Testing for Factor Price Equality in the Presence of Unobserved Factor Quality Differences

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Abstract

We develop a method for identifying departures from relative factor price equality across regions that is valid under general assumptions about production, markets and factors. Application of this method to the United States reveals substantial and increasing deviations in relative skilled wages across labor markets in both 1972 and 1992. These deviations vary systematically with labor markets’ industry structure both in the cross section and over time.

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1. Introduction

Variation in relative factor prices across labor markets is influential in determining workers’ susceptibility to international trade shocks, regional income convergence and the spatial location of industries. Persistence in relative factor reward variation over time also sheds light on the degree to which factor mobility is sufficient to arbitrage away factor-price gaps. This paper develops a method for identifying relative factor price differences across regions and uses it to test relative factor price equality in the United States.

Identifying relative factor price equality is a difficult problem for two reasons. First, any method must account for the possibility that factors vary in terms of unobservable quality or composition across labor markets. Regions with superior educational systems or worker training programs, for example, might possess higher-productivity skilled workers than regions without these attributes, thereby inducing higher observed relative skilled wages even if quality-adjusted skill premia are equal. Second, a useful method must correctly identify failures of relative factor price equality in the face of variation in market structure across industries and regions. Such variation in market structure is difficult for econometricians to discern.

Tests of relative factor price equality across countries are common in the international trade literature.\footnote{Empirical tests of factor price equality across countries include Trevenier (1993), Repetto and Ventura (1998), Davis and Weinstein (2001), Cunat (2000), Debaere and Demiroglu (2003) and Schott (2003). Tests for factor price equality within countries include Davis et al. (1997) and Debaere (2004) who study prefectures in Japan, Debaere (2004) who examines administrative regions in the United Kingdom, and Hanson and Slaughter (2002) who analyze U.S. states.} However, the scarcity of internationally comparable wage data has motivated the creation of tests that verify the implications of relative wage variation (e.g. production specialization) rather than differences in relative wages directly.\footnote{Theoretical conditions necessary for factor price equality have been explored by Samuelson (1949), McKenzie (1955), Dixit and Norman (1980), Wu (1987), Courant and Deardorff (1994) and Deardorff (1994).} The outcomes of these tests suggest significant relative factor price differences across developed and developing economies, but these studies do not typically control for unobserved differences in
factor quality and market structure.

Our method for identifying departures from relative factor price equality is based upon general optimality conditions for producer equilibrium. It possesses a number of important advantages over traditional methods. First, our method is valid under a wide range of assumptions regarding production, markets and factors, including imperfect competition and increasing returns to scale. Second, because it makes no assumptions about the preferences and costs of living faced by different groups of workers, it is robust to unobserved variation in consumer price indices specific to locations. Third, it controls for a variety of measurement issues that can cause observed factor prices to vary even if true, unobserved factor prices are identical, in particular region-factor-industry variation in the quality or composition of factors. Finally, it is easy to implement and can be used in a variety of contexts. The only data required are total payments to factors by industry and region.

Implementing our method, we find that the 181 local labor markets in the United States defined by the Bureau of Economic Analysis exhibit statistically significant and economically meaningful differences in non-production worker wages relative to production worker wages in both 1972 and 1992. In 1972, for example, differences in relative skilled wages between Nashville and New York City were around 30 percent. By 1992, this differential had risen to 36 percent. Overall, labor markets exhibit increasing relative-wage polarization: dividing U.S. labor markets into three groups according to the significance of their relative skilled wage differences in both years, we find that the number of labor markets in the “middle” declines with time as the two groups at either end expand.

We find these relative wage differences to be strongly related to labor markets’ industry structure. In the cross-section, we find that the larger the difference in two labor markets’ relative skilled wages, the smaller the number of industries they produce in common. Likewise, in the time-series, i.e. within labor markets across time, we find that greater changes in relative skilled wages are associated with a larger number of added and dropped industries: skill premia and industry mix evolve together.
Neoclassical trade theory provides a useful intuition for these trends. In that framework, sufficient heterogeneity of regional factor endowments combined with factor immobility across regions can give rise to an equilibrium in which regions offer different relative factor prices and attract different sets of industries: skill-scarce regions offer a high skill premium and attract industries intensive in unskilled labor, skill-abundant regions offer a low skill premium and attract skill-intensive industries.\(^3\) These relative wage differences can persist in equilibrium if factor mobility is imperfect or the prices of immobile amenities (e.g. housing) vary across regions and account for different shares of expenditure for skilled and unskilled workers (so that relative wages for the two groups of workers differ across regions but real wages for each group are equalized across regions).\(^4\)

Variation in labor markets’ industry participation is noteworthy because it implies potential asymmetric exposure of otherwise identical U.S. workers to domestic and international shocks. In particular, it may insulate unskilled workers in skill-intensive regions from the well-known distributional consequences of trade liberalization implied by the factor proportions framework. Because skill-scarce labor markets are more likely to produce goods in common with labor-abundant trading partners like Mexico and China, the wages of unskilled workers in these regions may respond more readily – and negatively – to the price declines associated with falling trade costs. These changes in wages will be in turn associated with either population flows across regions or changes in the price of immobile amenities such as housing.

Relative factor price inequality is also informative about the possibility of regional income convergence within countries. Research in the macroeconomic literature, for example, has found sluggish equilibration of relative per worker income levels across U.S. regions over time.\(^5\) Those findings suggest that either relative factor endowments or relative factor prices

\(^3\)In the neoclassical model, this outcome is referred to as a multiple cone equilibrium. See, for example, Leamer (1987).

\(^4\)For empirical evidence of imperfect labor mobility, see Bound and Holzer (2000). For empirical evidence on housing price differences, see for example Glaeser and Gyourko (2005).

\(^5\)See, for example, Barro and Sala-i-Martin (1991) and Carlino and Mills (1993).
are at best converging slowly. Our demonstration of persistent and increasing relative wage disparities provides evidence of the importance of factor prices, while our use of local labor market areas gives a much higher level of spatial resolution than is typical in the literature.

Our method and results also contribute to the large literature on U.S. income inequality. A number of papers have demonstrated that U.S. skill premia have risen precipitously over the past few decades. These studies generally document trends either for the U.S. as a whole or for relatively aggregate regions or states within the United States. Here we examine differences in relative wages across highly-spatially-disaggregated local labor market areas. Our findings of relative wage differences point to the relevance of regional heterogeneity in understanding the evolution of U.S. aggregate income inequality.

Our paper is organized as follows. Section 2 details the relevant propositions on relative factor price equality and develops their testable implications. In Section 3, we outline our empirical methodology. Section 4 provides an overview of U.S. regional variation and presents results for our test of relative factor price equality in 1972 and 1992. Section 5 offers evidence on the relation between industry structure and factor prices. Section 6 discusses possible explanations for our findings and Section 7 concludes.

2. Relative Factor Price Equality

Factor price equality can be either absolute or relative. If absolute factor price equality holds (AFPE), regions have identical nominal factor rewards for identical quality-adjusted factors at a point in time. If relative factor price equality holds (RFPE), regions have identical relative factor rewards for identical quality-adjusted factors even though absolute factor prices can differ.

We devote our theoretical and empirical attention in this paper to a test of relative factor price equality for three reasons. First, a test of relative factor price equality is more stringent

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6 See, for example, Katz and Murphy (1992) and Juhn et al. (1993).

7 Topel (1994), for example, documents a rise in U.S. income inequality across nine U.S. Census regions. An exception is Bound and Holzer (2000), which examines relative wage trends within U.S. metropolitan statistical areas (MSAs).
in the sense that relative factor prices can be equal even if absolute factor price equality fails. Second, as we demonstrate below, relative factor price equality can occur even in the presence of productivity differences across industries and regions. Finally, there is a natural and rich link between variation in regions’ relative factor prices and their industry structure, e.g., skill-intensive industries have an incentive to locate in skill-abundant regions. Nonetheless, in the Appendix, we provide a complementary test for absolute factor price equality.

Our method for identifying departures from factor price equality emphasizes the importance of potential unobserved variation in region-industry-factor quality that can bias traditional wage comparisons. We demonstrate how total payments to each factor, i.e., wagebills, can be exploited to control for this unobserved variation.

2.1. Production Structure

The value-added production function for industry $j$ and region $r$ is assumed to take the following form:

$$Y_{rj} = A_{rj} F_j (S_{rj}, U_{rj}, K_{rj}),$$

where $A_{rj}$ is a Hicks-neutral productivity shifter that allows technology to vary across regions and industries and $S_{rj}$, $U_{rj}$, and $K_{rj}$ are quality-adjusted inputs of skilled workers, unskilled workers, and capital, respectively. While to simplify the exposition, we consider only three factors of production and many industries, the analysis generalizes immediately to the case of arbitrary numbers of factors of production and industries. Individual factors enter production through the function $F_j$, which is assumed to vary across industries but to be the same across regions within an industry. To the extent that industries contain more-disaggregated products, we therefore assume that technology differences across these products within industries are well approximated by differences in the Hicks-neutral productivity shifter $A_{rj}$.

Firms in region $r$ and industry $j$ choose factor usage to minimize costs,

$$\min_{S_{rj}, U_{rj}, K_{rj}} \quad w_r S_{rj} + w_r^U U_{rj} + w_r^K K_{rj}$$

such that

$$A_{rj} F_j (S_{rj}, U_{rj}, K_{rj}) = Y_{rj}$$
which defines the total cost function,

\[ B_{rj} = A_{rj}^{-1} \Gamma_j (w^S_r, w^U_r, w^K_r) Y_{rj}. \]  

(3)

As our analysis exploits cost minimization, firms can act either as price-takers in product markets (perfect competition; this section) or choose prices subject to a downward sloping demand curve (imperfect competition; next section). While we begin by assuming constant returns to scale, later we extend the analysis to allow for internal and external increasing returns to scale. Similarly, while we begin by assuming that factor markets are perfectly competitive, our analysis can be extended to incorporate labor market imperfections, as long as employment continues to be chosen to minimize costs given factor prices.\(^8\)

We use a tilde (\(\tilde{\cdot}\)) to signify observed quantities that have not been adjusted for quality, and use \(\theta^z_{rj}\) to denote a quality adjustor for industry \(j\), region \(r\) and factor \(z\). Note that \(\theta^z_{rj}\) allows for unobserved variation in quality that is specific to factors, regions and industries.

The quality-adjusted employment level and wage of factor \(z \in (S, U, K)\) in region \(r\) equals the observed variable scaled by the quality adjustor, i.e.

\[ z_{rj} = \theta^z_{rj} \tilde{z}_{rj} \quad \text{and} \quad w^z_{rj} = \tilde{w}^z_{rj} / \theta^z_{rj}. \]  

(4)

where, without loss of generality, we choose units in which to measure the quality of each factor of production such that factor quality in a reference or base region \((b)\) is equal to one \((\theta^z_{bj} = 1)\).

In our baseline formulation in (1) and (4), we assume that output depends solely on quality-adjusted units of each factor of production \((z_{rj})\) and not on their composition between physical units of the factor of production \((\tilde{z}_{rj})\) and quality \((\theta^z_{rj})\). As a result, units of a given factor of production are perfect substitutes up to a vertical adjustment for differences in

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\(^8\)Our analysis is therefore consistent with ‘right to manage’ models of union behavior, where firms and unions bargain over wages within an industry but firms choose employment (see, for example, Farber 1986 and Layard et al. 1991). For clarity of exposition, we focus on the competitive case in the text, where wages are equalized across industries. With industry-specific bargaining, wages will generally vary across industries. As discussed further below, our empirical approach allows for inter-industry wage differentials, because it exploits variation across regions within industries.
factor quality. In a later section, we relax this assumption to allow each factor of production (e.g. skilled workers) to consist of many different types (e.g. managers and engineers), which are horizontally and vertically differentiated from one another. In that later extension, factor quality ($z_{rj}$) corresponds to an index number that controls for differences in both quality and composition.

The demand for quality-adjusted factor $z$ may be obtained using Shephard’s Lemma,

$$z_{rj} = A_{rj}^{-1} Y_{rj} \frac{\partial \Gamma_j(\cdot)}{\partial w_r^z}. \quad (5)$$

Dividing one first-order condition by another provides an expression for the relative demand for any two quality-adjusted factors of production. The relative demand for skilled workers in terms of unskilled workers is

$$\frac{S_{rj}}{U_{rj}} = \frac{\partial \Gamma_j(\cdot)/\partial w_r^S}{\partial \Gamma_j(\cdot)/\partial w_r^U}. \quad (6)$$

Notice that terms in region-industry productivity, $A_{rj}$, do not appear in equation (6), since with Hicks-neutral technology differences the direct effect of variation in technology on the marginal revenue product is identical for each factor. In contrast, if technology differences across regions within industries are non-neutral, they will affect the relative marginal revenues of skilled and unskilled labor, and induce variation in the relative demand for these factors of production. Region-industry variation in relative goods prices also has symmetric direct effects on the marginal revenue product of every factor and hence does not appear in equation (6). Using the relationship between quality-adjusted and observed values in (4), this implies the following relative demand for observed factors of production,

$$\frac{\tilde{S}_{rj}}{\tilde{U}_{rj}} = \frac{\theta^U_{rj} \partial \Gamma_j(\cdot)/\partial w_r^S}{\theta^S_{rj} \partial \Gamma_j(\cdot)/\partial w_r^U}. \quad (7)$$

\footnote{Of course, region-industry variation in productivity and relative goods prices has general equilibrium effects on relative factor prices through output of each good and hence relative factor demands. To the extent that these differences in relative factor demands induce differences in relative factor prices, our test below will reject relative factor price equality.}
2.2. Null Hypothesis of Relative Factor Price Equality (RFPE)

Under the null hypothesis that all relative factor prices are equalized (RFPE), *quality-adjusted* relative wages and factor usage across regions $r$ and $b$ must be equal,

$$\frac{w^S_r}{w^U_r} = \frac{w^S_b}{w^U_b} \text{ and } \frac{S_r}{U_r} = \frac{S_b}{U_b},$$

(8)

where the second equation follows directly from equation (6).\(^{10}\)

Under this null hypothesis of RFPE, *observed* relative wages and factor usage across regions are given by:

$$\frac{\bar{w}^S_r}{\bar{w}^U_r} = \frac{\theta^S_{rj} \bar{w}^S_{rj}}{\bar{U}_{rj}} \text{ and } \frac{\bar{S}_{rj}}{\bar{U}_{rj}} = \frac{\bar{S}_{bj}}{\bar{U}_{bj}}/\theta^S_{rj}.$$  

(9)

These relationships demonstrate the difficulty of using either observed relative wages or observed factor usages to test for factor price equality. Even under the null hypothesis of RFPE, observed relative wages and usages can vary across regions within industries as a result of differences in unobserved factor quality (i.e. $\theta^S_{rj} \neq 1$ or $\theta^U_{rj} \neq 1$).\(^{11}\) We solve this problem by combining observed wages and employment into wagebills, where the wagebill for factor $z$ is equal to $w^z_{rj} z_{rj} = \bar{w}^z_{rj} \bar{z}_{rj}$. As is evident from equation (9), multiplying wages and employment causes region-industry-factor quality adjustors to drop out. As a result, observed relative wagebills, which are generally available to empirical researchers, are equal under the null hypothesis of RFPE,

$$(H_0: \text{RFPE}) \quad \frac{\text{wagebill}^S_{rj}}{\text{wagebill}^U_{rj}} = \frac{\text{wagebill}^S_{bj}}{\text{wagebill}^U_{bj}}.$$  

(10)

\(^{10}\)Homogeneity of degree one of the cost function implies that the derivatives $\partial \Gamma_j/\partial w^z_r$ are homogenous of degree zero in factor prices. It follows immediately from equation (6) that, with identical quality-adjusted relative factor prices, regions will employ quality-adjusted factors of production in the same proportions.

\(^{11}\)As the factor quality of the base region has been normalized to equal one, $\theta^z_{bj} = 1$, $\theta^z_{rj} \neq 1$ indicates that factor quality differs in industry $j$ between the base region and region $r$. 
2.3. Alternative Hypothesis of Non-Relative Factor Price Equality (non-RFPE)

Under the alternative hypothesis of non-RFPE, the \( \text{quality-adjusted} \) relative \( \frac{w^S}{w^U} \) wage differs across regions \( r \) and \( b \) by a multiplicative factor, \( \gamma_{rb}^{SU} \),

\[
\frac{w_r^S}{w_r^U} = \gamma_{rb}^{SU} \frac{w_b^S}{w_b^U},
\]

\[\text{(11)}\]

where again we let region \( b \) be the benchmark region: \( \gamma_{rb}^{SU} = \gamma_{r}^{SU} / \gamma_{b}^{SU} \) and \( \gamma_{b}^{SU} = 1 \). Across regions, observed relative wages now vary because of differences in factor quality and because of variation in true wages,

\[
\frac{\bar{w}_r^S}{\bar{w}_r^U} = \gamma_{rb}^{SU} \frac{\theta_{rj}^S \bar{w}_b^S}{\theta_{rj}^U \bar{w}_b^U}.
\]

\[\text{(12)}\]

Additionally, observed factor usage now varies across regions because of both differences in factor quality and differences in factor demand driven by the variation in quality-adjusted relative factor prices:

\[
\frac{\bar{S}_{rj}}{\bar{U}_{rj}} = \gamma_{rb}^{SU} \frac{\partial \Gamma_{j}(\cdot)/\partial w_r^S}{\partial \Gamma_{j}(\cdot)/\partial w_r^U} \left( \frac{\partial \Gamma_{j}(\cdot)/\partial w_b^S}{\partial \Gamma_{j}(\cdot)/\partial w_b^U} \right) \frac{\bar{S}_{bj}}{\bar{U}_{bj}}.
\]

\[\text{(13)}\]

Multiplying the expressions for observed relative factor prices and observed relative employments (equations 12 and 13), the terms in unobserved factor quality again cancel. However, relative wagebills now generally vary across regions because of differences in factor prices and variation in factor usage,

\[
(H_1: \text{Non-RFPE}) \quad \frac{\text{wagebill}_{rj}}{\text{wagebill}_{rj}} = \eta_{rbj}^{SU} \frac{\text{wagebill}_{bj}}{\text{wagebill}_{bj}},
\]

\[\text{(14)}\]

where

\[
\eta_{rbj}^{SU} = \gamma_{rb}^{SU} \left( \frac{\partial \Gamma_{j}(\cdot)/\partial w_r^S}{\partial \Gamma_{j}(\cdot)/\partial w_r^U} \right) \left( \frac{\partial \Gamma_{j}(\cdot)/\partial w_b^U}{\partial \Gamma_{j}(\cdot)/\partial w_b^S} \right).
\]

\[\text{(15)}\]

2.4. Interpretation

Together equations (10) and (14) provide the basis for a test of the null hypothesis of RFPE that is robust to unobserved region-industry variation in factor quality. The
intuition underlying this method is that, although the empirical researcher cannot observe factor quality or quality-adjusted factor prices, observed factor prices contain information about the quality of observed factors when firms minimize costs. Multiplying observed factor prices by observed factor quantities enables us to control for unobserved variation in factor quality.

We note that our derivation of the relative wage bill test above makes a number of identifying assumptions: cost minimization, constant returns to scale, Hicks-neutral technology differences, and vertical differentiation of factors of production. Additionally, the null hypothesis that we are testing is that all relative factor prices are equalized.\textsuperscript{12} One potential explanation for differences in relative wage bills across regions is therefore differences in the prices of other factors of production that have varying degrees of complementarity and substitutability with skilled and unskilled workers. However, while our test of RFPE is a joint test of our identifying assumptions and the null hypothesis that all relative factor prices are equalized, its ability to control for unobserved differences in factor quality is an important advantage relative to other possible approaches. Furthermore, in subsequent sections below, we show how our identifying assumptions can be relaxed to allow for example for increasing returns to scale and for both horizontal and vertical differentiation of factors of production.

If RFPE fails, there are two effects on the relative wage bill for an industry across regions. The first is given in equation (15) directly by the difference in relative wages, $\gamma_{rb}^{SU}$. The second effect, inside the brackets, is due to differences in relative factor usage caused by the variation in relative wages, and thus is also a function of $\gamma_{rb}^{SU}$. Under the assumption that the production technology for a given industry exhibits a constant elasticity of substitution (CES) across all factors of production ($\sigma_j = 1/(1 - \rho_j)$, where $\rho_j$ is the CES parameter for industry $j$), we obtain the following expression for the differences in relative wage bills:

$$\eta_{rbj}^{SU} = \gamma_{rb}^{SU} \left[ (\gamma_{rb}^{SU})^{1/(\rho_j-1)} \right] = (\gamma_{rb}^{SU})^{\rho_j/(\rho_j-1)}.$$  \textsuperscript{(16)}

\textsuperscript{12}With perfect capital mobility, the rate of return to capital may be equalized across regions. However, as long as there is imperfect mobility of at least one other factor of production, quality-adjusted relative factor prices will in general vary.
Testing for Factor Price Equality

Therefore, while our test for RFPE using relative wage bills does not require us to make an assumption about the particular functional form of the production technology, it is possible to recover the underlying relative wage differences ($\gamma_{rb}^{SU}$) from the estimates of relative wage bill differences ($\eta_{rbj}^{SU}$) if one makes the additional assumption of a CES production technology and assumes a value for the elasticity of substitution.

We note that finding $\eta_{rbj}^{SU} \neq 1$ in our relative wage bill test is sufficient but not necessary to reject RFPE. This can be seen by considering the special case of a Cobb-Douglas production technology ($\rho_j = 0$ in equation 16), in which case relative wage bills are equalized ($\eta_{rbj}^{SU} = 1$) even in the presence of differences in quality-adjusted relative wages ($\gamma_{rb}^{SU} \neq 1$). Nevertheless, to the extent that we find relative wage bill differences ($\eta_{rbj}^{SU} \neq 1$), this result is sufficient for us to reject the null hypothesis of RFPE ($\gamma_{rb}^{SU} \neq 1$). As shown below, relative wage bills in fact vary substantially across U.S. local labor markets, and therefore the Cobb-Douglas assumption does not appear to provide a close approximation to the data.

2.5. Imperfect Competition

As our relative wage bill test exploits cost minimization, we show in this section that it is robust to the introduction of imperfect competition. Suppose that firms maximize profits subject to a downward sloping inverse demand curve, $v_{rj}(Y_{rj})$, under conditions of imperfect competition, which implies the following first-order condition for profit-maximization:

$$\frac{dv_{rj}(Y_{rj})}{dY_{rj}} Y_{rj} + v_{rj}(Y_{rj}) - \frac{\Gamma_j(\cdot)}{A_{rj}} = 0. \quad (17)$$

Defining the elasticity of demand as $\varepsilon_{rj}(Y_{rj}) \equiv -(dY_{rj}/dv_{rj}) v_{rj}/Y_{rj}$ where $v_{rj}$ denotes price, we obtain the standard result that equilibrium price is a mark-up over marginal cost,

$$v_{rj}(Y_{rj}) = \left(\frac{\varepsilon_{rj}(Y_{rj})}{\varepsilon_{rj}(Y_{rj}) - 1}\right) \frac{\Gamma_j(\cdot)}{A_{rj}}. \quad (18)$$

Applying Shephard’s Lemma, equilibrium demand for each quality-adjusted factor of production continues to be given by the derivative of the total cost function with respect to the

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13 Indeed, the fact that ($\gamma_{rb}^{SU} \rho_j/(\rho_j - 1)$) is close to 1 for $\rho_j$ close to 0 actually makes it harder to reject the null hypothesis of RFPE and strengthens any finding of a rejection.
factor price as specified in equation (5). Therefore the introduction of imperfect competition leaves the derivation of the test for relative factor price equality above unchanged.

2.6. External Economies of Scale

Our framework can also be extended to incorporate external economies of scale under either perfectly or imperfectly competitive market structures. External economies of scale correspond to the assumption that technical efficiency in a region-industry is a function of scale. In the most general case, we have,

\[ A_{rj} = A_{rj}(Y_{rj}, Y_{r,-j}, Y_{-r,j}, Y_{-r,-j}) \]  

(19)

where \( Y_{r,-j} \) is the vector of outputs in all other industries in a region, \( Y_{-r,j} \) is the vector of all other regions’ outputs in the industry, and \( Y_{-r,-j} \) is the vector of all other regions’ outputs in all other industries. Because the cost-minimization behavior of the firm is the same (see equation 2), the derivation of the test for relative factor price equality again remains unchanged.

2.7. Internal Economies of Scale

Our analysis can also incorporate internal economies of scale, which clearly must be combined with imperfect competition, and imply that the cost function (3) is no longer linearly homogenous of degree one in output. Under imperfect competition, equilibrium price continues to be a mark-up over marginal cost,

\[ v(Y) = \left( \frac{\varepsilon(Y)}{\varepsilon(Y) - 1} \right) \frac{1}{A_{rj}} \frac{\partial \Gamma_j(w_r^S, w_r^U, w_r^K, Y)}{\partial Y}. \]  

(20)

Furthermore, equilibrium demand for quality-adjusted factors of production can again be obtained from Shephard’s Lemma, and the relative demand for observed skilled and unskilled workers is given by,

\[ \frac{\tilde{S}}{\tilde{U}} = \frac{\theta_r^{U_j} \partial \Gamma_j(w_r^S, w_r^U, w_r^K, Y)/\partial w_r^S}{\theta_r^{S_j} \partial \Gamma_j(w_r^S, w_r^U, w_r^K, Y)/\partial w_r^U}. \]  

(21)
Multiplying the expressions for observed relative factor prices and observed relative employments, the terms in unobserved factor quality again cancel. The expression for relative wagebills becomes,

$$\frac{\widetilde{\text{wagebill}}_{rj}^S}{\widetilde{\text{wagebill}}_{rj}^U} = \gamma_{rb}^{SU} \left( \frac{\partial \Gamma_j(\cdot)}{\partial w_r^S} \right) \left( \frac{\partial \Gamma_j(\cdot)}{\partial w_r^U} \right) \frac{\widetilde{\text{wagebill}}_{bj}^S}{\widetilde{\text{wagebill}}_{bj}^U}$$

(22)

where the terms in brackets capturing relative unit factor input requirements are now a function of output, $Y$.

In the standard case of trade under internal economies of scale in the theoretical literature (Helpman and Krugman 1985), firms within an industry face the same constant elasticity of substitution $\varepsilon_j$, cost functions are homothetic and identical within industries, and there is free entry so that price equals average cost. Combining free entry with the pricing relationship in (20), the equilibrium ratio of average to marginal cost will equal a constant $\varepsilon_j/(\varepsilon_j - 1)$, which with homothetic cost functions defines a unique equilibrium value of output for all firms in the industry.

Under the null hypothesis of RFPE, $\gamma_{rb}^{SU} = 1$, and with all firms in the industry facing the same factor prices and producing the same output, the terms in parentheses in (22) cancel. Therefore we again obtain the prediction that relative wagebills are equalized under the null hypothesis of RFPE.\(^{14}\) More generally, in the presence of internal economies of scale, variation in firm size across regions and industries can influence relative factor demands and provides a potential explanation for rejections of RFPE.

2.8. Factor Quality and Factor Composition

While our analysis has so far assumed vertical differentiation of factors of production, in this section we show that the analysis can be extended to allow each factor of production (e.g. skilled workers) to consist of many different types (e.g. managers and engineers), which are horizontally and vertically differentiated from one another. We assume that the

\(^{14}\)See Helpman and Krugman (1985) for further analysis of theoretical models of monopolistic competition and increasing returns to scale with factor price equalization.
production technology is weakly separable in skilled and unskilled workers, so that firms first choose optimal quantities of skilled and unskilled workers before choosing optimal amounts of worker types within these categories. We demonstrate the point formally for skilled workers, but, without loss of generality, the argument applies for any factor of production. Though, for simplicity, we consider two types of skilled workers, the analysis goes through for any number of skill types. For notational convenience, we suppress region and industry subscripts throughout this section.

Assume the quality-adjusted flow of skilled labor services is a constant returns to scale function of the quality-adjusted flow of managerial and engineering services:

$$S = \phi(S_1, S_2)$$

$$= \phi \left( \frac{S_1}{(\bar{S}_1 + \bar{S}_2)}, \frac{S_2}{(\bar{S}_1 + \bar{S}_2)} \right) (\bar{S}_1 + \bar{S}_2)$$

$$= \phi \left( \theta^{S_1} \bar{n}_1, \theta^{S_2} \bar{n}_2 \right) \tilde{S},$$

where $S$ is quality-adjusted skilled labor services, $S_1$ is quality-adjusted managerial services, $S_2$ is quality-adjusted engineering services, $\phi(\cdot)$ is assumed to be linearly homogenous of degree one, $\tilde{S} = \bar{S}_1 + \bar{S}_2$ is the observed number of skilled workers, $\theta^{S_1}$ is the quality of managers, $\theta^{S_2}$ is the quality of engineers, and $\bar{n}_1 \equiv \bar{S}_1/(\bar{S}_1 + \bar{S}_2)$ and $\bar{n}_2 \equiv \bar{S}_2/(\bar{S}_1 + \bar{S}_2)$ are the observed shares of engineers and managers in skilled employment. Equation (23) may be re-written more compactly as:

$$S = \theta^S \tilde{S}, \quad \theta^S \equiv \phi \left( \theta^{S_1} \bar{n}_1, \theta^{S_2} \bar{n}_2 \right)$$

where the quality of skilled workers ($\theta^S$) is now an index number, which captures the quality of managers, the quality of engineers, and the composition of skilled workers between these two groups.

The quality-adjusted wage of skilled workers is now a price index, defined as the dual to equation (23):

$$w^S = \psi(\omega_1, \omega_2)$$

(25)
where $\omega_1$ is the quality-adjusted wage of managers and $\omega_2$ is the quality-adjusted wage of engineers.

Expenditure on quality-adjusted skilled worker services is equal to observed expenditure on skilled workers:

$$w^S S = \tilde{w}^S \tilde{S}$$  \hspace{1cm} (26)

where $w^S$ is the price index defined above and $\tilde{w}^S$ is the observed wage per skilled worker. It follows that the quality-adjusted skilled worker price index and the observed skilled worker wage are related according to:

$$w^S = \tilde{w}^S / \theta^S, \quad \theta^S \equiv \phi (\theta^{S_1 \tilde{n}_1}, \theta^{S_2 \tilde{n}_2}) .$$  \hspace{1cm} (27)

It is evident from equations (24) and (27) that the derivation of the test for relative factor price equality remains exactly the same as above and is unchanged by this extension.

3. Econometric Specification

In Section 2 we showed that under the null of RFPE the ratio of the skilled workers’ wagebill to the unskilled workers’ wagebill is the same across regions within an industry. This implies that, for an industry $j$, each region’s relative wagebill equals the value for any base region $b$ and, in particular, for the United States as a whole,

$$\frac{\text{wagebill}^S_{rj}}{\text{wagebill}^U_{rj}} = \frac{\text{wagebill}^S_{bj}}{\text{wagebill}^U_{bj}} = \frac{\text{wagebill}^S_{US,j}}{\text{wagebill}^U_{US,j}}.$$  \hspace{1cm} (28)

The simplest test of the null hypothesis is therefore to regress the log of the ratio of wagebills for region $r$ relative to the ratio for the U.S. on a set of region dummies,

$$\ln \left( \frac{RB_{rj}^{SU}}{RB_{US,j}^{SU}} \right) = \sum_r \alpha^{SU}_r d_r + \varepsilon^{SU}_{rj}$$  \hspace{1cm} (29)

where $RB_{rj}^{SU}$ denotes the relative wagebill in industry $j$ and region $r$ for skilled workers and unskilled workers ($RB_{rj}^{SU} = \text{wagebill}^S_{rj} / \text{wagebill}^U_{rj}$); $RB_{US,j}^{SU}$ is the corresponding relative wagebill for the U.S. as a whole; and the $\alpha^{SU}_r$ correspond to the coefficients on the
Testing for Factor Price Equality

regional dummies $d_r$. Note that we exclude the own region $r$ when defining the relative wagebill for the U.S. as a whole. Under the null hypothesis of RFPE, $\alpha_r^{SU} = 0$ for all regions and factor pairs, and a test of whether the $\alpha_r^{SU}$ are jointly equal to zero therefore provides a test of RFPE.

The regression in equation (29) corresponds to a differences in means test. We choose the aggregate U.S. as a base region and test RFPE by comparing the relative wagebill for an industry $j$ across all regions $r$ to the value for the aggregate U.S. in the same industry.

We also test RFPE by allowing individual regions to be the base region. That is, we begin by choosing a region $b$ to be the base (where $\gamma_b^{SU} = 1$) and run a regression analogous to equation (29),

$$\ln \left( \frac{RWB^{SU}_{rj}}{RWB^{SU}_{bj}} \right) = \sum_r \alpha^{SU}_{rb} d_r + \varepsilon^{SU}_{rjb}. \quad (30)$$

A test of whether the $\alpha^{SU}_{rb}$ are jointly equal to zero provides a test of the null hypothesis of RFPE. Rejecting $\alpha^{SU}_{rb} = 0$ is sufficient to reject the null hypothesis of RFPE, and any pair of regions $r$ and $r'$ face the same relative factor prices if $\alpha^{SU}_{rb} = \alpha^{SU}_{r'b}$. To avoid problems with the choice of the base region, we estimate equation (30) for all possible choices of base region $b$.

Although regions have the same relative wagebills under the null hypothesis of RFPE (hence $\alpha^{SU}_{rb} = 0$), the theoretical analysis of Section 2 suggests that, under the alternative hypothesis, the coefficients on the regional dummies ($\gamma^{SU}_{rb}$ in equation 14 and $\alpha^{SU}_{rb}$ in equations 29 and 30) may vary across industries. With a constant elasticity of substitution (CES) production technology, this cross-industry variation is associated with different elasticities of substitution between skilled and unskilled workers (equation 16). We have no strong priors on the industry variation in the elasticity of substitution between different types of labor or in other features of the operator $\Gamma_j$ in the cost function (equation 3), and therefore we pool observations across industries. Since under the null hypothesis, $\alpha^{SU}_{r'bj} = 0$, holds for all industries $j$, a finding of statistically significant coefficients on the regional dummies when pooling observations is sufficient to reject RFPE.
While our test for relative factor price equality does not require us to assume a particular functional form for the production technology, under the assumption of a CES production technology and given a choice for the elasticity of substitution $\sigma$, the estimated coefficients on the regional dummies may be used to derive implied quality-adjusted relative wages and unobserved factor quality across regions via equation (16). We use this result to interpret our empirical results below.

Note that equations (29) and (30) compare the relative wagebill for skilled and unskilled workers in region $r$ to the value in a base region within each industry $j$. This is a ‘difference in differences’ specification with a number of attractive statistical properties. Any industry-specific determinant of relative wagebills that is common across regions is ‘difference-out’ when we normalize relative to the base region on the left-hand side of the equations (e.g. compensating differentials across industries, other inter-industry wage differentials, industry-specific labor market institutions such as the degree of unionization, differences across industries in the classification of workers between skilled and unskilled). The analysis thus explicitly controls for observed and unobserved heterogeneity in the determinants of relative wagebills across industries.

Similarly, in both region $r$ and the base region we analyze the wagebill of skilled workers relative to unskilled workers. Therefore, any region-specific determinant of wagebills that is common to both skilled and production workers is ‘difference-out’ when we construct a region’s relative wagebill ($RWB_{rj}^{SU} = \text{wagebill}_{sj}^{S} / \text{wagebill}_{uj}^{U}$). Here potential examples include neutral regional technology differences and compensating differentials across regions that are common to skilled and unskilled workers, e.g. region-specific differences in the cost of living.

4. **Empirical Implementation**

In this section we apply our methodology to test for relative factor price equality across 181 U.S. labor markets in 1972 and 1992.
4.1. Data

We examine wagebills across the 181 Labor Market Areas (LMAs) that make up the continental United States (Alaska and Hawaii are excluded). LMAs, constructed by the Bureau of Economic Analysis, are aggregations of counties that are based on commuting patterns and therefore correspond closely to the concept of regional labor markets where wages are determined (see Johnson and Spatz 1993 for more detail). LMAs are permitted to cross state lines, and more than one labor market may appear in each state. As a result, LMAs provide greater resolution of relative factor price variation than more aggregate geographic units such as states or Census regions.\(^\text{15}\)

Data on total payments to production (unskilled) and non-production (skilled) workers for 1972 and 1992 by industry and labor market area are obtained from the Censuses of Manufactures in the Longitudinal Research Database (LRD) collected by the U.S. Bureau of the Census.\(^\text{16}\) As noted above, our methodology explicitly controls for differences in the quality and composition of these two categories of workers across regions and industries. We use four-digit Standard Industrial Classification (SIC4) industry classification to focus on narrowly-defined industries for which the assumption of a common production technology up to a Hicks-neutral productivity shifter is likely to be a more reasonable approximation. For the same reason, we exclude any four-digit industry code that explicitly includes miscellaneous products (i.e., SIC4 codes ending in ‘9’). This pruning leaves us with 401 of the original 458 SIC4 industries covering 88 percent of manufacturing output and 86 percent of manufacturing employment.

\(^{15}\)A number of studies (e.g. Topel 1986; Lee 1999, Bound and Holzer 2000, Hanson and Slaughter 2002, and Bernard and Jensen 2000) document variation in income inequality or wages across either the nine U.S. Census regions or across U.S. states. Related work using wage regressions by Heckman et al. (1996) finds that worker characteristics are priced differently across U.S. Census regions.

\(^{16}\)Our sample covers all manufacturing establishments in the continental United States for which information on production and non-production workers is available. This sample excludes very small plants that do not report information on their inputs. Other data sources, such as the Decennial Census, collect more detailed information on worker wages and observed characteristics than does the LRD. However, these surveys generally record the industry of the worker at a very aggregate level of activity. Furthermore, sampling in these datasets does not ensure proportional representation by region-industry limiting their usefulness for testing relative factor price equality.
4.2. Testing RFPE

Table 1 reports the results of testing for relative factor price equality across LMAs using the U.S. average as the base region (equation 29). The data easily reject the null hypothesis of RFPE across regions within the United States for both 1972 and for 1992.\footnote{The hypothesis that all the LMA coefficients are equal to zero is rejected at the 1 percent level in both 1972 and 1992.} In 1972, 37 (55) regions have relative wagebills significantly different from the U.S. average at the 5 (10) percent level of significance. In 1992, 64 regions reject at the 5 percent level and 74 at the 10 percent level.

The relative wagebill results in Table 1 can be used to estimate relative skilled wage differences in individual labor markets by assuming CES production, as noted in equations (15) and (16) above. Nashville and New York City, for example, have significantly different relative wagebills for non-production and production workers, and thus significantly different relative wages. In 1972 the average relative wagebill across all industries in Nashville is 10 percent below the U.S. average while that for New York is 15 percent above. Twenty years later, the gap between the two labor markets had widened to 34 percent. Assuming CES production technologies and an elasticity of substitution of 2 between production and non-production workers (i.e. $\rho = 0.5$) in both years, these wagebill differences imply that quality-adjusted relative wages were 1.30 and 1.36 times higher in Nashville than in New York in 1972 and 1992, respectively.\footnote{An elasticity of substitution between skilled and unskilled workers greater than unity is consistent with empirical estimates in the labor literature (Katz and Autor 1999).} Therefore, while the absolute level of skilled wages can be higher in New York than in Nashville, and while factor quality and composition can vary between the two locations, our estimates imply that skilled workers of the same quality receive lower relative wages in New York than in Nashville.

We assign LMAs to factor-price cohorts based on the sign and significance (at the 10 percent level) of the coefficients reported in Table 1. Figures 1 and 2 display the distribution of regions across cohorts for 1972 and 1992, respectively. Regions with relative skilled...
wagebills that are significantly higher than those for the aggregate United States are grouped in cohort A (black shading), while those with relative skilled wagebills that significantly lower are assigned to cohort C (cross-hatching). The remaining labor markets, with relative skilled wagebills that are not significantly different from the U.S. as a whole, are placed in cohort B. Regions in cohort A have higher relative wagebills and thus lower relative wages for skilled workers of the same quality, while regions in cohort C have higher relative wages for skilled workers of the same quality. Using the same assumption for the elasticity of substitution between factors as above, we estimate the average quality-adjusted relative skilled wage to be 11 percent higher and 21 percent lower than the national average in cohorts C and A, respectively, in 1972. The comparable percentages for 1992 are, respectively, 10 percent higher for C and 16 percent lower for A.

As indicated in Table 1 and highlighted in Figures 1 and 2, there is substantial movement of labor markets across cohorts between 1972 and 1992. In 1972 there are 9, 126, and 46 labor markets in the A, B, and C cohorts, respectively. The corresponding figures are 16, 107, and 58 for 1992. Twenty-seven labor markets jump to a higher relative wagebill cohort over the sample period, while 34 regions drop to a lower relative wagebill cohort. These movements suggest a polarization of U.S. labor markets across wage cohorts over time.

Our second specification for testing for relative factor price equality is the complete set of bivariate regressions captured by equation (30). Because there are far too many coefficients to report (32,580 per year when every region is used as a base), we report a summary of rejections in Table 2. In 1972, 19 percent of the region-pairs reject relative factor price equality at the 10 percent level, while 13 percent reject at the 5 percent level. Every region rejects with at least 3 other regions. In 1992, 24 percent of the region pairs reject relative factor price equality at the 10 percent level, 17 percent reject at the 5 percent level. Every region rejects with at least 3 other regions.

Both specifications provide strong evidence against the hypothesis that all regions in the United States have the same factor price. The extent of this evidence is supported by the hypothesis that there is no significant difference in the factor prices across regions. The results from both specifications are consistent with the hypothesis that there are significant differences in factor prices across regions, which is consistent with the hypothesis that there is significant polarization of U.S. labor markets across wage cohorts over time.

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19Disclosure of individual coefficients from Table 2 is also not possible under Title XIII of the Bureau of Census.
United States face the same relative factor prices in either 1972 or 1992. After controlling for unobserved variation in factor quality, we find that labor markets in the U.S. vary significantly in terms of relative wages, and that relative wages for workers of the same quality are lower in areas with greater supplies of skilled workers. In the next section we explore the link between relative wage variation and industry structure.

5. RFPE and Industry Specialization

As an additional check on our estimates of relative factor price differences, we examine whether they are systematically related to differences in production structure, as implied by neoclassical theories of trade and production. If there is a single cone of diversification where all regions produce all goods with the same production technologies and facing the same goods prices, neoclassical theory implies that factor prices are equalized when the number of goods is equal to or greater than the number of factors of production. In contrast, if there are multiple cones of diversification, neoclassical theory implies that regions with different relative factor prices specialize in different sets of industries. Skill-abundant regions with lower quality-adjusted relative skilled wages specialize in skill-intensive industries, while labor-abundant regions with higher quality-adjusted relative skilled wages specialize in labor-intensive industries.

To investigate whether our estimates of differences in quality-adjusted relative wages are systematically related to production structure, we begin in Table 3 by examining the extent of industry overlap among labor markets. The first two rows report the minimum, median and maximum percent of regions per industry, i.e., the breadth of industry production across regions. The median industry is produced in 34 percent of regions in 1992, up from 28 percent of regions in 1972. As indicated in the final column, some industries, like cement, are produced in every region.

The middle two rows of the table report the minimum, median and maximum percent of industries per region, i.e., the variety of industrial production within regions. No region
produces all industries in either year; the most ‘diverse’ region manufactures 84 percent of all industries in 1972 and 86 percent in 1992. The median region increases its scope from 13 to 20 percent of all industries between 1972 and 1992.

The final two rows of Table 3 characterize the extent of bilateral industry overlap among regions. The percent of industries that two regions have in common is defined as the number of industries produced in both regions divided by the number of industries produced in the region with the larger number of industries. As indicated in the table, no two regions produce the same set of industries, though the extent of overlap increases with time.

We now test whether larger differences in relative skilled wages across labor markets are associated with smaller overlaps in the industries they produce, both in the cross-section and over time.

5.1. Industry Mix Across Regions

We examine whether the overlap in industry mix between two regions is negatively correlated with the difference in their quality-adjusted relative factor prices by running an OLS regression of the number of industries two regions have in common on the distance between regions’ relative wagebills,

$$COMMON_{rb} = \delta_0 + \delta_1|\alpha_{rb}^{SU}| + \beta_r I_r + \beta_b I_b + \epsilon_{rb},$$

(31)

where $|\alpha_{rb}^{SU}|$ is the absolute value of the regression coefficient from equation (30), $COMMON_{rb}$ is the number of industries that regions $r$ and $b$ produce in common, and $I_r$ and $I_b$ are the number of industries produced by region $r$ and $b$, respectively.\(^{20}\) Separate estimation results for 1972 and 1992 are reported in Table 4. These results indicate that regions with more dissimilar wagebill ratios have fewer industries in common. The point estimates suggest that a pair of regions with the maximum estimated differences in relative wages would have 17 and 28 fewer industries in common in 1972 and 1992, respectively. Two regions with the

\(^{20}\)While the relative wage bill differences $\alpha_{rb}^{SU}$ are estimated from industries that exist in both regions, we now examine the extent to which there are other industries that are present or one or other of two regions but not both.
median number of industries would have few, if any, industries in common if they exhibited
the maximum differences in relative wages in each year.\footnote{We note that regional product mixes may not be mutually exclusive because some goods with very high transport costs, such as cement, are largely untraded.}

5.2. Industry Mix Over Time

If differences in quality-adjusted relative wages influence production location decisions, regions experiencing larger changes in their relative wages over time are likely to exhibit greater churning of their industry mix in terms of adding and dropping industries. To examine this relationship, we run an OLS regression of the form,

\[
CHURN_r = \alpha + \beta_d \left| \alpha_{r,92}^{SU} - \alpha_{r,72}^{SU} \right| + \epsilon_r,
\]

where the dependent variable, \(CHURN_r\), is the percent of industries either added or dropped by region \(r\) between 1972 and 1992 relative to its number of industries in 1972, and \(\left| \alpha_{r,92}^{SU} - \alpha_{r,72}^{SU} \right|\) is the absolute value of the change in region \(r\)’s wagebill ratio relative to the U.S. between 1972 and 1992 (Table 1). These changes range from 0.005 to 0.6 with a median of 0.07. Results are reported in Table 5. Consistent with the idea that relative wages influence production location decisions, we find that industry churning and changes in estimated wagebill ratios are positively and significantly correlated. The implied value of \(CHURN_r\) for the median change in relative wagebill ratios is 7.5 percentage points.

Taken together, the results of Tables 4 and 5 indicate that our estimates of differences in quality-adjusted relative wages are indeed systematically related to differences in production structure, as expected from the neoclassical theory of trade and production.

6. Conclusions

This paper develops a methodology for testing whether factor prices are equal across geographic regions. It is based on cost minimization by firms and invokes only general assumptions about production, markets and factors. In particular, the method can identify departures from relative factor price equality in the presence of unobserved variation in the
quality and composition of factors of production across both industries and regions. The test is relatively easy to implement in that it requires data only on the total payments to factors (e.g. wagebills) by industry and region. Even though the researcher cannot observe factor quality or quality-adjusted factor prices, observed relative wage bills contain information on unobserved quality-adjusted relative factor prices when firms minimize costs.

We use our methodology to test for relative factor price equality across 181 U.S. labor markets areas in 1972 and 1992. The data reject the null hypothesis that all regions offer the same relative factor prices in both years. Results indicate substantial relative wage variation across skill-scarce and skill-abundant labor markets. We also find that the estimated differences in relative wages are systematically related to industrial structure: the greater the difference in relative wages across a region pair, the greater the difference in the pair’s industry structure. This relationship is also evident within regions across time: regions experiencing larger changes in relative wages between 1972 and 1992 undergo larger changes in the set of industries they produce.

The association we find between regions’ relative wages and their industry structure suggests U.S. labor markets may be asymmetrically exposed to domestic and external shocks: a common industry shock can have heterogeneous effects across regions depending on the industries in which they are specialized. Further examination of this link has potential to shed light on several issues in economics, including the ability of skill-scarce regions to catch up with skill-abundant regions, the impact of trade liberalization on U.S. relative wages, and the effects of asymmetric shocks in optimum currency areas.

Finally, we note that our approach to characterizing factor price inequality might usefully be applied to other settings where unobserved variation in quality is an important problem for identification. A similar test based on consumer expenditure minimization, for example, could be developed to test the law of one price across geographic areas.
References


A Appendix A: Absolute Factor Price Equalization (AFPE)

This appendix develops a test for absolute factor price equality that controls for unobserved factor quality. Like our test for relative factor price equality, it makes use of the result that factor quality terms cancel when observed wages and observed employment levels are multiplied.

To test absolute factor price equalization (AFPE) we analyze variation across regions in the share of total payments to a factor of production in output. Though our demonstration here is for skilled workers, the analysis for other factors of production is analogous. Observed employment of skilled workers may be obtained from equations (4) and (2). Multiplying observed employment by observed wages and dividing by output, we obtain,

\[
\frac{\bar{w}_r^S s_{rj}}{Y_{rj}} = \frac{w_r^S s_{rj}}{Y_{rj}} = \frac{w_r^S \partial \Gamma_j(\cdot)}{A_{rj} \partial w_r^S}.
\]

where, from the total cost function (3), \( A_{rj}^{-1} \Gamma_j(\cdot) \) is the unit cost function and \( A_{rj}^{-1} \partial \Gamma_j(\cdot)/\partial w_r^S \) corresponds to the unit input requirement for quality-adjusted skilled labor.

Under the null hypothesis of AFPE, quality-adjusted wages are equal across regions \((w_r^S = w_b^S)\) and observed wages vary in direct proportion to unobserved factor quality \((\bar{w}_r^S = \theta_{rj}^S w_b^S)\), where we again choose region \(b\) as a reference region so that \(\theta_{bj} = 1 \ \forall j\). Additionally, the equality of the absolute level of factor prices requires identical production technologies across regions and industries \((A_{rj} = A_{bj})\). Therefore, combining identical production technologies and identical quality-adjusted factor prices, we obtain the prediction that unit input requirements for quality-adjusted factors are the same across regions. Therefore, under the null hypothesis of AFPE, factor shares in equation (33) are equalized across regions,

\[
(H_0 : \text{AFPE}), \quad \frac{w_r^S s_{rj}}{Y_{rj}} = \frac{w_b^S s_{bj}}{Y_{bj}}.
\]

Under the alternative hypothesis of non-AFPE, technical efficiency may vary across region-industry pairs, and regions may be characterized by different quality-adjusted factor prices.
and hence different unit input requirements for quality-adjusted factors. As a result, from equation (34), factor shares in the two regions are related as follows:

\[
(H_1 : \text{non-AFPE}), \quad \frac{w_r^S S_{rj}}{Y_{rj}} = \gamma_{rb}^S \left( \frac{A_{bj}}{A_{rj}} \right) \left( \frac{\partial \Gamma_j(\cdot)}{\partial \Gamma_j(\cdot)/\partial w_r^S} \right) \left( w_b^S S_{bj} \right) \left( \frac{Y_{bj}}{Y_{rj}} \right). \tag{35}
\]

Together, equations (34) and (35) provide the basis for a test of the null hypothesis of AFPE, with AFPE implying a testable parameter restriction in equation (35).
Figure 1: Labor Market Areas and Relative Wagebill Groups - 1972
Figure 2: Labor Market Areas and Relative Wagebill Groups - 1992
Table 1: Coefficients of Regression of Region Relative Wagebill on US Average Relative Wagebill

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**Significant at the 10% level; ***Significant at the 5% level; ****Significant at the 1% level.**
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Notes: Table summarizes rejections of relative factor price equality from estimation of equation 30.

Table 2: Summary of Bilateral Region-Pair RFPEQ Rejections from Estimation of Equation 18

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<td>100</td>
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<tr>
<td></td>
<td>1992</td>
<td>3</td>
<td>34</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Industries per Region as a Percent of All Industries</th>
<th>Year</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1972</td>
<td>2</td>
<td>13</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>1992</td>
<td>2</td>
<td>20</td>
<td>86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bilateral Overlap as a Percent of the Larger Region’s Industries</th>
<th>Year</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1972</td>
<td>5</td>
<td>32</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>1992</td>
<td>6</td>
<td>34</td>
<td>93</td>
</tr>
</tbody>
</table>

Table 3: Overlap of Four-Digit SIC Industries Across US Labor Market Areas
### Table 4: Regional Industry Overlap As a Function of Relative Wagebill Disparity

<table>
<thead>
<tr>
<th></th>
<th>1972</th>
<th>1992</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Industries Common to Regions r and s</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Wagebill Disparity</td>
<td>-12.9</td>
<td>-23.1</td>
</tr>
<tr>
<td></td>
<td>-0.8</td>
<td>-1.3</td>
</tr>
<tr>
<td>Industries in Region r</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>-0.003</td>
<td>-0.003</td>
</tr>
<tr>
<td>Industries in Region s</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td>Constant</td>
<td>-4.9</td>
<td>-13.3</td>
</tr>
<tr>
<td></td>
<td>-0.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>Observations</td>
<td>16,290</td>
<td>16,290</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.68</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Notes: OLS regression results. Dependent variable is number of industries produced in common by regions r and s. Robust standard errors noted below each coefficient.
<table>
<thead>
<tr>
<th>1972 to 1992 Change in Wagebill Ratio</th>
<th>107.6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-33.3</td>
</tr>
</tbody>
</table>

Constant

<table>
<thead>
<tr>
<th>Constant</th>
<th>79.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-4.4</td>
</tr>
</tbody>
</table>

Observations

<table>
<thead>
<tr>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>181</td>
</tr>
</tbody>
</table>

R^2

<table>
<thead>
<tr>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
</tr>
</tbody>
</table>

Notes: OLS regression results of changes in region industry structure on changes in relative wagebill ratio over time. Dependent variable is the percent of industries added or dropped by region r between 1972 and 1992 relative to its number of industries in 1972. The first independent variable is the absolute value of the change in region r's wagebill ratio relative to the U.S. between 1972 and 1992 (i.e., the coefficients listed in Table 3). Robust standard errors noted below each coefficient.

Table 5: Industry Churning versus Relative Wagebill Changes, 1992 versus 1972