1\} \), and where \( M_1 \) and \( M_2 \) were determined above. Payments to labor used in entry are:

\[
L_e = M_e f_e
\]

where \( M_e \) was determined above.

(c) Rewriting the first-order conditions for consumer maximization, we
obtain

\[
\begin{align*}
C_1 &= \frac{\alpha R}{P_1} = \frac{\alpha \bar{L}}{P_1} \\
C_2 &= \frac{(1 - \alpha)R}{P_2} = \frac{(1 - \alpha)L}{P_2}
\end{align*}
\]

where \(\{P_1, P_2\}\) were determined as part of the equilibrium sextuple. ■