1. Introduction

Monitoring and detection of compliance violations are major challenges to the enforcement of environmental regulations. Each year, a large number of pollution events go undetected because they occur sporadically in dispersed locations; it has been estimated that as much as 95% of total toxic chemical emissions may go unreported (U.S. GAO 1991). Compounding this problem is the fact that many of these violations occur unexpectedly, making it difficult for regulatory bodies such as the U.S. Environmental Protection Agency (EPA) to plan systematic enforcement actions. A recent study of emissions at refineries and chemical plants reveals a large amount of accidental pollutant releases, caused by equipment malfunctions such as leaks, tripped compressors, and power outages (Environmental Integrity Project 2012). The study also notes that most of these violations occur unexpectedly, making it difficult for regulatory bodies such as the U.S. Environmental Protection Agency (EPA) to plan systematic enforcement actions. A recent study of emissions at refineries and chemical plants reveals a large amount of accidental pollutant releases, caused by equipment malfunctions such as leaks, tripped compressors, and power outages (Environmental Integrity Project 2012).

These “upset emissions” have been a source of controversy for many years, especially because the EPA has maintained a loose policy for regulating them, allowing for various exceptions and delegating enforcement actions to state and local authorities (U.S. EPA 1983). Whereas advocacy groups have labeled this a loophole and demanded an overhaul of the policy (see U.S. EPA 2013, 2014a, b for recent updates), that the regulations have not been rigorously enforced reflects the difficulties associated with monitoring random violations with limited budget and resources. Therefore, it is important to devise a cost-effective monitoring mechanism tailored to detecting random violations and mitigating their impact.

The main monitoring tool employed by regulators— inspections—is costly to implement. Moreover, the unpredictable nature of violations limits the effectiveness of inspections. As a way of alleviating the burden of inspections and saving costs, regulators have adopted policies that encourage voluntary disclosure by offering legal and financial incentives to violating firms (U.S. Environmental Protection Agency 2000). A commonly used incentive is eased sanctions, i.e., if a firm discloses noncompliance, it is rewarded with reduced penalty. (Although the incentive role is not explicitly stated, the EPA policy governing upset emissions grants penalty exemptions if a violation is promptly self-reported (U.S. EPA 2013); see §7 for more discussions.) For such incentive-based schemes to be effective, however, they should be combined with appropriate levels of inspections and penalties that act as credible threats.

The academic literature has studied this incentive problem from the perspective of economic theory. Building on the notion of probabilistic law enforcement (Becker 1968), a number of researchers have developed game-theoretic models to investigate how incentives can be designed to elicit voluntary disclosure and complement enforcement actions. A subset of them focus on how they operate in stochastic settings, motivated by the examples of equipment malfunctions that cause accidental pollution (Malik 1993,
Livernois and McKenna 1999). While they capture the realities of random violations to some degree, these studies are based on simplifying assumptions that ignore important details on the nature of stochasticity and how they impact the efficiency of enforcement actions. In particular, the timing aspect has not received the attention that it deserves: in the presence of stochastically evolving states of compliance, when should a regulator perform an inspection, and when should a firm disclose noncompliance?

In this paper, we examine these questions by developing a novel analytical framework, inspired by “inspection models” found in the theory of reliability. These model features are naturally built in a production setting where a firm experiences occasional equipment breakdowns that expose the environment to pollution. We enrich this model base by adding an element of incentives, namely, the firm’s voluntary disclosure of noncompliance. To represent this, we build a dynamic model of disclosure in which the firm makes a self-interested decision to either keep silence about noncompliant equipment or disclose the state preemptively to avoid the severe penalty that may follow if he is “caught” in the future. Anticipating such behavior, the regulator employs one of two inspection policies: random inspections (RI) and periodic inspections (PI). Under RI, the regulator randomizes the inspection intervals by sampling from a probability distribution, whereas under PI, the regulator performs inspections according to a set schedule of constant intervals. This new model framework allows us to address the following research questions that have been overlooked in the literature. Given that environmental violations occur randomly over time and that the firm may disclose noncompliance selectively, should the regulator perform random or periodic inspections? How does the firm’s opportunistic disclosure behavior impact the design of two main enforcement levers, inspection intensity and penalties?

We find that, contrary to the common belief found in the literature and in practice, there exist situations in which performing PI is more efficient than performing RI. Even though PI allow the firm to perfectly anticipate the upcoming inspections and act more opportunistically, such an adverse effect does not necessarily lead to a net loss in efficiency. We also find that the firm’s opportunistic disclosure timing behavior may lead to a partial disclosure equilibrium in which the classic result from Becker (1968) is reversed; instead of penalty substituting for inspections, these two enforcement levers may act as complements. Further analysis reveals that these insights are robust to a number of model variations. The timing aspect of disclosure and inspections has been ignored in the literature of environmental law enforcement, and we add a new dimension to the body of knowledge by focusing on its impacts.

2. Related Literature

Our model integrates elements from two distinct streams of literature which, to the best of our knowledge, have never been put together in a single problem setting, which are: reliability theory and law enforcement economics. As we demonstrate, ideas from each of these areas—developed in isolation over the years—bring new perspectives to the topic of environmental regulation once they are combined.

Among many models that have been proposed in the literature of reliability theory (see Rausand and Hoyland 2004 for an overview), the ones that have direct relevance to our problem are the models of inspection policies in the context of machine maintenance (Barlow and Proschan 1996, pp. 107–118). These models assume that the state of a system, i.e., whether the system is functioning or not, is normally invisible to a system operator. As a result, a system failure is not reported unless it is discovered in a costly inspection. The models suggest optimal inspection schedules based on probabilistic representations of random failure processes. A variant of the inspection models particularly relevant to our problem is the “intermittent faults” model, in which a system restores itself after each instance of repeated failures (Su et al. 1978; Nakagawa 2005, pp. 220–224). Because of the random process, detection of a failure is not guaranteed.

The inspection models provide the mathematical foundations that are well suited for analyzing the problem of environmental compliance monitoring. First, a direct analogy can be made between loss of output due to random system failures and pollution caused by random malfunctions of pollution control equipment. Second, inspections are necessary for discovering compliance violations, which are often hidden from the public’s view. Third, the repeated nature of compliance violations is captured in the inspection models, which evaluate long-term strategies for managing recurring failures. What the inspection models lack is the dimension of incentives, which we discuss next.

The basic premise of the inspection models, namely, that constant monitoring is prohibitive because inspections are costly, was recognized in the economics literature by Becker (1968). In his model of probabilistic law enforcement, an enforcement authority combines random audits with sanctions (e.g., fines or imprisonment) to maximize the probability of apprehending the violators. This combination influences the potential violator’s behavior and determines the level of criminal activities, an element missing in the inspection models. (For a recent survey of law enforcement economic theories, see Polinsky and Shavell 2007.)

Becker’s (1968) seminal work has been extended to the context of regulating environmental violations; see Cohen (1999) for a survey of this area. As Cropper and Oates (1992) note, there are two types of environmental violations that the literature discusses: willful and stochastic. Our work falls into the latter category, as the firm in our model takes actions after an unintended random violation occurs. Other papers that consider stochastic environmental violations include Beavis and Walker (1982), Harrington (1988), Malik (1993), Innes (1999a, b), and Livernois and McKenna (1999). Among these, Harrington (1988) and
Livernois and McKenna (1999) are closest to our paper in problem motivation, as stochasticity in their model originates from imperfect reliability of a “pollution abatement device.” However, their stylized representation of reliability does not capture the timing decisions, a central feature of our model. Russell (1990) is one of the few papers that represent stochasticity as a Markov process, but the focus is on random errors in audits instead of random occurrences of noncompliance. Harrington (1988) considers a game between a regulator and firms in a dynamic setting and pays special attention to the case where penalties are restricted, as we do, but he does not discuss the main issues we focus on in our paper.

In environmental and nonenvironmental applications, a stream of literature examines firms’ self-reporting behavior when they are subject to inspections and sanctions imposed by a regulator, a key feature of our model. Harford (1987) and Malik (1993) consider firms under mandatory disclosure requirements who may misrepresent the severity of violations. Our paper is more in line with Kaplow and Shavell (1994) in that they incorporate voluntary disclosure; similar to our model but unlike in Harford (1987) and Malik (1993), individuals may choose not to report violations depending on the economic trade-offs they face. In contrast to our paper, however, Kaplow and Shavell (1994) focus on self-reporting of premeditated violations instead of stochastic violations. Innes (1999a, b) and Friesen (2006) combine self-reporting with cleanup efforts and study how they interact with each other. Although our model does not include such post-violation remediation, a similar mitigating role is played in our model by the suspension of production that follows a discovery of noncompliance, which stops ongoing or impending pollution. This preventive benefit of disclosure has received little attention in the literature, and it is naturally built in our model because disclosure decisions are made in continuous time. Livernois and McKenna (1999) consider a setting similar to ours and explain the coexistence of high compliance and low expected fines. Our analysis shows a similar result, but we focus on other aspects of the problem. A number of researchers have empirically examined regulated firms’ self-reporting behavior, including Pfaff and Sanchirico (2004), Short and Toffel (2007), Stafford (2007), and Toffel and Short (2011). A topic related to self-reporting is self-auditing (e.g., Mishra et al. 1997, Pfaff and Sanchirico 2000, Khanna and Widyawati 2011), which we do not address because our model assumes that the firm can costlessly monitor its compliance state.

Despite some commonalities shared with these works, our paper distinguishes itself in a number of important aspects. First, the way we model stochastic environmental violations is more realistic than what has appeared in the literature. Unlike the stylized representations found in these works (e.g., “low” and “high” emissions with assigned probabilities), our model features a Markov chain of compliance or noncompliance states that alternate over time. Second, in contrast to other existing works that represent audits or inspections simply as an exogenously given point measure of probability, we bring precision by modeling repeated inspections, which endogenously determine the probability of detection. Finally, the firm in our model decides not only whether noncompliance should be disclosed but also when it should be disclosed, taking into account random state transitions and future inspections. Such opportunistic disclosure timing behavior has been largely ignored in the literature, and as we show in our analysis, it generates dynamics that bring new insights.

The problem of environmental compliance monitoring and disclosure has been studied by researchers in operations management in recent years, including Kalkanci et al. (2012), Plambeck and Taylor (2012, 2014), and Jira and Toffel (2013). They investigate various topics in this area, but none of them have compared the merits of random versus periodic (or unannounced versus announced) inspections or developed a dynamic model of disclosure, as we do in our paper. From a modeling perspective, our paper shares similarities with Kim et al. (2010) and Kim and Tomlin (2013) in that game-theoretic analysis is combined with elements of reliability theory to represent low-probability, high-consequence events. In sum, our paper is uniquely positioned in terms of research focus, insights, and model features that bridge the gap between distinct areas of research.

3. Model

3.1. Overview

A firm (“he”) produces a good and generates revenue from its sales. A regulator (“she”) monitors the firm’s production activity and enforces environmental regulations. Both are risk neutral. The firm’s production requires use of an environmentally harmful substance (“pollutant”), which normally does not enter into the environment thanks to emission control equipment (“equipment”) that the firm operates. However, this equipment occasionally breaks down. If production continues while equipment is down, the pollutant is emitted and causes environmental damage. Production and sales start at time zero and last over an infinite horizon. Production runs continuously at a constant rate unless it is suspended by the regulator to prevent environmental damage. While production is suspended, the firm loses sales opportunities.

In addition to suspending production, the regulator wields two enforcement levers: inspections and penalties. Inspections enable the regulator to discover the equipment status. Penalties are imposed on the firm after equipment malfunction is reported. To avoid the severe penalty associated with a nonreport, the firm may disclose equipment malfunction preemptively.

Before time zero, the regulator sets the inspection frequency and penalty amounts, subsequently announcing them to the firm. The regulator does not update these decisions once they are announced. In response, the firm...
chooses whether or not he should disclose equipment malfunction. We seek a subgame perfect equilibrium of this two-stage game in which the regulator moves first as the Stackelberg leader. The regulator makes decisions to maximize the expected social welfare per unit time, while the firm makes decisions to maximize his expected profit per unit time. Note that we do not consider the firm’s premeditated polluting actions or attempts to falsify evidence of pollution, subjects beyond the scope of this paper.

### 3.2. Compliance States

At any given moment, the firm’s emission control equipment is in one of two states: equipment is said to be in compliance if it is functioning, whereas equipment is in noncompliance if it is not. The equipment compliance and noncompliance states alternate as a two-state continuous-time Markov chain with the rates $\lambda$ and $\mu$, respectively. Hence the firm stays in the compliance state for an exponentially distributed amount of time with mean $1/\lambda$ and in the noncompliance state for an exponentially distributed amount of time with mean $1/\mu$. The rate $\mu$ is interpreted as the capacity to restore compliance, which the firm has installed before production starts. We assume that $\lambda \ll \mu$, i.e., noncompliance is a rare occurrence. We refer to each occurrence of noncompliance as a noncompliance episode, defined by start and end times. Initially at time zero, equipment is in compliance. The probability that equipment is in noncompliance at time $t \geq 0$ given that it is in compliance at time zero is (Nakagawa 2005, p. 221)

$$\theta(t) = \frac{\lambda}{\lambda + \mu} (1 - e^{(-\lambda+\mu)t}).$$

(1)

In the main part of the paper (§§4 and 5), we assume that the transition rates $\lambda$ and $\mu$ are exogenously given and unaffected by managerial interventions. This simplifying assumption allows us to isolate the impact of the firm’s disclosure, the focus of our paper. In §6.2, we relax this assumption and consider a scenario where the firm invests in compliance improvement, thereby determining $\lambda$ endogenously.

If equipment is in compliance during production or if production is suspended following a report of noncompliance, no pollutant is emitted to the environment. (Allowing for nonzero emission does not alter the insights from analysis.) In contrast, if equipment is in noncompliance but production continues ("noncompliant production"), the pollutant is emitted at a constant rate. Because a noncompliance episode may be reported and suspended after some time has elapsed since it started—for example, when it is detected in an inspection—an episode may consist of two successive portions: unsuspended noncompliance and suspended noncompliance. Pollutant is emitted in the first portion but not in the second portion, during which there is no production output.

### 3.3. Production and Sales

To highlight the dynamics arising from stochastic compliance state transitions, we keep the representation of the production process to a minimum. It is assumed that the firm produces its good at a constant rate, normalized to one, unless production is suspended by the regulator. No inventory is held by the firm to compensate for the loss of output during suspended production. While these assumptions are introduced as simplifications, they are reasonable representations of the practices in continuous production industries cited in the motivating examples (petroleum refining, chemical production, etc.).

The firm earns revenue $r$ for each unit sold. (To focus on the relationship between the firm and the regulator, we do not take into account the utility of the firm’s customers.) Production cost is normalized to zero. If the pollutant is emitted, it causes environmental damage valued at $h$ per unit time. Damage is avoided if production is suspended. We assume that $h$ is greater than the opportunity cost of lost sales, i.e., $h > r$. This assumption implies that, from society’s standpoint, pollution prevention takes precedence over revenue generation.

### 3.4. Information Structure

The firm has complete visibility to the state of his equipment at all times, but the regulator does not. There are two ways in which a noncompliance episode is reported to the regulator: either the regulator discovers it after she performs an inspection (detection) or the firm preemptively informs it to the regulator (disclosure). Both events reveal the noncompliance state immediately. Because inspections are costly, the regulator does not perform continuous inspections. As a result, some noncompliance episodes may escape detection and go unreported; see Figure 1 for illustrations. The firm does not disclose past noncompliance episodes that went unreported.

We assume that a noncompliance report informs the regulator of whether the report has been delayed, but not how long the delay has been. Thus the regulator can obtain only limited information about the extent of past environmental damage. This assumption captures the practical difficulty that the regulator may encounter in verifying the firm’s claim about exact timing of equipment malfunction or assessing the actual damage done. On the other hand, the regulator can determine whether the report has been delayed, for example, by finding out if traces of pollution exist.

Under this assumption, there are three different types of noncompliance report: immediate disclosure at the onset of noncompliance occurrence ("early disclosure"), late disclosure, and detection. (Focusing on these three types of report facilitates analysis, which becomes intractable otherwise.) Detection of a noncompliance episode happens only if the firm has not disclosed it by the time of inspection, and as a result, it is always delayed.
Figure 1. Illustrations of detection and disclosure instances.

Notes. Compliance and noncompliance states alternate over time, while the regulator performs an inspection every $T$ time units unless it is suspended because of a noncompliance report via detection or disclosure. The inspection process restarts upon compliance restoration. The independent and identically distributed durations of compliance and noncompliance episodes are denoted by $U$ and $D$, respectively.

We also assume that, once a noncompliance episode is reported, the firm (who is “put on notice”) keeps the regulator informed of the noncompliance state until compliance is restored. Hence, no inspections are needed from the time a noncompliance episode is reported until it concludes.

As we discuss further in §4.2, a pathological situation may arise in which the firm times his disclosure to take place immediately before an inspection. Since this “last-minute disclosure” is made practically at the same instance when detection happens, as a tie-breaking rule, we regard this as a detection event. In §6.1, we study the impact of treating this event as a late disclosure instead of a detection.

3.5. Regulator’s Enforcement Actions

The regulator is empowered to suspend the firm’s production after emission control equipment is found to be in noncompliance. Since the cost of environmental damage outweighs the firm’s opportunity cost of lost sales ($h > r$), it is optimal for the regulator, a social welfare maximizer, to suspend production immediately after noncompliance is reported. In addition, no inspections are needed once noncompliance is reported until the moment compliance is restored, as we have assumed in §3.4. Hence the regulator suspends production and inspections as soon as noncompliance is reported to her via detection or disclosure. Both activities resume when compliance is restored.

The regulator incurs a fixed cost $\chi$ each time she performs an inspection. We assume that the cost of performing an inspection is sufficiently small, so that the condition $\chi < (h - r)\lambda / \mu^2$ is satisfied. This condition ensures that the regulator has an incentive to perform inspections; if it is violated, she may find inspections too costly to justify the potential social benefit.

We assume that the regulator assigns distinct penalties corresponding to the three types of noncompliance reports that were identified in §3.4: $\kappa_1$ for early (immediate) disclosure, $\kappa_2$ for late disclosure, and $\kappa_d$ for detection that follows no disclosure. Reflecting the increasing degree of untruthfulness, they satisfy $0 \leq \kappa_1 \leq \kappa_2 \leq \kappa_d$. (Reduced penalty for disclosure is a key feature of the EPA’s Audit Policy; see U.S. Environmental Protection Agency 2000.) We assume that there is an upper bound on the penalties, denoted by $K$, whose value is exogenously given. This rules out an unrealistic scenario where the regulator imposes an arbitrarily large penalty on the firm. The same assumption is commonly found in the literature and it is often interpreted as the wealth constraint (see Heyes 2000 for a discussion about this assumption). Because the largest penalty imposed on the firm is $\kappa_d$, the upper bound is expressed by $\kappa_d \leq K$, which we refer to as the maximum penalty constraint. We make the following mild technical assumption on $K$ that further rules out unlikely situations: $K < r \hat{x}/\mu$, where $\hat{x} > 0$ is the unique solution to the equation $(x/2)(x/\ln(1 + x) - 1)^{-1} = \mu/\lambda - 1$. When combined with the earlier assumption $\lambda \ll \mu$, this condition is easily satisfied by most reasonable values of $K$; in practical scenarios the condition is effectively equivalent to $K < \infty$.

3.6. Inspection Policies

We focus on two inspection policies employed by the regulator that exhibit opposite characteristics: random inspections (RI) and periodic inspections (PI). They differ mainly by whether the firm can anticipate the timing of inspections, because PI are performed at fixed intervals, whereas the intervals are uncertain under RI. In both cases, the regulator performs inspections every $T$ time units—a variable we call inspection interval—unless an inspection is suspended following a noncompliance report, in which case the inspection process restarts on compliance restoration. Under the RI policy $T$ is a random variable with mean $\tau$, i.e., $E[T] = \tau$. Under the PI policy, in contrast, $T = \tau$ since $T$ is deterministic. The regulator sets $\tau$ in both cases, thus determining the inspection frequency $\nu = 1/\tau$. (Note that, despite the constant value $T = \tau$ under the PI policy, in general, the realized intervals are not uniformly spaced because an inspection may be suspended for a random amount of time following a report of noncompliance. See Figure 1 for illustrations.)

We assume that $T$ under the RI policy is exponentially distributed. This assumption is made to maximize the contrast between RI and PI. Because of the memoryless property, a firm facing RI with exponential intervals has identical outlook of the future at each point in time; the “surprise factor” of RI is maximal under this policy, because the timing of the past inspection is irrelevant in predicting when the next inspection will take place. This is in sharp contrast to the case of PI, because under this policy, the past inspection informs the firm with perfect knowledge about the timing of the next inspection; in this case, the surprise factor is zero. In this sense, RI and PI capture the notions of unannounced and announced inspections (Farmer 2007).


3.7. Firm’s Disclosure Decision

We assume that the firm makes his disclosure decision continuously over time after noncompliance occurs, until either compliance is restored or noncompliance is reported. Since disclosure is irreversible and the firm’s expected profit depends on when it is made, disclosure is a timing decision that answers the following question: Once noncompliance occurs, should the firm disclose it now or delay the decision to a moment later? We assume that the firm solves this recursive continuous-time dynamic decision problem with the goal of maximizing his expected profit until compliance restoration. His decision is based on the inspection frequency \( \nu \) and the penalties he faces (\( \kappa_e, \kappa_i, \) and \( \kappa_d \)), as well as the opportunity cost of lost revenue due to suspended production that follows disclosure or detection.

Notice that the disclosure policy derived from this analysis is myopically optimal, because the stated goal is to maximize the firm’s expected profit for the duration of a single noncompliance episode. In general, a myopic policy does not guarantee maximizing the firm’s long-run average profit. However, as we demonstrate in §4, the myopic optimum coincides with the long-run average optimum in the limiting case we focus on: \( \lambda \ll \mu \). For this reason and for tractability that it brings, we solve the firm’s disclosure decision problem based on the myopic objective.

3.8. Performance Measures

Let \( \bar{I} \), \( \bar{R} \), and \( \bar{B} \) denote the long-run time averages of the following performance measures, in the presented order: number of inspections performed per unit time, cumulative duration of suspended noncompliance per unit time, and cumulative duration of unsuspended noncompliance per unit time. Under the assumptions outlined above, the long-run average social welfare is then equal to \( r(1−\bar{R})−\chi\bar{I}−h\bar{B} \): the firm earns the revenue \( r \) per unit time unless production is suspended, which lasts for \( \bar{R} \) per unit time in the long run; the regulator incurs the fixed cost \( \chi \) each time she performs an inspection, doing so \( \bar{I} \) times per unit time in the long run; the damage valued at \( h \) is done to the environment, whereas noncompliant production lasts for \( \bar{B} \) per unit time in the long run. Note that the penalties \( \kappa_e, \kappa_i, \) and \( \kappa_d \) do not appear in the social welfare function because they are monetary transfers between the firm and regulator that, in the absence of taxation, cancel out within the social boundaries.

From the expression above, it is clear that maximizing the long-run average social welfare is equivalent to minimizing the long-run average social cost \( \bar{C} \equiv \chi\bar{I}+r\bar{R}+h\bar{B} \), the convention we adopt in the remainder of the paper. Similarly, the firm’s profit-maximization problem is formulated as the equivalent cost minimization problem, \( \Psi \) denoting his long-run average cost. The expressions for \( \bar{C} \) and \( \Psi \) are evaluated in the next section, where we characterize the equilibria under the RI and PI policies.

4. Equilibria Under Random and Periodic Inspections

In this section, we characterize the equilibria that emerge under the RI and PI policies. In each case, we evaluate the long-run average performance measures \( \bar{I}, \bar{R}, \bar{B}, \bar{C}, \) and \( \Psi \), derive the firm’s optimal response to the regulator’s announcement of inspection frequency and the penalty amounts, and solve the regulator’s social cost minimization problem. The comparisons of equilibria are presented in §5. Because the equilibrium under RI is simpler to characterize, we present our analysis of this policy first.

4.1. Random Inspections

Recall that the regulator employing the RI policy sets the inspection frequency \( \nu = 1/\tau \) but randomizes the actual inspection times by sampling from the exponential distribution to determine the inspection interval \( T \). The following result identifies the firm’s optimal disclosure policy under RI, based on an analysis of the dynamic disclosure timing decision problem described in §3.7. (Throughout the paper, we adopt the convention that the firm facing a tie between disclosure and delay chooses disclosure.)

**Lemma 1.** Under RI with frequency \( \nu = 1/\tau \) and the penalties \( \kappa_e, \kappa_i, \) and \( \kappa_d \), the firm either never discloses noncompliance or discloses every noncompliance episode immediately after it occurs. The former is optimal if \( \tau > (\kappa_d−\kappa_e)/(r+\kappa_i\mu) \), whereas the latter is optimal if \( \tau \leq (\kappa_d−\kappa_e)/(r+\kappa_i\mu) \).

All proofs are found in the appendix. A brief description of the dynamic disclosure problem is as follows. At time \( t \geq 0 \) after the onset of a noncompliance episode, the firm chooses between disclosing it then and delaying the decision by \( \delta \) time units to face three possible outcomes by time \( t+\delta \): compliance restoration, detection, and the same situation as before. In addition, the delay allows the firm to earn revenue generated from noncompliance production. Each of these actions and probabilistic outcomes is associated with revenue and cost, and the firm weighs these values in making his disclosure decision at any time \( t \geq 0 \) until either compliance restoration or detection occurs. The optimal disclosure policy is identified by solving this recursive problem and taking a limit \( \delta \to 0 \).

As Lemma 1 demonstrates, the optimal disclosure policy under RI exhibits a binary structure in which the firm either never discloses noncompliance or fully discloses noncompliance. We refer to these as the nondisclosure policy and the full disclosure policy. (Unlike the extant literature, which assumes the dichotomous disclosure behavior, we derive its optimality as a solution to the firm’s dynamic disclosure decision problem.) Such a simple decision rule is enabled by two features of our model: fixed penalty \( \kappa_i \) for late disclosure and memorylessness of the inspection interval \( T \). The combination of these two implies that, once the firm has decided not to disclose noncompliance at its...
onset, at each instance afterward, he faces the same trade-off between the disclosure and delay options. Hence, he either discloses noncompliance at the first opportunity or keeps delaying disclosure. Whichever action is optimal for a particular noncompliance episode, the same applies to all episodes because of time symmetry introduced by memorylessness. Therefore the firm discloses either no episode or every episode in full.

Notice from Lemma 1 that the late disclosure penalty $\kappa_i$ does not appear in the description of the firm’s optimal disclosure behavior. This is because the firm is induced to choose between two extremes of either disclosing a noncompliance episode at its onset or never disclosing it. Consequently, only the early disclosure penalty $\kappa_e$ and the detection penalty $\kappa_d$ remain at the optimum.

Next, based on Lemma 1 that identifies nondisclosure and full disclosure as the two classes of disclosure policies employed by the firm, we now evaluate the long-run average performance measures $I$, $\bar{R}$, $\bar{B}$, and $\bar{Y}$ assuming that the firm employs the same policies. To enable a tractable analysis later in §5, we evaluate these measures in first-order approximations using the earlier assumption that noncompliance occurs rarely, i.e., $\lambda \ll \mu$. These approximations are obtained by first deriving the exact measures using the renewal-reward theorem (Tijms 2003, p. 41), expanding them with respect to the ratio $\lambda/\mu$, then retaining up to the first-order terms.

**Lemma 2 (Approximate Measures Under Random Inspections).** With nondisclosure: $I = (1/\tau)(1 - (\lambda/\mu) \cdot (1/(\mu \tau + 1)), \bar{R} = (\lambda/\mu)/(1/(\mu \tau + 1)), \bar{B} = (\lambda/\mu) \cdot (\mu \tau / (\mu \tau + 1))$, and $\bar{Y} = (\lambda/\mu)((r + \kappa_e, \mu / (\mu \tau + 1))$. With full disclosure: $I = (1/\tau)(1 - (\lambda / \mu)), \bar{R} = \lambda / \mu, \bar{B} = 0$, and $\bar{Y} = (\lambda / \mu)(r + \kappa_d)\mu$.

Comparing the two values for $\bar{Y}$ from Lemma 2, we see that the conditions that result in either nondisclosure or full disclosure under the long-run average objectives match exactly with those appearing in Lemma 1 that were derived under the myopic objective: $\tau > (\kappa_e - \kappa_d)/(r + \kappa_e, \mu)$ for nondisclosure and $\tau \leq (\kappa_d - \kappa_e)/(r + \kappa_e, \mu)$ for full disclosure. This observation verifies our assertion from §3.7 that the myopic optimum coincides with the long-run average optimum. Hence the firm that employs the myopically optimal disclosure policies also maximizes his long-run average profit under the same classes of policies.6

Now we turn our attention to the regulator’s decisions. Taking into account the firm’s optimal disclosure response and using the expressions derived in Lemma 2, we can write the long-run average social cost as

$$\bar{C}^* = \begin{cases} \frac{r \lambda}{\mu} + \frac{\mu}{\lambda} \left( 1 - \frac{\lambda}{\mu} \right) \frac{1}{\mu \tau + 1} & \text{if } \tau \leq \frac{\kappa_d - \kappa_e}{r + \kappa_e, \mu}, \\ \frac{h \lambda}{\mu} - \frac{\lambda}{\mu} \left( \frac{\lambda}{\tau} + h - r \right) \frac{1}{\mu \tau + 1} & \text{if } \tau > \frac{\kappa_d - \kappa_e}{r + \kappa_e, \mu}. \end{cases}$$

The regulator chooses $\kappa_e, \kappa_i, \kappa_d$, and $\tau$ that together minimize this function subject to the constraint $0 \leq \kappa_e \leq \kappa_i \leq \kappa_d \leq K$. The equilibrium decisions, denoted by the superscript $*,$ are as follows.

**Proposition 1 (Equilibrium Under Random Inspections).** Let

$$\tau^* = \left( 2\sqrt{\frac{\mu - \lambda}{\chi - \mu}} \right) \lambda \left( 1 - \frac{\lambda}{\mu} \right)^{-1} \text{ and}$$

$$\tau^* = \left( 2\sqrt{\frac{\mu - \lambda}{\chi - \mu}} \right) \lambda \left( \frac{1 - \frac{\lambda}{\mu}}{\lambda} \right)^{-1}.$$

In equilibrium, the regulator employing the RI policy chooses any $\kappa_e, \kappa_i, \kappa_d$ satisfying $0 \leq \kappa_e \leq \kappa_i \leq \kappa_d \leq K$ with $\tau^* = \tau^*$ if $K < \tau^*$, whereas she chooses $\kappa_e = 0, \kappa_i \in [0, K]$, and $\kappa_d = K$ with $\tau^* = K / \tau^*$ if $K \geq \tau^*$. In response, the firm chooses nondisclosure if $K < \tau^*$, whereas he chooses full disclosure if $K \geq \tau^*$.

The proposition confirms that RI result in a dichotomous equilibrium structure, which follows directly from the firm’s binary response. There exists a cutoff value for $K$, the maximum penalty amount allowed, under which nondisclosure is induced and over which full disclosure is induced. The proposition also reveals that the regulator’s optimal mix of enforcement levers changes qualitatively as $K$ varies. We postpone the discussion of this observation to §5. Notice that the full disclosure equilibrium is characterized by the penalties that are set at opposite extremes: $\kappa_e = 0$ and $\kappa_d = K$. In particular, $\kappa_i = 0$ implies that the firm is rewarded with a penalty waiver if he discloses noncompliance early.7 In our setting, maximal differentiation of $\kappa_e$ and $\kappa_d$ is optimal because it maximizes the gap between the benefit of early disclosure and the downside of no disclosure that may result in detection. Because early disclosure is the socially preferred outcome, the regulator encourages this behavior to the fullest extent by offering a penalty waiver.

Finally, combining (2) with the results from Proposition 1 yields the closed-form expressions for the equilibrium long-run average social cost:

$$\bar{C}^* = \begin{cases} \frac{r \lambda}{\mu} - \frac{\mu}{\lambda} \left( 2\sqrt{\frac{\mu - \lambda}{\chi - \mu}} \right) \lambda \left( 1 - \frac{\lambda}{\mu} \right) & \text{if } K < \tau^*, \\ \frac{r \lambda}{\mu} + \frac{\lambda}{\mu} \left( \frac{1 - \frac{\lambda}{\mu}}{\lambda} \right)^{-1} & \text{if } K \geq \tau^*. \end{cases}$$

### 4.2. Periodic Inspections

Under PI, the inspection interval $T$ is set to a constant $\tau$. We first analyze the firm’s dynamic disclosure timing decision problem described in §3.7 to identify the optimal disclosure policy. This problem is similar to the one under RI,
Lemma 3. Under PI with frequency $v = 1/\tau$ and the penalties $\kappa_c$, $\kappa_d$, and $\kappa_\mu$, the firm discloses a noncompliance episode immediately after it occurs if the time remaining until the next inspection is smaller than or equal to $\min\{1/(\mu_\tau)\ln(r + \kappa_\mu)/r + \kappa_\mu, \tau\}$. Otherwise, the firm keeps delaying disclosure until either the noncompliance episode concludes without disclosure or he discloses it immediately before the next inspection arrives, whichever happens first.

Unlike RI that induce a binary response of disclosing either all noncompliance episodes or none, PI may lead to partial disclosure. That is, the firm may selectively disclose some episodes and omit others. An important factor that shapes such opportunistic behavior is the timing of a noncompliance occurrence relative to the time of the next inspection. If noncompliance occurs shortly before the next inspection arrives, the firm is better off disclosing it preemptively, so that he receives a reduced penalty for early disclosure rather than being detected and charged with a larger penalty. In contrast, if the next inspection is to arrive far into the future, the firm is willing to take a chance and not disclose the current noncompliance because compliance is likely to be restored before the next inspection and the firm may escape detection.

Lemma 3 formalizes this reasoning, presented as the optimal solution of the firm’s dynamic disclosure decision problem. We refer to the decision rule described in the lemma as the threshold disclosure policy. This policy is characterized by a single parameter $s \in [0, \tau]$, which denotes the size of the disclosure window time interval that precedes the time of next inspection. Any noncompliance episode that starts within this time window is disclosed as soon as it occurs, whereas the episodes that start outside of the window are either never disclosed (if compliance is restored before the next inspection) or disclosed immediately before the inspection arrives (if compliance is not restored by then), an event regarded as detection by the assumption in §3.4. Hence the larger the window, the more noncompliance disclosed. This policy includes nondisclosure and full disclosure as special cases, corresponding to $s = 0$ and $s = \tau$, respectively. Note that the last-minute disclosure behavior is another manifestation of the firm’s opportunism created by the certainty inherent in PI.

Lemma 3 also identifies the optimal disclosure window size as $s = \min\{\mu^{-1}\ln((r + \kappa_\mu)/r + \kappa_\mu), \tau\}$. Similar to what we observed under RI, the firm’s optimal response is independent of the late disclosure penalty $\kappa_\mu$, because he ends up choosing between early disclosure that incurs the penalty $\kappa_c$ and the last-minute disclosure that incurs the detection penalty $\kappa_d$. This window size represents myopic optimum, derived under the assumption that the firm maximizes his expected profit for the duration of a single noncompliance episode. In contrast to the case of RI, it turns out that this myoptimally optimal window size does not necessarily maximize the firm’s long-run average profit, under the same class of threshold disclosure policy. However, we can prove that equivalence between the two objectives is restored under the condition $\lambda \ll \mu$, after first-order approximations are applied. Details of performance measure evaluations are found in §A of the online appendix (available as supplemental material at http://dx.doi.org/10.1287/opre.2015.1345). In what follows, we outline the evaluation steps.

First, we assume that the firm employs the threshold disclosure policy as described above, with an arbitrary disclosure window size $s$ and no restrictions on $\lambda$ and $\mu$. Second, we show that the threshold disclosure policy gives rise to five possible stochastic outcomes in an inspection cycle, a renewal cycle that defines the time unit of analysis. Third, based on the probabilities of these outcomes, we evaluate the expected values of performance measures in a single inspection cycle (e.g., $E[I]$ for expected number of inspections in a cycle) as well as the expected cycle length, denoted by $E[X]$, all as functions of $s$. The exact long-run time averages are found by forming ratios of these values, using the renewal-reward theorem. For example, the long-run average number of inspections per unit time is

$$\bar{I} = \frac{E[I]}{E[X]} = \left( e^{-\lambda s} + \lambda \left( e^{-\lambda s} - e^{-\lambda s - e^{-\lambda s}} \right) \theta(\tau - s)/(\mu - \lambda) \right) \cdot \left( \frac{1}{1 + 1/\mu} \right) \left( 1 + 1/\mu \right) \left( 1 - e^{-\lambda s} + 1/\mu \right) \left( 1 + 1/\mu \right) \left( 1 - e^{-\lambda s} + 1/\mu \right) \theta(\tau - s)^{-1}$$

where $\theta(t)$ is defined in (1). Finally, we obtain approximations by expanding these expressions with respect to the ratio $\lambda/\mu$ and retaining up to the first-order terms. The results are summarized in the following lemma.

Lemma 4 (Approximate Measures Under Periodic Inspections). With the disclosure window size $s$: $\bar{I} = (1/\tau) \cdot (1 - (\lambda/\mu)(s/\tau + (e^{-\mu s} - e^{-\mu s} + e^{-\mu s})) / (\mu s/\tau)) \cdot \lambda(s/\tau - e^{-\mu s} / (\mu s/\tau)) - \lambda(s/\tau - e^{-\mu s} / (\mu s/\tau))$. $\bar{R} = (\lambda/\mu)(s/\tau + (e^{-\mu s} - e^{-\mu s} + e^{-\mu s})) / (\mu s/\tau)$. $\bar{B} = (\lambda/\mu)(1 - s/\tau - (e^{-\mu s} - e^{-\mu s}) / (\mu s/\tau))$. $\bar{B} = (\lambda/\mu)(s/\tau + (e^{-\mu s} - e^{-\mu s} + e^{-\mu s}) / (\mu s/\tau))$. It is straightforward to show that the firm’s long-run average cost $\bar{\Psi}$ in Lemma 4 is minimized at $s = \min\{\mu^{-1}\ln((r + \kappa_\mu)/r + \kappa_\mu, \tau)\}$, the same optimal value that we obtained in Lemma 3. Therefore the long-run average optimum matches exactly with the myopic optimum when first-order approximations apply under the condition $\lambda \ll \mu$, as we asserted. (In Lemma B.1 in the online appendix, we provide a heuristic argument that explains this equivalence.) Using the results from Lemma 4, we can
write the long-run average social cost \( \bar{C} = \chi I + r \bar{R} + h \bar{B} \) as follows:

\[
\bar{C} = \frac{h \lambda}{\mu} + \chi - \chi \left( \frac{s^2}{2 \tau^2} \right) - \frac{\lambda}{\tau} (\chi + h - r) \left( \frac{s + e^{-\mu s} - e^{-\mu r}}{\mu \tau} \right).
\] (4)

The regulator’s problem is to minimize this function by adjusting \( \kappa_e, \kappa_i, \kappa_d, \) and \( \tau \) that satisfy the constraint \( 0 \leq \kappa_e, \kappa_i, \kappa_d \leq K \), anticipating the firm’s optimal response \( s^* \) specified above. For notational convenience, let

\[
\sigma = \frac{1}{\mu} \ln \left( 1 + \frac{K \mu}{r} \right).
\] (5)

The equilibrium decisions, denoted by the superscript \( p \), are as follows.

**Proposition 2 (Equilibrium Under Periodic Inspections).** Let \( G(\tau | \sigma) = (1 + ((h - r)/\chi) \tau) (1 - e^{-\mu \tau}) - (1 - (h - r)/\chi) \tau (1 - \sigma/\tau - (e^{-\mu s} - e^{-\mu r})/(\mu \tau)) + \mu \sigma(1 - \sigma/\tau) \). In equilibrium, the regulator employing the PI policy chooses \( \kappa_d^p = 0, \kappa_i^p \in [0, K], \) and \( \kappa_e^p = K \) with \( \tau^p = \max \{ \sigma, \hat{\tau}(\sigma) \} \), where \( \hat{\tau}(\sigma) > 0 \) is the unique solution to the equation \( G(\tau | \sigma) = \mu/\lambda - 1 \). In response, the firm chooses \( s^p = \sigma \).

Reflecting the firm’s optimal response, the equilibrium described in the proposition permits a complete range of disclosure behavior: nondisclosure, partial disclosure, and full disclosure. They are induced, in turn, as \( K \) increases, starting with nondisclosure at \( K = 0 \) and moving to greater degrees of partial disclosure (larger disclosure window), reaching full disclosure after \( K \) becomes sufficiently large. This gradual change contrasts with what we observed in the case of RI, under which only nondisclosure and full disclosure emerge in discrete steps. It also contrasts with the binary disclosure response commonly assumed in the literature, which does not take timing decisions into account. We postpone the detailed comparison of the equilibria under RI and PI policies to §5. Note that maximal differentiation of two penalties \( \kappa_e, \kappa_d \) continues under PI, for the same reason as the one under RI.

## 5. Comparison of Equilibria

In this section, we compare the equilibria under RI and PI that we derived in the last section. In §5.1, we focus on the equilibrium enforcement levers under each inspection policy, and in §5.2 we compare the equilibrium social costs under the two policies.

### 5.1. Equilibrium Enforcement Levers

We first examine how the two main enforcement levers, namely, detection penalty \( \kappa_d \) and inspection frequency \( \nu = 1/\tau \), interact with each other in equilibrium. The equilibrium values of these two variables are jointly determined by \( K \), the maximum penalty amount allowed. Thus we focus on the impact of varying \( K \). Henceforth we call \( \kappa_d \) simply as “penalty” because the early disclosure penalty \( \kappa_e \) does not interact with \( \nu \) in equilibrium and is omitted from the discussion.

As we found in Propositions 1 and 2, the maximum penalty constraint \( \kappa_d \leq K \) binds in equilibrium under both inspection policies, except when \( K \) is sufficiently small under RI (see §4.1). That the maximum penalty limit is reached reflects the fact that the penalty is a more efficient instrument than inspections; unlike costly inspections, the regulator does not incur any direct cost by using monetary penalty as an incentive. It also implies that relaxing the binding maximum penalty constraint (larger \( K \)) leads to a new equilibrium in which the penalty is increased accordingly. Thus, \( \kappa_d^e \) and \( \kappa_d^p \) increase as \( K \) becomes larger except when \( K \) is kept low under RI.

By contrast, the behaviors of \( \tau^e \) and \( \tau^p \)—or equivalently the inspection frequencies under RI and PI—are not straightforward, as the next proposition reveals. (Note that \( \tau^e \) and \( \hat{\tau}(\sigma) \) appearing in the proposition are defined in Propositions 1 and 2.)

**Proposition 3.** As \( K \) increases from zero to infinity, \( \tau^e \) initially stays constant until it jumps downward at \( K = r \tau^e \), after which it increases. In contrast, \( \tau^p \) initially decreases but increases after reaching a point at which \( \sigma = \hat{\tau}(\sigma) \). \( \tau^e \) and \( \tau^p \) increase if and only if the firm is induced to choose full disclosure.

According to the proposition, the inspection frequencies and \( K \) exhibit nonmonotonic relationships. See Figure 2(a) that illustrates this observation. Nonmonotonicity manifests itself also in how the long-run average cost of inspections \( \chi I \) changes in \( K \); see Figure 2(b).

When \( K \) is large enough to induce full disclosure, \( \tau^e \) and \( \tau^p \) increase in \( K \) (see Figure 2(a)). That is, the regulator performs less frequent inspections if she is able to charge a higher penalty for detection, set to the maximum value \( K \). Thus, in this case, we recover the classic result from Becker (1968) that penalty and inspection intensity act as substitutes; the higher the penalty, the lower the need to perform costly inspections. The same relationship does not hold, however, when \( K \) is sufficiently small. Namely, substitutability between the penalty and inspection frequency is replaced by independence under RI, whereas it is replaced by complementarity under PI. Complementarity implies that a higher penalty should be accompanied by more frequent inspections, instead of lessening the need for inspections. Therefore the nature of interaction between the two variables fundamentally changes depending on how much penalty can be levied.

To develop intuition, let us examine the equilibrium under each inspection policy. First, consider RI. Recall from Proposition 1 that the firm is induced to choose either nondisclosure or full disclosure in equilibrium. With only a limited amount of penalty that can be charged to the firm...
(small $K$), the regulator is unable to find a cost-effective combination of enforcement levers that converts the firm’s response from nondisclosure to full disclosure. Because of this ineffectiveness, the regulator abandons inducing full disclosure and instead focuses on detecting noncompliance by choosing an inspection frequency that balances the cost of inspections and the social benefit gained from detection. (The assumption $\chi < (h - r)\lambda/\mu^2$ from §3.5 guarantees that such a trade-off exists.) Because the detection probability is unaffected by penalties when the firm keeps practicing nondisclosure, the regulator maintains the same inspection frequency even as $K$ increases, up to the point where it becomes large enough to induce full disclosure. In this range, the maximum penalty constraint $\kappa_d \leq K$ does not necessarily bind.

Next, consider PI. Contrary to what we observed under RI, the maximum penalty constraint always binds in equilibrium: $\kappa_d^p = K$ even for small $K$ (see Proposition 2). This happens because, unlike the binary response under RI, transparency of the inspection schedule allows the firm to fine-tune his disclosure timing. As long as full disclosure has not been attained, any small change in the penalty leads the firm to incrementally adjust the disclosure window size, so that his trade-off is balanced. The regulator takes advantage of this continuous response, increasing the penalty until the maximum value $K$ is reached.

Complementarity between the penalty $\kappa_d^p = K$ and inspection frequency arises when the firm’s fine-tuning behavior leads to a partial disclosure equilibrium. Consider the chain of events that unfolds after the penalty is increased incrementally. To compensate for the expected loss because of increased penalty, the firm responds to this change by disclosing more early noncompliance occurrences (i.e., choose a larger disclosure window), thereby preempting inspections. This leads to fewer inspections, which benefit the regulator as the costs of inspections are reduced. However, she does not simply absorb this saving. The regulator reinvests the saving in performing more inspections, because she can induce even more disclosure by doing so as long as full disclosure has not been reached. Hence, a reinforcement mechanism is in effect; unless the firm fully discloses noncompliance, an increased penalty leads to a new equilibrium in which the regulator schedules more frequent inspections.

A similar effect has not been mentioned in the literature because it is commonly assumed that an individual’s action set is binary: either disclose or do not disclose noncompliance, or either commit or do not commit a harmful act. Such a binary structure makes the individual’s response to a gradual shift in enforcement levers completely inelastic, unless a regime change happens when the optimal action jumps from one to the other. We observe a similar discontinuity under RI, but under PI the discontinuity is replaced by a continuous response that exhibits complementarity between the two levers (see Figure 2(a)). Note that Innes (1999a, b) and others have found cases in which substitutability does not result in maximally set penalty. Our finding complements this by documenting a case where an equilibrium with maximally set penalty does not necessarily imply substitutability.

Finally, observe from Figure 2(a) that equilibrium inspection frequency is greater under PI than under RI (i.e., $\tau^p < \tau^r$) when both policies induce the same disclosure behavior: nondisclosure or full disclosure. This suggests that it is less expensive to perform PI. The insight behind this observation is provided in the next discussion.

### 5.2. Equilibrium Social Costs

We now compare the equilibrium long-run average social costs under RI and PI, denoted by $\bar{C}^r$ and $\bar{C}^p$. As before, we pay special attention to $K$, the maximum penalty allowed, by examining how the social cost varies with $K$ under the two inspection policies.

**Proposition 4.** $\bar{C}^r > \bar{C}^p$ for sufficiently small $K$, whereas $\bar{C}^r < \bar{C}^p$ for sufficiently large $K$. Moreover, $\bar{C}^r$ and $\bar{C}^p$...
cross at a value of $K$, which induces full disclosure under RI and partial disclosure under PI.

Although the proposition does not state that $\tilde{C}^r$ and $\tilde{C}^p$ cross exactly once, such single crossing is observed numerically. With this property, the proposition implies that RI are preferred if and only if the regulator can charge a large penalty to the firm (large $K$); otherwise, PI are preferred. The numerical example presented in Figure 3(a) confirms this finding. As the figure shows, the advantage of RI disappears as $K$ approaches zero, the limit at which a lower social cost is attained under PI than under RI.

Intuition suggests that performing RI is more effective than performing PI. Consider PI. Under this policy, the firm perfectly anticipates when the next inspection will arrive, and therefore his disclosure behavior is maximally opportunistic. RI mitigate any efficiency loss because of this behavior, because memorylessness inherent in that policy minimizes the firm’s ability to plan ahead. According to this reasoning, then, the RI policy should dominate the PI policy. This is indeed what happens when $K$ is large, but Proposition 4 reveals that the opposite is true when $K$ is small: the RI policy is dominated by the PI policy. Hence, a reversal occurs with small $K$.

The reason for this reversal is that, ceteris paribus, PI are more efficient than RI when the compliance or noncompliance states alternate as a Markov process. The next lemma supports this assertion.

**Lemma 5.** With fixed inspection frequency, the long-run average social cost is lower under PI than under RI if the firm’s response is normalized to either nondisclosure or full disclosure under both inspection policies.

Note that the lemma is proved using exact performance measures; the result is general and does not depend on approximations. The assumptions in Lemma 5 are chosen to isolate the impact of randomization, controlling for other confounding factors that affect the equilibrium by normalizing the firm’s disclosure behavior and fixing inspection frequency. With these factors controlled for, the lemma states that performing PI is more efficient than performing RI.

To understand the reason for this result, consider the case where the firm never discloses noncompliance. In this case, noncompliance can only be discovered via detection, and therefore the efficiency of an inspection policy is determined entirely by detection probability. This is precisely $\theta(t)$ given in (1), namely, the probability that an inspection at time $t$ finds noncompliance after compliance is observed at time zero. Under the PI policy, inspections are performed every constant time units equal to $\tau$; hence, detection probability is $\theta(\tau)$. In contrast, under the RI policy, inspections are performed every $T$ random time units, which has the mean $E[T] = \tau$; hence detection probability is $E[\theta(T)]$ in this case. Notice from (1) that $\theta(t)$ is concave increasing. Concavity arises because finite inspection intervals capture the transient behavior of the underlying Markov process. Then, by Jensen’s inequality, we have $\theta(\tau) > E[\theta(T)]$, i.e., probability of detection is greater under PI than under RI; due to concavity, randomization assigns more weight to an early nondetection than to a late detection. Therefore PI are more efficient than RI at detecting noncompliance.

Lemma 5 also shows that the PI policy continues to outperform the RI policy even when the firm fully discloses noncompliance. Because no detection occurs in this case, higher probability of noncompliance detection is translated into a smaller expected number of inspections needed while compliance is maintained. Therefore, the long-run average inspection cost is lower under periodic inspections than under random inspections even with full disclosure.

Returning to the result of Proposition 4, we now see that our earlier description of the result—that RI are preferred if and only if $K$ is large—originates from the tension between efficiency of noncompliance detection and the firm’s opportunistic disclosure timing behavior. Relative to RI, performing PI brings higher efficiency of detection but at the same time allows the firm to fine tune his disclosure timing, thus softening the impact of an increased penalty. When $K$ is small, the detection advantage dominates because the small amount of penalty suppresses the impact of the firm’s opportunistic disclosure behavior. When $K$ is large, by contrast, the opposite happens because the impact of opportunistic behavior is amplified.

Because of this trade-off, the regulator who considers implementing a cost-effective inspection policy would have to make a choice. One key determinant of such a choice is the maximum penalty $K$, which has been our focus so far. Another is $\chi$, the cost incurred in each inspection. Both are important economic factors that directly influence the effectiveness of an inspection policy. The following result identifies the condition under which one inspection policy is preferred to the other as $\chi$ varies.

**Lemma 6.** For fixed $K$, there exists a unique value $\hat{\chi} \geq 0$ such that $\tilde{C}^r < \tilde{C}^p$ for all $\chi < \hat{\chi}$ and $\tilde{C}^r > \tilde{C}^p$ for all $\chi > \hat{\chi}$.

Hence, PI are preferred when it is expensive to perform an inspection; this is another manifestation of the relative efficiency that this inspection policy brings. Combining this with the earlier finding, we conclude that PI are more cost effective when the regulator operates in restrictive conditions marked by a limited amount of penalty (small $K$) and a high inspection cost (large $\chi$). This is confirmed in a numerical example presented in Figure 3(b), which identifies the regions in the $(K, \chi)$ space where one inspection policy dominates the other.

### 6. Extensions and Robustness

#### 6.1. Alternative Assumption on Last-Minute Disclosure

As we discussed in §4.2, PI create an incentive for the firm to disclose noncompliance immediately before a scheduled inspection. For a number of reasons, we have regarded this
Figure 3. The left panel shows the equilibrium long-run average social costs $\bar{C}_r$ (dashed line) and $\bar{C}_p$ (solid line) as a function of the maximum penalty $K$. The right panel shows the regions in the $(K, \chi)$ space in which one of the two inspection policies is preferred from the social cost perspective.

Note. The same parameter values as in Figure 2 are chosen, except that $\chi$ in the second figure is varied between 0 and 0.04.

“last-minute disclosure” as a detection event (see Endnote #2). We now consider an alternative in which this type of disclosure is treated as late disclosure instead of detection.

This alternative assumption does not change what happens under RI, because the firm does not practice last-minute disclosure in that case (see Lemma 1). Under PI, in contrast, two changes are made. First, the firm incurs the late disclosure penalty $\kappa_i$ after last-minute disclosure, instead of the detection penalty $\kappa_f$. Second, the regulator saves the cost $\chi$ he would have incurred in an inspection resulting in detection, because last-minute disclosure preempts such an event. In equilibrium, however, only the second change matters. This is because the firm’s disclosure behavior is unaffected except that he incurs $\kappa_i$ instead of $\kappa_f$ after last-minute disclosure, which the regulator takes advantage of by setting $\kappa_i$ to the maximum value $K$; the maximum penalty constraint continues to bind with $\kappa_f = K$. Consequently, the firm’s choice of disclosure window size $s$ in equilibrium remains exactly the same as before.

To account for the inspection cost savings, the social cost $\bar{C}$ in (4) under PI needs to be modified. It can be shown that this modification—adding an extra term $-\lambda(e^{-\mu s} - e^{-\mu t})/\mu \tau$ in (4)—does not change the qualitative insights we derived in the last section (see Proposition B.1 in the online appendix for details). Specifically, complementarity between the penalty $\kappa_i = K$ and inspection frequency continues to exist under PI, and PI continue to be preferred to RI when $K$ is small. The modification, in fact, amplifies the latter effect, because reduced number of inspections with no change in the firm’s disclosure behavior favors PI in a wider range of parameter combinations. Therefore we conclude that the main insights from our analysis are not significantly impacted by an alternative modeling assumption on last-minute disclosure.

6.2. Investment in Compliance Improvement

Thus far, all results have been derived under the assumption that the only action the firm takes in response to the regulator’s inspections is disclosure of noncompliance. This allows us to isolate the impact of the firm’s disclosure decision from other confounding factors, but in reality, disclosure may be combined with other actions. In this section, we consider one such scenario where the firm makes an up-front investment to improve compliance, leading to an increased mean compliance duration $1/\lambda$. A similar assumption is made by Innes (1999a, b) and others.

We modify the model as follows. To capture compliance improvement, we redefine the transition rate $\lambda$ as $\lambda = \lambda_0/(1 + a)$, where $a \geq 0$ is the “effort” variable that denotes the percentage increase in mean compliance duration. Therefore, exerting an effort of amount $a$ results in an increase in mean compliance duration from the default value $1/\lambda_0$ to $(1 + a)/\lambda_0$. For simplicity, we assume that the cost of effort is linearly increasing in $a$ such that it is equal to $ca$, where $c$ is the effort cost coefficient. Assume $c < r\lambda_0/\mu$, which rules out a trivial case where the firm never finds it optimal to exert an effort. Finally, we insert an additional step in the game sequence: after the regulator has announced the penalties and inspection frequency but before the firm starts making his disclosure decision dynamically, the firm optimally chooses the effort level. This sequence reflects the fact that compliance improvement typically requires an advance commitment (e.g., up-front investment to enhance equipment reliability).

A complete characterization of the equilibrium of this modified game turns out to be analytically intractable, since adding one more decision variable results in multimodal objective functions that substantially increase complexity. As such, we investigate the new equilibrium numerically after specifying the firm’s optimal response in the following lemma. As the lemma shows, under the assumption $\lambda \ll \mu$, it turns out that the firm makes his compliance improvement decision independently from the disclosure decision. This is true because the firm’s dynamic disclosure decision is made ex post with a myopic goal (which coincides with the long-run objective under $\lambda \ll \mu$), and therefore the
duration of compliance preceding noncompliance does not factor into his disclosure decision.

Lemma 7. Let \( \rho \equiv (r + K_d \mu) / (r + K_s \mu) \). The firm makes the disclosure decision as described in Lemmas 1 and 3. Moreover, the firm sets \( \lambda \) as follows: (a) under random inspections, \( \lambda = \min \{ \lambda_0, \sqrt{c_L \mu / (r + K_d \mu)}(\mu \tau_1 + 1) \} \) if \( \rho > \mu \tau_1 + 1 \) and \( \lambda = \sqrt{c_L \mu / (r + K_s \mu)} \) if \( \rho \geq \mu \tau_1 + 1 \) and (b) under periodic inspections, \( \lambda = \min \{ \lambda_0, \sqrt{c_L \mu / (r + K_s \mu)}(\mu \tau / (\ln p + 1 - pe^{-\mu \tau})) \} \) if \( \rho < e^{\mu \tau} \) and \( \lambda = \sqrt{c_L \mu / (r + K_s \mu)} \) if \( \rho \geq e^{\mu \tau} \).

Lemma 7 also shows that the firm exerts more compliance effort (lower \( \lambda \)) in response to a greater degree of enforcement, i.e., higher penalties and higher inspection frequency. Although this is expected, we find that a new trade-off is introduced as the early disclosure penalty \( K_s \) now plays an additional role as an incentive for compliance improvement. This permits a possibility that a nonzero value for \( K_s \) is optimal, but numerical examples show that such instances occur only in extreme situations (very small values for \( c \) and \( \chi \)). Thus, in most cases, equilibria are found with \( K_s = 0 \) and \( K_d = K \) as before.

Even with the equilibrium penalties unchanged, adding compliance improvement decision creates new dynamics. This is illustrated in Figure 4(a). Comparing this figure to its counterpart Figure 2(a), we see that a more complex pattern emerges. This is evident from the nonmonotonic shape of \( \tau^* \) (solid line in the figure) in the small-\( K \) region, where complementarity between the penalty \( K_d = K \) and inspection frequency \( 1 / \tau^* \) is now augmented by substitutability around \( K = 0 \). The new substitutability at a very low \( K \) appears because the firm’s compliance improvement effort lessens the regulator’s need to perform costly inspections; adding the effort attenuates complementarity between the penalty and inspection frequency and may even reverse it.

Despite this change, our earlier conclusion in Proposition 4—that the PI policy is preferred to the RI policy for small \( K \), whereas the opposite is true for large \( K \) —remains the same; see Figure 4(b). Hence, this key result is robust to the model variation that includes endogenous determination of the transition rate \( \lambda \).

7. Conclusions

In this paper, we offer new perspectives on the problem of environmental regulation enforcement by developing a novel analytical framework that combines law enforcement economics with reliability theory. We find that, contrary to common belief, periodic inspections (PI) may outperform random inspections (RI) even though PI offer perfect information about the inspection schedule that allows the firm to choose the timing of noncompliance disclosure to his advantage. This opportunistic behavior limits the effectiveness of PI, yet the net efficiency may actually be higher under PI than under RI. As it turns out, PI are more efficient at detecting noncompliance when compliance or noncompliance states alternate stochastically. Therefore, there exists a trade-off between efficient noncompliance detection and the firm’s opportunistic disclosure behavior. Depending on which is more significant, either inspection policy may be preferred. We find that PI are preferred when the regulator operates in restrictive conditions, marked by limited penalties and a high inspection cost. If these conditions are not in place, RI are preferred.

Furthermore, we find that the firm’s opportunistic disclosure behavior may lead to an equilibrium in which substitutability between inspection intensity and detection penalty, a classic insight by Becker (1968), is reversed. This happens when a partial disclosure equilibrium is established under PI in which the firm discloses only the late occurrences of noncompliance. In such a case, the regulator complements a higher penalty with more frequent inspections to steer the firm toward more disclosure.

Recently, there have been heated debates about upset emissions, such as those caused by unforeseen equipment malfunctions during production. The existing EPA policy on regulating these types of violations have been criticized due in large part to inconsistencies created by a combination of escape clauses, reliance on self-reporting, and light penalties (Environmental Integrity Project 2004, 2012; Ozymy and Jarrell 2011). Following a series of lawsuits brought to correct this situation, in September 2014, the EPA proposed to eliminate penalty exemptions that had been routinely granted for self-reported violations, a practice perceived by advocacy groups as a loophole (U.S. EPA 2014a, b). Although it is too early to tell whether this change will bring about the desired environmental benefits, one unintended consequence that might follow is exacerbated under-reporting of violations (which is believed to be prevalent already; see McCoy et al. 2010, Lombardi and Fuller 2013), since the firms’ economic benefit of reporting a violation—penalty exemption—is now removed. As a result, the regulators may have to rely more on inspections to discover violations and enforce the policy. In this new regulatory setting, our analysis suggests that it may be more cost-effective to perform scheduled inspections in lieu of random inspections. This insight is valuable especially because practitioners and scholars alike have assumed that RI are superior, focusing only on behavioral benefits.

Our findings suggest that enforcement strategies for mitigating the impact of environmental violations should be tailored to operating conditions. Conventional practices that perform well in controlling routine emissions may be less effective when they are employed to control sporadic and random emissions. Finally, the analytical framework developed in this paper can be used as a basis for estimating the amount of emissions that go unreported. Such inferences are widespread in other areas (for example, see Kaplan 2010), and our model paves the way for similar applications.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/opre.2015.1345.
Figure 4. Equilibrium mean inspection intervals $\tau^r$ and $\tau^p$ (left) and the social costs $C^r$ and $C^p$ (right) when the firm invests in compliance improvement with $\lambda_0 = 1.0$ and $c = 0.05$.

Notes. All other parameters are set to the same values as those in Figures 2 and 3. The left panel parallels Figure 2(a) and the right panel parallels Figure 3(a).

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Appendix. Proofs of Main Results

Proof of Lemma 1. Let $D \sim \exp(\mu)$ and $T \sim \exp(\nu)$ denote the duration of a noncompliance episode and the time between two successive and uninterrupted inspections, respectively. Without loss of generality, focus on a randomly selected noncompliance episode. As stated in §3.7, the firm’s objective is to maximize his expected profit for the duration of the episode after it occurs. Let $t \geq 0$ be the time since the onset of noncompliance. We consider $t = 0$ and $t > 0$ separately; consider $t > 0$ first. Let $\mathcal{N}_t$ denote the event [No disclosure and no detection by time $t \in (0, D)$]. Let $D_t$ and $T_t$ denote the remaining time until compliance restoration and until the next inspection, respectively, measured at time $t > 0$ given $\mathcal{N}_t$. Because of memorylessness, $D_t \sim \exp(\mu)$ and $T_t \sim \exp(\nu)$. Suppose that, given $\mathcal{N}_t$, the firm at time $t > 0$ decides whether to disclose noncompliance at that moment or to delay the decision by a time interval $\delta$. Thus the firm’s action choice consists of two options: (a) disclose immediately at time $t$ or (b) delay the decision to time $t + \delta$. If the firm chooses the disclosure option, then he incurs the late disclosure penalty $\kappa_1$ (since $t > 0$) and forgos subsequent revenue since production is suspended immediately after disclosure. If the firm chooses the delay option, in contrast, he earns additional expected revenue $rE[\min\{D_t, T_t, \delta\}]$ and faces three possibilities at time $t + \delta$: (i) compliance restoration if $D_t < \min\{T_t, \delta\}$; (ii) detection if $T_t < \min\{D_t, \delta\}$; (iii) the same choice as before if $\delta < \min\{D_t, T_t\}$. In case (iii), the firm’s disclosure/delay choice at time $t + \delta$ (given $\mathcal{N}_{t+\delta}$) is identical to the one he faced at time $t$ because of time symmetry, due to memorylessness of $D_t$ and $T_t$. Hence, the firm’s expected profit for the remaining duration of the current noncompliance episode after the disclosure or delay choice at any time $t > 0$ conditional on $\mathcal{N}_t$ is a constant; denote this constant “profit to go” as $V$. Then, in cases (i)–(iii) above, the firm’s delay choice expects to bring him additional profits of $0$, $-\kappa_1$, and $V$, respectively. Combining all possibilities, we see that the profit to go at time $t > 0$ after the delay choice is $V = rE[\min\{D_t, T_t, \delta\}] - \kappa_1 \Pr(T_t < \min\{D_t, \delta\}) + V \Pr(\delta < \min\{D_t, T_t\})$. Noting $\Pr(T_t < \min\{D_t, \delta\}) = \Pr(T_t < D_t) \Pr(\min\{D_t, T_t\} < \delta)$ using the fact that $1(T_t < D_t)$ and $\min\{D_t, T_t\}$ are independent random variables, we evaluate $V_t$ as $V_t = (r - \kappa_1 \nu/\mu) + (1 - e^{-r(\mu + \nu)} + V e^{-r(\mu + \nu)})$. Therefore, at any time $t > 0$ given $\mathcal{N}_t$, he earns the delay option at time $t > 0$ if $V_t > 0$, otherwise, he chooses the disclosure option if $V_t \leq 0$. In case (a), the firm chooses the delay option if $V_t = 0$, whereas an infinitesimal delay $\delta$ is optimal at time $t = 0$ under the two identified conditions. First, assume $\kappa_1 < (\kappa_1 - r\nu)/\mu + 1$. If the firm chooses disclosure at time $t = 0$, he incurs the penalty $\kappa_1$ and forgoses subsequent revenue. If the firm chooses to delay the decision by an infinitesimal amount of time, in contrast, he incurs the penalty $\kappa_1$ and forgoses subsequent revenue because it is optimal for him to choose disclosure at time $t > 0$ under the condition $\kappa_1 \leq (\kappa_1 - r\nu)/\mu + 1$. Since $\kappa_1 \leq \kappa_1$, the firm chooses immediate disclosure at time $t = 0$ and earns the profit of $-\kappa_1$. Hence, under the condition $\kappa_1 \leq (\kappa_1 - r\nu)/\mu + 1$, full disclosure is optimal. Note that this condition implies $\kappa_1 \leq (\kappa_1 - r\nu)/\mu + 1). Second, assume $\kappa_1 > (\kappa_1 - r\nu)/\mu + 1$. Immediate disclosure at time $t = 0$ gives the firm the expected profit equal to $-\kappa_1$, whereas an infinitesimal delay gives him the expected profit of $V = (r - \kappa_1 \nu)/\mu + 1$, which is earned by continuing to delay the disclosure decision. Hence, under the condition $\kappa_1 > (\kappa_1 - r\nu)/\mu + 1$, full disclosure is optimal if $-\kappa_1 \geq V = (r - \kappa_1 \nu)/\mu + 1$, whereas nondisclosure is optimal otherwise. Combining all cases, we conclude that nondisclosure is optimal if $(\kappa_1 - r\nu)/\mu + 1 < \kappa_1$, whereas
whereas full disclosure is optimal if $\kappa_i \leq (\kappa_i - r \tau)/(\mu \tau + 1)$. These conditions can be rewritten as $\tau > (\kappa_i - \kappa_i)/(r + \mu + \kappa)$ and $\tau \leq (\kappa_i - \kappa_i)/(r + \mu + \kappa)$, respectively. □

Proof of Lemma 2. Consider a single compliance-noncompliance cycle (“cycle”), which forms a renewal. Each cycle lasts $U + D$ time units, where $U \sim \exp(\lambda)$ and $D \sim \exp(\mu)$. Set the cycle start time to zero. Let $T_i$ be the remaining time until the next inspection since time $U$, which marks the onset of noncompliance. Because of memorylessness, $T_i$ and $T_i$ are identically distributed: $T_i \sim \exp(\nu)$. Recall $\nu = 1/\tau$. First, consider full disclosure. Since all noncompliance episodes are disclosed, there exist no suspended noncompliance: hence the expected durations of suspended and unsuspended noncompliance in a cycle are, respectively, $E[R] = E[D] = 1/\mu$ and $E[B] = 0$. Applying the renewal-reward theorem, we evaluate the long-run averages as $\bar{R} = E[R]/(E[U] + E[D]) = \lambda/(\lambda + \mu)$ and $\bar{B} = 0$. To compute $\bar{I}$, note that inspections are performed only for the duration of $U$ in a cycle since inspections are suspended as soon as noncompliance occurs. Hence the expected number of inspections performed in a cycle is equal to $E[I] = E[U]/(1/\tau)$ and it follows that $\bar{I} = E[I]/(E[U] + E[D]) = (\mu/(\mu + \lambda)) \lambda/(\lambda + \mu)$. Next, consider nondisclosure. Since the probability of detection in a cycle is $Pr(T_i \leq D)$ and the duration of noncompliance since detection is distributed identically to $D$ because of memorylessness, $E[R] = E[D] = 1/\mu$ and the firm’s long-run average cost is $\bar{\Psi} = r - E[U] + E[D] = (\lambda/(\lambda + \mu)) \lambda/(\lambda + \mu)$. Hence, $\bar{R} = E[R]/(E[U] + E[D]) = (\lambda/(\lambda + \mu)) \lambda/(\lambda + \mu)$. Moreover, $E[B] = E[\min(T_i, D)] = 0$. In the next $D$ time units the regulator performs at most one inspection: zero in the event $T_i > D$ and one in the event $T_i \leq D$ since inspections are suspended following a detected violation. Therefore, the expected number of inspections in a cycle is $E[I] = 0 + E[U]/(1/\tau) + 1/(\mu + \tau + 1)$. Then, $\bar{I} = E[I]/E[U] + E[D] = (\mu/(\mu + \lambda)) \lambda/(\lambda + \mu)$. The firm’s expected profit per cycle is $E[\Pi] = rE[U] + rE[\min(T_i, D)] - \kappa_i Pr(T_i \leq D) = r/(\lambda + (r - \kappa_i)/(\mu + \nu))$, where the event $T_i \leq D$ represents detection. Then, the firm’s long-run average cost is $\bar{\Psi} = r - E[U]/(E[U] + E[D]) = (\lambda/(\lambda + \mu))(r/(\mu + \lambda) + \kappa_i/(\mu + \lambda))$. The approximations are obtained by expanding the expressions with respect to the ratio $\lambda/\mu$ and retaining up to the first-order terms. □

Proof of Proposition 1. Fix $\kappa_i$ and $\kappa_d$, and let $\phi \equiv (\kappa_d - \kappa_i)/(r + \mu + \kappa)$. Since $\phi$ increases in $\kappa_d$, and decreases in $\kappa_i$, with $0 \leq \kappa_i \leq \kappa_i \leq K$, the lower bound on $\phi$ is found by setting $\kappa_i = \kappa_i$ and $\kappa_d = 0$, resulting in $0 \leq \phi \leq K/r$. Let $C_D$ and $C_D$ denote $C_D$ defined in (2) for the regions $\tau \leq \phi$ and $\tau > \phi$, respectively. Evaluating $C_D$ and $C_D$ at the boundary $\tau = \phi$ and subtracting them yield $C_D - C_D = (\lambda/(\mu + \phi - h + r)\kappa_i/(\mu + (\phi - h + r)) > 0$, implying that $C_D$ is discontinuous at $\tau = \phi$, jumping upward as $\tau$ crosses $\phi$ from left to right. Hence $C_D < C_D$ in the vicinity of $\tau = \phi$. Suppose $\tau \leq \phi$. From (2) we see $\partial C_D/\partial \tau < 0$, which implies that $C_D$ is minimized at the boundary $\tau = \phi$. Now, suppose $\tau > \phi$. Differentiating $C_D$ and setting it to zero yields the solution $\tau^* = (\sqrt{(h - r)/(\mu + \phi - h + r)}\kappa_i/(\mu + (\phi - h + r)) - 1. Substituting $\tau^*$ in the second derivative, we get $(\partial^2 C_D/\partial \tau^2)|_{\tau^*} = (2/(\mu + (\phi - h + r)\kappa_i/(\mu + (\phi - h + r)) > 0$, which implies that the unique critical point $\tau^*$ is a local minimizer of $C_D$. It can be verified that $\lim_{\tau \to \tau^*} C_D^* = \infty$, $\lim_{\tau \to \tau^*} C_D^* = (h\lambda)/(\mu + (\phi - h + r))$ and $\lim_{\tau \to \tau^*} \partial C_D^*/\partial \tau = -\infty$; hence $\tau^*$ is the unique interior minimizer of $C_D$ in the expanded region $\tau > 0$. Whether $\tau^*$ is also the global minimizer depends on the conditions $\tau^* \leq \phi$ and $\tau^* > \phi$. In the former case $C_D^*$, defined in the region $\tau > \phi$, is increasing in $\tau$; together with the facts that $C_D^*$ is minimized at $\tau = \phi$ and that $C_D^* < C_D^*$ in the vicinity of $\tau = \phi$, this implies that $C_D^*$ is minimized at $\tau = \phi$. In the latter case, $C_D^*$ is minimized at the interior point $\tau^* > \phi$, and $C_D^*$ is minimized at $\tau = \phi$. Given that $C_D^* < C_D^*$ in the vicinity of $\tau = \phi$, there are two candidates for the global minimizer: $\tau = \phi$ and $\tau^*$. Evaluating $C_D^*$ at these values, we get $C_D^* = r\lambda/\mu + (\chi/\phi)(1 - \lambda/\mu)$ at $\tau = \phi$ and $C_D^* = r\lambda/\mu - \chi(\mu - 2\lambda) + 2\chi((h - r)/(\mu + \phi - h + r))/\mu$ at $\tau = \tau^*$. $\tau^*$ is the global minimizer if and only if $C_D^* < C_D^*$, which is equivalent to the condition $\phi < \tau^*$, where $\tau^* = (2\lambda((\mu/\lambda - \phi)))/(\mu + \lambda)$. Note that, since $\tau^* < \tau^*$ (which follows from the assumptions $\chi < ((h - r)/\mu)^2$ and $\lambda \ll \mu$ introduced in §3), the condition $\phi < \tau^*$ implies $\phi < \tau^*$, which ensures the local minimum at $\tau = \tau^*$. Summarizing, the global minimizer of $C_D$ is found at $\tau = \tau^*$ if $\phi < \tau^*$ and is at $\tau = \phi$ if $\phi \geq \tau^*$. The corresponding reduced social cost is $C_D^* = r\lambda/\mu - \chi(\mu - 2\lambda) + 2\chi((h - r)/(\mu + \phi - h + r))/\mu \lambda$ at $\phi < \tau^*$ and $C_D^* = r\lambda/\mu - \chi(\phi)(1 - \lambda/\mu)$ at $\phi \geq \tau^*$. Notice that, since $\tau^* < \tau^*$ independent of $\phi$, whereas for $\phi \geq \tau^*$ it decreases in $\phi$. Hence, as $\phi$ increases from zero to infinity, $C_D^*$ stays constant for $\phi < \tau^*$ and then decreases afterward. Next, we incorporate the bounds $0 \leq \phi \leq K/r$ in finding $\phi$ that minimizes $C_D^*$. Two cases need to be considered separately: $K/r < \tau^*$ and $K/r \geq \tau^*$. If $K/r < \tau^*$, the constraint $\phi \leq K/r$ implies $\phi < \tau^*$, the region in which $C_D^*$ does not vary with $\phi$. Hence, the minimum is found at any value of $\phi$ satisfying $\phi \leq K/r$, which implies that any values of $\kappa_i$ and $\kappa_d$ satisfying $0 \leq \kappa_i \leq \kappa_i \leq K$ are permitted. Recall from above that the optimal $\tau$ in this case is $\tau = \tau^*$ since $\phi < \tau^*$. If $K/r \geq \tau^*$, in contrast, the constraint $\phi \leq K/r$ includes the region in which $C_D^*$ decreases in $\phi$. Hence $C_D^*$ is minimized at the constraint boundary $\phi = K/r$, which implies $\kappa_i = 0$ and $\kappa_d = K$. Recall from above that the optimal $\tau$ in this case is $\tau = \phi = K/r$ since $\phi = K/r \geq \tau^*$. Summarizing, it is optimal for the regulator to choose any $\kappa_i$ and $\kappa_d$ satisfying $0 \leq \kappa_i \leq \kappa_i \leq K$ with $\tau = \tau^*$ if $K/r < \tau^*$, whereas it is optimal to choose $\kappa_i = 0$ and $\kappa_d = K$ with $\tau = K/r$ if $K/r \geq \tau^*$. The firm’s optimal response to this choice follows directly from Lemma 1. □

Proof of Lemma 3. The proof proceeds similarly like that of Lemma 1. We use the same notations and conventions used there. Assume that, upon transition to noncompliance, the firm finds $s \in (0, \tau)$ time units remaining until the next scheduled inspection. Let $t \geq 0$ be the time since the onset of noncompliance. Suppose that, given $N$, the firm at time $t > 0$ decides whether to disclose noncompliance at that moment or to delay the decision by a time interval $\delta = s/n$, where $n$ is sufficiently large so that $\delta \ll s$. Thus the firm’s action choice consists of two options: (a) disclose immediately at time $t$ or (b) delay the decision to time $t + \delta$. Let $V(t|X)$ be the firm’s expected profit for the remaining duration of the current noncompliance episode after this choice is made.
at time $t$ ("profit to go"), conditional on $N_t$. Suppose the condition $N_t$ is met at time $t \in (0, s)$. If the firm chooses to disclose noncompliance at time $t$, then his conditional profit to go at that moment is $-\kappa_t$; he incurs the late disclosure penalty $\kappa_t$ (since $t > 0$) and foregoes the potential revenue after time $t$ since production is suspended immediately after disclosure. If the firm chooses to delay the disclosure decision by $\delta$ time units, in contrast, he earns an additional expected revenue $rE[\min(D_t, \delta)]$ and faces the same choice as before at time $t + \delta$ if $D_t \geq \delta$, i.e., if noncompliance does not end before time $t + \delta$. Thus, his conditional profit to go at time $t$ after the delay choice is $V_t(N_t) = rE[\min(D_t, \delta)] + V(t + \delta \mid N_{t+\delta})Pr(D_t \geq \delta) = r/\mu(1 - e^{-\mu t}) + V(t + \delta \mid N_{t+\delta})e^{-\mu \delta}$. Then, the firm's disclosure or delay choice at time $t \in (0, s)$ given $N_t$ is expressed by the recursion $V_t(N_t) = \max(-\kappa_t, V_t(N_t))$: the first argument corresponds to the disclosure option and the second argument to the delay option. The terminal value for $V(t \mid N_t)$ is $V(s \mid N_t) = -\kappa_t$, since at time $t = s$ given $N_t$ the firm is detected and pays the penalty $\kappa_t$ with additional revenue forfeited because production is suspended then. Given this value, at time $t = s - \delta$, the firm's choice is expressed as $V(s - \delta \mid N_{s-\delta}) = \max(-\kappa_t, (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu \delta})$ and (b) $-\kappa_t \geq (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu \delta}$ are two possible cases: (a) $-\kappa_t < (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu \delta}$ and $\kappa_t e^{-\mu \delta}$ in case (a), the firm chooses the delay option at time $t = s - \delta$. Then, $V(s - \delta \mid N_{s-\delta}) = (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu \delta}$. Since $(r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu \delta} > -\kappa_t$, by the condition in (a), from $V(s - \delta \mid N_{s-\delta}) = \max(-\kappa_t, (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu \delta})$, we see that the firm chooses the delay option at time $t = s - \delta$. Repeating the same argument iteratively, we see that the firm at time $t = s - n\delta$ faces the problem $V(s - n\delta \mid N_{N_{s-n\delta}}) = \max(-\kappa_t, (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu n\delta})$ and chooses the delay option for all $n = 1, 2, \ldots, n - 1$ whenever the condition in (a) is met. Next, consider case (b). In this case the firm chooses the disclosure option at time $t = s - \delta$. Then, $V(s - \delta \mid N_{s-\delta}) = -\kappa_t$ and therefore $V(s - n\delta \mid N_{s-n\delta}) = (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu n\delta}$. Since $(r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu n\delta} > -\kappa_t$, we have $V(s - n\delta \mid N_{s-n\delta}) \geq \max(-\kappa_t, (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu n\delta}) = (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu n\delta}$, implying that the firm chooses the disclosure option at time $t = s - \delta$. Repeating the same argument iteratively, we see that the firm at time $t = s - n\delta$ faces the problem $V(s - n\delta \mid N_{s-n\delta}) = \max(-\kappa_t, (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu n\delta})$ and chooses the delay option for all $n = 2, 3, \ldots, n - 1$ whenever the condition in (b) is met. The only exception is $j = 1$, when the firm chooses disclosure at time $t = s - n\delta$ as we proved above. Letting $n \to \infty$ and thus $\delta = s/n \to 0$ reduces the conditions in the cases (a) and (b) to $\kappa_t > \kappa_d$ and $\kappa_t \leq \kappa_d$, respectively. Therefore, since $\kappa_t \leq \kappa_d$ by assumption, only (b) survives for all $t > 0$, the firm keeps delaying the disclosure decision until either compliance is restored or he discloses noncompliance immediately before time $t = s$, an event considered detection by the assumption in $\S 3.5$. Now, consider the firm's disclosure or delay choice at time $t = 0$. Immediate disclosure at time $t = 0$ gives the firm the expected profit equal to $-\kappa_t$, the early disclosure penalty. In contrast, an infinitesimal delay leads to continued disclosure delay until time $t = \min(D, s)$, at which either compliance is restored (if $D > s$) or detection occurs at time $t = s$ (if $D > s$). The expected profit of the delay choice at $t = 0$ is then $rE[\min(D, s)] - \kappa_t Pr(D > s) = (r/\mu)(1 - e^{-\mu t}) - \kappa_t e^{-\mu t}$. Comparing the expected profits of the two options, we conclude that the firm keeps delaying disclosure if $(1/\mu)\ln(r + \kappa_t)/(r + \kappa_d) < s$, whereas he discloses the current episode at the onset if $s \leq (1/\mu)\ln(r + \kappa_t)/(r + \kappa_d)$.

**Proof of Lemma 4.** See $\S A$ in the online appendix.

**Proof of Proposition 2.** For notational convenience, let us suppose the portfolio $\sigma$ in $G(t \mid \sigma)$. It can be proved that the equilibrium exists in the region $t \geq \sigma$ satisfying $\kappa_t = 0$ and $\kappa_d = K$, and that the firm sets $s' = \sigma$ in equilibrium. (See Lemma B.2 in the online appendix.) It then remains to find $t \geq \sigma$ that minimizes $C(4)$ for $s' = \sigma$ substituted. Let $C^*(t)$ be the reduced cost function with $s' = \sigma$. First, we show that $C^*(t)$ defined in the expanded region $t > 0$ has a unique interior minimizer. Differentiating $C^*(t)$ and setting it to zero yields the first-order condition $G(t) = \mu/\lambda - 1$. Let us rewrite $G(t)$ as

$$G(t) = \left(1 - \frac{h - r}{\chi \mu}\right)(1 - e^{-\mu t}) + \left(1 - \frac{2h - r}{\chi \mu}\right)\frac{2}{\mu t}. $$

Moreover, the following properties hold: (i) in the limit $t \to 0$, $G(t), G'(t),$ and $G''(t)$ approach 0, 0, and $(h - r)/\chi \mu - \mu^2/3$ if $\sigma = 0$, while they approach $-\infty$, $-\infty$, and $-\infty$ if $\sigma > 0$; (ii) in the limit $t \to \infty$, $G(t)$ and $G'(t)$ approach $h/r(\chi \mu) - 1 + \alpha$ and $0$. Since $G(t)$ exhibits behaviors that are qualitatively different around $t = 0$ depending on whether $\sigma = 0$ or $\sigma > 0$, we consider these two cases separately. Assume $\sigma = 0$ and let $G_0(t)$ be $G(t)$ with $\sigma = 0$. The limits shown above imply that $G_0(t)$ initially increases from zero at $t = 0$ if $h/r(\chi \mu) > 1/2$, converging to $h/r(\chi \mu) - 1$ as $t \to \infty$. Let $t^0$ be the solution of $G_0(t^0) = 0$, i.e., $t^0$ is a critical point of $G_0(t)$. Note that $G_0(t^0) = 0$ can be written as $\varphi_0(\mu t^0) = ((h - r)/(\chi \mu))(3 - \mu^2 t^0)$, where $\varphi_0(x) = (1/x^3)(2(e^{-x} - 1 - x))$ is decreasing. It can be proved that $\varphi_0(x) < 1$ for $x > 0$. (See Lemma B.3 in the online appendix.) Then, evaluating $G_0(t^0)$ at $t^0$, we get $G_0(t^0) = (\varphi_0(\mu t^0) - 1)^2 < 0$. That $G_0(t^0) < 0$ at a critical point $t^0$ implies $G_0(t)$ is quasi-concave. Recall the assumptions $\mu/\lambda > 1$ and $h/r(\chi \mu) > \mu/\lambda$ stated in $\S 3.3$, which together imply $h/r(\chi \mu) > 1 + \alpha$ when $\sigma = 0$, with $h/r(\chi \mu) > 1 + \alpha$ when $\sigma = 0$, with $h/r(\chi \mu) > 1 + \alpha$. Combined with the earlier finding that $G_0(t)$ initially increases if $h/r(\chi \mu) > 1/2$ and that $\lim_{t \to -\infty} G_0(t) = (h/r(\chi \mu) - 1) > 0$, we can show that $G_0(t)$ crosses $\mu/\lambda - 1$ exactly once from below at $t^0 > 0$. Therefore the optimal solution is unique when $\sigma = 0$. Now, assume $\sigma > 0$. Consider two cases: $\beta \geq 2$ and $0 < \beta < 2$. If $\beta \geq 2$, it is straightforward to show that $G'(t) > 0$ for all $t \geq 0$. Therefore, in this case, $G(t)$ monotonically increases from $-\infty$ to $h/r(\chi \mu) - 1 + \alpha$ as $t$ goes from zero to infinity. Since $\sigma > 0$ when $\sigma = 0$ and $h/r(\chi \mu) > \mu/\lambda$, the limit $h/r(\chi \mu) - 1 + \alpha$ is greater than $\mu/\lambda - 1$; hence, $G(t)$ crosses $\mu/\lambda - 1$ exactly once.
from below and therefore the solution is unique. Now, assume \( \beta < 2 \). In this case, \( G(\tau) \) may not be monotonically increasing, i.e., it may peak before it converges to \((h - r)/(\mu \lambda) - 1 + \alpha \) as \( \tau \to \infty \). Let \( \tau^* \) be the solution of \( G(\tau) = 0 \), i.e., \( \tau^* \) is a critical point of \( G(\tau) \). Note that \( G(\tau^*) = 0 \) can be written as \( e^\alpha \frac{1}{\mu \lambda - 1} \). The function \( \phi(x) \) defined above is a special case of \( \phi(x) \).\) It can be proved that \( \phi(x) < 0 \). Then, the argument similar to the conclusion of \( \alpha = 0 \) leads to the conclusion that the solution is unique. In all cases \( \alpha = 0 \), \( \alpha > 0 \) with \( \beta > 2 \), \( \alpha > 0 \) with \( \beta < 2 \), we found that the equation \( G(\tau) = \mu \lambda - 1 \) has a unique solution for which \( G(\tau) < \mu \lambda - 1 \) and \( G(\tau) > \mu \lambda - 1 \) to its right. Since the solution depends on \( \sigma \), we denote it as \( \tilde{\sigma} \), which satisfies \( G(\tilde{\sigma}(\sigma)) > 0 \). Thus, \( \tilde{\sigma}(\sigma) \) defined in the expanded region \( \tau > 0 \) has a unique interior minimizer \( \tilde{\sigma}(\sigma) \), the statement we set out to prove. Restricting the region to that in which the equilibrium exists \( \tau > 0 \), we finally conclude that in equilibrium, the regulator chooses \( \tau^* = \max\{\tilde{\sigma}(\sigma), \tilde{\sigma}(\sigma)\} \).

**Proof of Proposition 3.** First, consider RI. Proposition 1 states that \( \tau^* = \bar{\tau} \) if \( K < h - r \) and \( \tau^* = K\bar{r} \) if \( K > h - r \), where \( \bar{\tau}^* \) is independent of \( K \). Hence \( \tau^* \) does not vary in \( K \) for \( K > h - r \) whereas it increases in \( K \) for \( K > h - r \). From the same proposition, we also see that the latter condition is sufficient and necessary for full disclosure. Note also that \( \tau^* < \bar{\tau}^* \). Then, \( \tau^* \) jumps downward at \( K = h - r \) because, at that boundary, \( \lim_{h - r \to h - r^+} \tau^* = \bar{\tau}^* \). Next, consider PI. Recall from Proposition 2 that \( \tau^* = \max\{\tilde{\sigma}, \tilde{\sigma}(\sigma)\} \), where we suppressed the argument in \( \tilde{\sigma}(\sigma) \) for notational convenience. Suppose \( \sigma < \tilde{\sigma} \). Then, \( \tau^* = \bar{\tau} \), which satisfies \( G(\bar{\tau}(\sigma)) = \mu \lambda - 1 \) (Proposition 2). Implicit differentiation of this equation yields \( d\bar{\tau}/d\sigma = -((\delta G(\bar{\tau}(\sigma))/\delta(\sigma))/\delta G(\bar{\tau}(\sigma))/\delta(\sigma))_{\sigma=\bar{\sigma}} \), which we now show to be negative. In the proof of Proposition 2, we showed that the numerator satisfies \( \delta G(\sigma)/\delta(\sigma)_{\sigma=\bar{\sigma}} > 0 \). Next, observe that the denominator satisfies

\[
\frac{\delta G(\sigma)/\delta(\sigma)_{\sigma=\bar{\sigma}}}{\alpha(\tau - r - \chi(\mu + \alpha)) (1 + \mu \sigma)^2} > 0,
\]

where the first inequality follows from \( 1 - e^{-\mu(\tau - r - \chi(\mu + \alpha))} > x/(1 + x) \), the second inequality from the assumption \( \sigma < \bar{\sigma} \) stated above, and the third from the assumptions \( (h - r+ r)/(\mu \lambda) > \mu \lambda - 1 \) stated in §3, which together imply \( h - r+ r)/(\mu \lambda) > 2 \). Therefore \( d\bar{\tau}/d\sigma < 0 \), i.e., \( \bar{\tau} \) decreases in \( \sigma \) if \( \sigma < \bar{\sigma} \). Next, suppose \( \sigma > \bar{\sigma} \). Then, \( \tau^* = \tilde{\sigma} \), which increases in \( \sigma \). In summary, \( \tau^* \) decreases in \( \sigma \) for \( \sigma < \bar{\sigma} \) and it increases for \( \sigma > \bar{\sigma} \), reaching the minimum at \( \sigma = \bar{\sigma} \). According to Proposition 2 full disclosure occurs if and only if \( \sigma \geq \bar{\sigma} \), when \( \sigma^* = \tau^* = \sigma \). Since \( \sigma \) defined in (5) is an increasing function of \( K \), the statements in the proposition about \( \tau^* \) follow.

**Proof of Proposition 4.** From Propositions 1–3, we infer that four separate regions exist based on the firm’s disclosure response: (I) nondisclosure under RI and PI; (II) nondisclosure under RI and partial disclosure under PI; (III) full disclosure under RI and partial disclosure under PI; (IV) full disclosure under RI and PI. Region I corresponds to \( K = 0 \), whereas the remaining regions appear in the presented order as \( K \) increases. We compare \( \tilde{C} \) and \( \tilde{C}^\tau \) in each of these regions. First, consider Region I. With nondisclosure, \( \tau^* \) and \( \tau^p \) are unique local minimizers of the corresponding long-run average social costs, denoted by \( \tilde{C}^\tau(\tau) \) and \( \tilde{C}^p(\tau) \). Under RI, \( \tilde{C}^\tau(\tau) = h\lambda \mu + \chi \tau / (1 + \mu \lambda + 1) \) (see (2)). Under PI, \( \tilde{C}^p(\tau) = h\lambda \mu + \chi \tau / (1 + \mu \lambda + 1) \). Obtaining the result, \( s = 0 \) in (4).

Using the inequality \( 1 - e^{-\alpha} > x/(x + 1) \), we see that \( \tilde{C}^\tau(\tau)^{(\tau)} = (\lambda \mu)(\chi \tau / (1 + \mu \lambda + 1)) > (\mu \lambda + 1)^{-1} \). Since \( \tilde{C}^\tau(\tau) < \tilde{C}^\tau(\tau) \) for any given \( \tau > 0 \), we have \( \tilde{C}^\tau(\tau) < \tilde{C}^\tau(\tau)^{(\tau)} \), i.e., the minimum of \( \tilde{C}^\tau(\tau) \) is smaller than the minimum of \( \tilde{C}^\tau(\tau) \). Hence \( \tilde{C} \). Consider Region II. From Proposition 1 and (3), we see that \( \tilde{C} \) does not change in \( K \) with full disclosure under RI. By contrast, \( \tilde{C} \) decreases in \( K \) with partial disclosure under PI; this can be shown from the observations that \( \tilde{C} \), evaluated by substituting \( s = \sigma^* \) and \( \tau = \tau^* = \sigma^* \) from (5) Proposition 2 in (4), decreases in \( \sigma \) (by the envelope theorem) and that \( \sigma^* = 1/\mu \ln(1 + K_\mu / r) \) in (5) increases in \( K \). Combined with the earlier finding that \( \tilde{C} \) decreases in \( K \) when \( K = 0 \), these results together imply that the difference \( \tilde{C} \) becomes larger as \( K \) increases in this region. Hence, \( \tilde{C} > \tilde{C} \). Third, consider Region IV. From (3), we have \( \tilde{C} = r\lambda \mu + \chi \mu / (1 + \mu \lambda + 1) \) with full disclosure under RI. Under PI, full disclosure implies \( s = \sigma^* = 1/\mu \ln(1 + K_\mu / r) \) (see Proposition 2). Substituting these in (4) yields \( \tilde{C} = r\lambda \mu + \chi \mu / (1 + K_\mu / r) \). Then,

\[
\tilde{C} - \tilde{C} = \chi \lambda \left( \frac{1}{\ln(1 + K_\mu / r)} - \frac{1}{K_\mu / r} \right) \left( \frac{\mu \sigma}{\lambda} - 1 \right) - \gamma \left( \frac{K_\mu / r}{\tau} \right),
\]

where \( \gamma(x) \equiv (x/2)/(x/\ln(1 + x)) - 1 \) is an increasing function. Recall the assumption \( K < r / \mu \) from §3, where \( \bar{\mu} \) uniquely solves the equation \( \gamma(x) = \bar{K} \). Then, \( \gamma(K_\mu / r) < \gamma(x) = \bar{K} \). With \( K_\mu / r < \ln(1 + K_\mu / r) \), this implies \( \tilde{C} > \tilde{C} \). Finally, consider Region III. We see from (3) that \( \tilde{C} \) with full disclosure decreases in \( K \). In addition, as we have shown in Region II, \( \tilde{C} \) with partial disclosure decreases in \( K \). Therefore, \( \tilde{C} \) and \( \tilde{C} \) decrease in \( K \) in this region. Combined with the earlier findings that \( \tilde{C} > \tilde{C} \) in Region II (to the left of Region III in the scale of \( K \)) and \( \tilde{C} < \tilde{C} \) in Region IV (to the right of Region III), this implies that \( \tilde{C} \) and \( \tilde{C} \) cross in this region.
it follows that $\theta(t)$ defined in (1) satisfies $\lambda/(\lambda + \mu + 1/t) < \theta(t) < \lambda$. Then,
\[
\tilde{I}_p - \bar{I}_c = \frac{\mu}{\mu + \lambda + 1} - \frac{\mu}{\lambda + \mu} + \left(1 + \frac{\lambda}{\mu + 1} \right)
\]
\[
< \frac{\mu}{\lambda + \mu} - \frac{\lambda + 1}{\mu + \lambda + 1} \frac{\lambda + \mu + 1}{\mu + \lambda + 1} = 0
\]
and
\[
\tilde{R}_p - \bar{R}_c = \frac{1}{\mu + \lambda + 1} - \frac{\lambda + 1}{\mu + \lambda + 1} \frac{\lambda + \mu + 1}{\mu + \lambda + 1} > \frac{\lambda + 1}{\mu + \lambda + 1} - \frac{\lambda + \mu + 1}{\mu + \lambda + 1} = 0
\]

Since $\tilde{I}_p < \bar{I}_c$ and $\tilde{R}_p > \bar{R}_c$, we have $\bar{C}_p < \bar{C}_c$, i.e., the long-run average social cost is lower under PI. Next, consider the case of full disclosure. This corresponds to $s^* = \tau$ under PI. Setting $s^* = \tau$ in the expressions derived in §A yields $\tilde{I}_p = (\lambda \mu / (\lambda + \mu)) \cdot (e^{-\lambda \tau} / (1 - e^{-\lambda \tau}))$ and $\bar{R}_c = \lambda / (\lambda + \mu)$. Under RI, in contrast, $\tilde{I}_p = (\mu / (\lambda + \mu)) \cdot \tau$. $\tilde{R}_c = \lambda / (\lambda + \mu)$ (see the proof of Lemma 2). Since $e^{-\lambda \tau} / (1 - e^{-\lambda \tau}) < 1 / \lambda \tau$, we have $\tilde{I}_p < \tilde{I}_c$. Combined with $\tilde{R}_p > \bar{R}_c$, this implies $\tilde{C}_p < \tilde{C}_c$. □

**Proof of Lemma 6.** Recall from the proof of Proposition 4 that $\tilde{C}^c$ and $\tilde{C}^p$ cross when full disclosure is induced under RI and partial disclosure is induced under PI. Hence $\tilde{C}^c = \lambda / \mu + (\chi / K)(1 - 1 / \lambda \mu)$ from (3) whereas $\tilde{C}^p$ is evaluated by substituting $s = \bar{s}$ and $\tau = \bar{\tau}$ > $\bar{s}$ from Proposition 2 in (4). Using the definition $\bar{s} = 1 / \mu \ln(1 + \lambda \mu / r)$ from (5), $\tilde{C}^c$ can be rewritten as $\tilde{C}^c = \lambda / \mu + (\chi / K)(1 - 1 / \lambda \mu)$. Define $\Delta(\chi) = \tilde{C}^c - \tilde{C}^p$, expressed as a function of $\chi$. Setting $\chi = 0$ yields $\Delta(0) = -((\bar{s} - \bar{s}) / \lambda \mu) \eta(\bar{s}, \bar{\tau})$, where $\eta(\bar{s}, \bar{\tau}) = 1 - \bar{s} / \bar{\tau} - e^{-\bar{s} / \bar{\tau}} - e^{-\bar{\tau} / \lambda \mu}$ is defined for $\bar{s} < \bar{\tau}$, observe $\eta(0, \bar{\tau}) > 0$, $\lim_{\bar{s} \to \infty} \eta(\bar{s}, \bar{\tau}) = 0$, and $\eta(\bar{s}, \bar{\tau}) / \delta \bar{s} < 0$. This implies $\eta(\bar{s}, \bar{\tau}) > 0$ for all $s < \bar{s}$, and hence $\Delta(0) < 0$. Moreover, using the envelope theorem, we get
\[
\Delta(\chi) = \frac{\mu}{e^{\chi / \bar{\tau}} - 1} \left(1 - \frac{\lambda}{\mu}\right) - \frac{\lambda}{\bar{\tau}} + \left(\frac{\mu}{\bar{\tau}} \bar{\tau} - \frac{\sigma^2}{\bar{\tau}^2}\right)
\]
\[
\left(\frac{\sigma}{\bar{\tau}} - \frac{e^{-\mu \bar{\tau}} - e^{-\bar{\tau} / \lambda \mu}}{\mu \bar{\tau}}\right).
\]
Let $\hat{\chi}$ be the value at which $\Delta = 0$. Rearranging the terms of the equation $\Delta(\chi) = 0$ and substituting them in $\Delta(\chi) = (\bar{s} - \bar{s}) / \lambda \mu \eta(\bar{s}, \bar{\tau}) > 0$. Combined with the earlier finding $\Delta(0) < 0$, this implies that $\Delta(\chi)$ crosses zero from negative to positive as $\chi$ increases. Since this is true at any crossing point $\hat{\chi}$ and $\Delta(\chi)$ is continuous, $\hat{\chi}$ is unique; if it were not unique, then the opposite direction of crossing should have also existed. Hence $\tilde{C}^p < \tilde{C}^c$ for all $\chi < \hat{\chi}$ and $\tilde{C}^p > \tilde{C}^c$ for all $\chi > \hat{\chi}$, where $\hat{\chi} \in [0, \infty)$. □

**Proof of Lemma 7.** The results are obtained by straightforward differentiation. □

**Endnotes**

1. For controversies surrounding accidental oil spills from hydraulic fracturing (reported and unreported), see Kusnetz (2012). Sahagun (2014) and Nearing (2014) document other recent examples of accidental pollution.

2. We make this assumption for three reasons. First, in practical situations, last-minute disclosure may come too late, after the regulator’s inspection planning is complete and the related expenses have already been spent. Second, honoring penalty reduction for this type of disclosure (see §3.5 for the penalty structure) that exhibits extreme opportunism contradicts the idea of awarding “good” behavior. Third, this assumption allows for a fair comparison between two inspection policies that we examine in §5.

3. Such a nonmonetary punitive action taken by a regulator is a key feature in Livernois and McKenna (1999), who assume that the regulator is empowered to force a firm to restore compliance. See Li and Miller (2014) who report government-led suspension of production because of environmental concerns.

4. For example, setting $\mu / \lambda = 10$ in the given equation yields $\hat{x} = 6.57 \times 10^7$, implying that the maximum penalty that can be levied should satisfy $K < r / \mu = 0.1 \times \hat{x} \times (r / \lambda)$, i.e., the penalty should be smaller than 6.57 million times the revenue earned for the expected duration of compliance.

5. Although the EPA conducts announced and unannounced inspections, most inspections are unannounced (U.S. EPA 2004). The advantages and disadvantages of each approach are viewed mainly from a behavioral perspective: “If operators are aware of inspections, they can prepare for them in a way that adds value to the visit. An unannounced visit is more likely to detect noncompliant behaviour that the operator wishes to hide” (Farmer 2007, p. 115). Complementing this insight, we document an efficiency advantage of periodic (or announced) inspections.

6. Under RI, this finding, in fact, does not depend on approximations; the same conclusion is made with exact values for $\Psi$. This is true because the myopic and long-run objectives are equivalent because of time symmetry introduced by memorylessness. Under PI, in contrast, equivalence is assured only when approximations are made (see §4.2).

7. A similar observation is made by Livernois and McKenna (1999) in a setting that has a continuum of heterogeneous firms (p. 432): “the solution involves making the difference between [the fines for truthful and untruthful reporting] as large as possible in order to maximize the proportion of firms that truthfully report violations.”

8. Observe that the threshold disclosure policy and optimal disclosure policies under RI share a common feature: for a given noncompliance episode, the firm either discloses it at the onset or keeps delaying disclosure. As we discussed in §4.1, under RI it was the combination of fixed penalty $\kappa_r$ for late disclosure and the firm’s constant uncertainty about the timing of next inspection that led to this behavior. Under PI, by contrast, it is the combination of fixed $\kappa_r$ and the firm’s perfect knowledge about the timing of next inspection that leads to the same behavior. Knowing exactly when the next inspection arrives, the firm delays his disclosure as much as possible, so that he accumulates revenue through noncompliant production without risking unexpected detection. Hence the same behavior is induced by opposite characteristics of the two inspection policies: maximal versus minimal uncertainty about inspection timing. The only difference is that perfect knowledge under PI allows the firm to disclose noncompliance at the last minute.
9. This is because the firm’s disclosure action impacts not only its profitability but also the length of a renewal cycle (“per unit time” part in the definition of long-run averages), as disclosure prompts suspension of inspections. By contrast, this effect is suppressed under RI because of memorylessness of inspection intervals.

References


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