Product Quality in a Distribution Channel With Inventory Risk

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Abstract

We analyze a situation in which a product has to be designed and sold under demand uncertainty. The design of the product refers to its “quality,” which comprises of any attribute (or a combination of attributes) more of which increases selling price as well as marginal cost. Product quality and inventory must be determined prior to the start of the selling season. Therefore, inventory risk is present (i.e., there is a possible mismatch between supply and demand) and newsvendor considerations arise. In a centralized channel, in which a single firm determines quality and inventory, at the optimal quality-inventory pair the elasticity of the unit quality cost function equals the reciprocal of the elasticity of the expected sales function. In a decentralized channel, in which a manufacturer determines product quality and wholesale price and a retailer determines inventory, at the optimal quality-inventory pair the elasticity of the unit quality cost function equals the reciprocal of the elasticity of the expected sales function conditional on a stockout. For both centralized and decentralized channels, quality and inventory are substitutes, and inventory risk leads to lowering of quality as compared to the case without inventory risk — the possibility of having leftover inventory leads to underinvestment in quality as the cost of increasing quality is incurred on every unit produced while revenue is realized only on the units actually sold. We further show that under inventory risk, decentralization introduces an inefficiency that distorts the optimal quality level, and total channel expected profit decreases. Interestingly, even though double marginalization reduces manufacturer incentives to invest in quality, in a decentralized system quality can be higher than in a centralized system. This is because in a decentralized channel with a wholesale price contract, the full inventory risk lies with the retailer. Therefore, the manufacturer, being shielded from inventory risk, overinvests in quality. However, consumer surplus decreases in a decentralized channel despite the fact that quality is higher. Finally, we consider a “buyback” contract in which the manufacturer shares risk with the retailer by promising to buy back unsold inventory from the retailer, and show that quality decreases as the manufacturer assumes greater risk by increasing the buyback price.

Keywords: supply chain; product design; newsvendor; channel coordination; buyback contracts; marketing and operations interface.
1 Introduction

A key decision that a firm has to make while designing a product is the following: At what level should the firm set the quality of its product? Broadly speaking, “quality” refers to an attribute (or a combination of attributes) more of which increases selling price but also marginal cost. A number of existing papers address this basic question and its variants in different contexts (some well-known papers include Mussa & Rosen 1978, Moorthy 1984 and Villas-Boas 1998). However, this literature has largely overlooked issues arising from operational considerations.

Consider a general situation with long production lead-time and a limited selling season for a product. The product has to be produced at the beginning of the selling season, i.e., the quantity to be produced has to be decided early on and cannot be adjusted subsequently. Furthermore, assume that the demand realization for the product is uncertain. Such a situation represents the case for seasonal products such as apparel that typically take time to design and manufacture, and are also produced at locations far from where they are eventually sold leading to a long transportation lead-time (e.g., apparel sold in the USA and Europe is often produced in Asian countries such as China and India). In such situations, supply-demand mismatch is a central problem, leading to stock-outs and unsold inventory. In other words, newsvendor dynamics are operative in these types of situations with long lead-times and uncertain demand.

The above example is representative of many real-world situations. While significant attention has been paid in the literature to coordinating supply chain inventory decisions in such a setting (Cachon 2003), there has been far less focus on product design decisions. In this paper, we explore this topic by analyzing a stylized model in which a firm sells a seasonal good under newsvendor dynamics. We modify this classic inventory framework by including a key product design decision: value-enhancing product quality. This may include new product features, enhanced performance of existing features, improved product durability by upgrading raw materials or components (e.g., the quality of fabric in an article of clothing), or improved quality of the manufacturing process (e.g., handcrafted items versus mass produced items). All else being equal, consumers value quality and are willing to pay more for a higher quality product; however, quality comes at the expense of increased marginal manufacturing costs. Prior to the start of the selling season, both the product quality and the inventory level have to be determined. We thus merge a classic inventory model
(the newsvendor) with a quality choice model in which a higher quality product leads to a higher selling price, but also a higher marginal production cost. Because quality directly impacts both cost and price, it clearly interacts with optimal inventory decisions that, in the newsvendor framework for seasonal goods, depend crucially on the cost of having too little inventory (the lost margin on a sale) and the cost of having too much inventory (the loss on each unsold unit). We consider both a centralized supply chain, in which a single firm determines quality and inventory, and a decentralized supply chain, in which a manufacturer determines product quality and a retailer determines inventory.¹

Using our model, we provide insights into how inventory issues and quality choice interact. Specifically, we answer the following questions. First, if a single firm decides both quality and inventory, how do inventory issues influence quality choice? Does quality increase or decrease compared to the case of no inventory considerations, and why? Second, in a distribution channel with a manufacturer who decides quality and a retailer who decides quantity that it will stock given the quality, how do inventory issues influence quality choice, and why? How does this quality level compare with the quality chosen by a centralized firm facing inventory issues? Finally, how do simple versus advanced contracts influence quality and inventory decisions in a decentralized channel?

For a centralized channel, we characterize the optimal inventory and quality, and demonstrate that at the optimal inventory-quality pair the elasticity of the unit quality cost function equals the reciprocal of the elasticity of the expected sales function. We find that quality and inventory are substitutes. Intuitively, increasing quality leads to a higher marginal cost of production (while retail price increases slower than marginal cost). Therefore, to reduce the probability that it has leftover inventory (which is costlier at a higher quality level), the firm produces less. We also find that the optimal quality is lower than in the no-inventory case.

In a decentralized channel, we demonstrate that at the optimal quality-inventory pair the elasticity of the unit quality cost function equals the reciprocal of the elasticity of the expected sales function conditional on a stockout. We show that a standard wholesale price contract cannot coordinate the channel in inventory or quality, and discuss the impact of these contracts on quality and inventory levels. In our model, in the case without inventory, a standard wholesale price con-

¹Throughout the paper, we use the terms “supply chain,” “distribution channel” and “channel” interchangeably.
tract can coordinate the channel quality decision; in other words, introducing newsvendor dynamics introduces inefficiency in the decentralized supply chain.

Furthermore, we show that in a decentralized system, quality can be higher than in a centralized system (we characterize the exact condition for this to hold), despite the fact that double marginalization reduces the manufacturer’s incentives to invest in quality. The insight behind this result is that in a decentralized channel, the manufacturer (who makes the quality decision) only faces an indirect effect of demand uncertainty through the retailer. In other words, the manufacturer is shielded from inventory risk. Therefore, the manufacturer is less sensitive to the possible cost incurred due to leftover inventory and has an incentive to set quality at a level higher than in the centralized case (where it directly faces demand uncertainty).

In our main model, we assume demand to be inelastic in price. We extend our analysis to a more standard model in which consumers have heterogeneous willingness to pay for quality, which leads to demand being elastic in price. In this case, in a centralized channel the firm decides quality, quantity and price. In a decentralized channel, the manufacturer decides the quality and the wholesale price, and the retailer decides the quantity and the retail price. This model is not amenable to a full analytical treatment. Using numerical simulations, we find that all the results of our simpler basic model are reproduced in this model as well. In addition, this model helps us to analyze consumer surplus and we find that consumer surplus decreases in a decentralized channel despite the fact that quality is higher.

As discussed above, in a wholesale price contract the whole inventory risk rests with the retailer. To understand how risk sharing between the manufacturer and the retailer influences quality and inventory decisions, we use a “buyback” contract under which the manufacturer sells inventory to the retailer at a wholesale price, but promises to buy back unsold inventory at the end of the season at a buyback price. Varying the buyback price allows different amounts of risk to be allocated to the manufacturer. We find that as the buyback price increases, i.e., the manufacturer assumes more risk, quality decreases and the inventory ordered by the retailer increases. This confirms the insight obtained from the main model that overinvestment in quality in the decentralized channel is due to the manufacturer being shielded from risk. Counter-intuitively, as the manufacturer assumes more risk in the buyback contract, the profit of the retailer decreases, and this happens because the manufacturer can charge a higher wholesale price.
The rest of the paper is organized as follows. In the next section, we discuss the related literature. In Section 3, we describe our model. In Section 4, we establish the benchmark model without newsvendor considerations, for which we solve both the centralized and decentralized channel scenarios. In Section 5, we analyze the focal model in which quality and quantity decisions have to be made in the presence of newsvendor considerations (i.e., mismatch between supply and demand) for both centralized and decentralized channels. In Section 6, we conduct numerical analyses to confirm the robustness of the results in a situation where consumers are heterogeneous in the valuations for quality. In Section 7, we analyze the impact of the buyback contract, which provides a framework for systematically varying the amount of risk shared by the manufacturer and the retailer, on quality choice. In Section 8, we conclude with a discussion.

2 Related Literature

We consider the simultaneous choice of product quality and inventory in centralized and decentralized channel. Consequently, there are several streams of research that bear particular relevance to this paper. We discuss these below.


While much of this literature considers the optimal design of a product line to serve multiple heterogeneous customer segments, we focus instead on the quality decision for a single product intended to serve a single, homogenous customer segment — however, in our model the product is seasonal and the size of the market is uncertain, necessitating a non-trivial inventory decision. Specifically, Carlton & Dana (2008) study product line design and find that demand uncertainty leads to lower quality provision. We obtain this result as well, and further generalize it by relating the optimal inventory-quality pair to the relationship between cost and sales elasticities. In addition, our primary focus is on exploring quality and inventory decisions in a decentralized supply chain (which Carlton & Dana 2008 do not consider).
**Impact of Decentralization on Quality.** Existing work has examined the impact of decentralization on vertical product quality choice, typically demonstrating that decentralization and the associated double marginalization reduces product quality (Jeuland & Shugan 1983; Villas-Boas 1998; Economides 1999). In contrast, Xu (2009) shows that the shape of the marginal revenue function, which depends on consumer valuations, is key to determining whether quality increases or decreases upon decentralization. Shi et al. (2013) demonstrate that if consumer valuations follow non-uniform distributions or if consumers are two-dimensionally heterogeneous, it’s possible for product quality to increase as a result of decentralization. The papers in this stream do not model supply and demand mismatch, a key feature of our model, and a key driver of our results.

**Channel Coordination.** A large body of research focuses on the coordination of decentralized distribution channels (Jeuland & Shugan 1983, Moorthy 1987, Lal 1990, Iyer 1998, Raju & Zhang 2005, Kuksov & Pazgal 2007). These papers, however, assume quality to be exogenously determined and do not consider inventory risk. There is also a rich literature on coordinating inventory decisions assuming quality (and, often, retail price) to be exogenously determined. This literature studies wholesale price contracts (Lariviere & Porteus 2001, Gerchak & Wang 2004, Bernstein & Kök 2009), revenue sharing contracts (Bernstein & Federgruen 2005, Cachon & Lariviere 2005) and buyback contracts (Pasternack 1985), among others (see Cachon 2003 for a review). In this paper, we assume that quality is specified by the manufacturer.

**Marketing and Operations Management Interface.** More generally, there is a broad literature on the marketing and operations management interface that studies how decisions in both marketing and operations management are impacted by considering phenomena typically studied by the other stream (Eliashberg & Steinberg 1987, Desai et al. 2007, Iyer et al. 2007, Netessine & Taylor 2007, Shulman et al. 2009, Su 2009, Jerath et al. 2010, Swinney 2011, Kim & Netessine 2013, Dai & Jerath 2013, Biyalogorsky & Koenigsberg 2014). Our paper adds to this literature, especially the growing literature analyzing the interaction of marketing considerations, such as pricing (Roels 2013) and conspicuous consumption (Tereyagoglu & Veeraraghavan 2012), with newsvendor decisions.
3 Model

A firm (the “manufacturer”) designs and manufactures a single seasonal product. The product is either sold directly to consumers by the manufacturer (the “centralized” supply chain) or sold to a retailer who sells to consumers (the “decentralized” supply chain). The product is sold over a single selling season and, at the conclusion of the season, excess inventory is salvaged for zero revenue.\(^2\)

Design and production leadtimes are sufficiently long that the manufacturer must design the product and determine inventory levels in advance of the revelation of demand information, and there is no opportunity to procure additional inventory after the selling season begins. Consequently, the model setup is that of a single period newsvendor with zero salvage value, and is reflective of seasonal or short lifecycle consumer products such as apparel or seasonal sporting goods.

Prior to the start of the selling season, two decisions must be made concerning this product:

1. The product quality level \(\theta\).
2. The inventory level \(q\).

Thus, we augment the classical newsvendor model with an additional decision, the choice of quality \(\theta\). In the context of this paper, we define quality as an attribute of the product that increases the consumers’ willingness to pay. While increased willingness-to-pay allows the firm to charge a higher price, higher quality also requires a higher manufacturing cost. As a result, when choosing the quality of a product, firms must strike a balance between consumer utility and marginal production cost (Mussa & Rosen 1978, Krishnan & Zhu 2006, Netessine & Taylor 2007).

We assume that all consumers have a willingness-to-pay \(v_0 \theta\) for a product of quality \(\theta\), where \(v_0 > 0\) is a value coefficient. We assume that the customer-facing agent in the supply chain decides the retail price, i.e., in a centralized supply chain the manufacturer makes this decision, and in a decentralized supply chain the retailer makes this decision. Suppose the retail price is \(r\). Then the utility that a consumer obtains from purchasing a product of quality \(\theta\) is given by \(v_0 \theta - r\).

Assume, without loss of generality, that consumers have an outside option that provides utility zero. Therefore, the consumer will purchase the product if \(v_0 \theta - r \geq 0\) and the optimal retail price

\(^2\)Positive salvage value may be readily introduced to the model, although the resulting analysis is more cumbersome, and analytically proving that the equilibrium is unique in the decentralized supply chain is challenging (though we have numerically observed all results continue to hold with non-zero salvage value). Hence, for ease of exposition, we assume zero salvage value.
will make this constraint binding, i.e., the optimal retail price will be given by $v_0 \theta$. From this point onwards, for simplicity, we will directly use the retail price $v_0 \theta$ in our analysis. We note at the outset that since pricing is at value, consumer surplus is zero for this model.

The assumption of homogeneous consumer valuation for quality allows analytical tractability of our model without sacrificing the key insights that we want to convey in the paper. In Section 6, we extend our model to consider heterogeneous consumer tastes for quality (which is arguably more standard in the quality decision literature, see Moorthy 1984 and Villas-Boas 1998) and show that our major insights continue to hold. In addition, this model allows us to address the question of the impact of decentralization on consumer surplus.

The unit production cost is an increasing function of product quality, $c(\theta)$. We assume it to be twice differentiable and denote its elasticity (the percentage change in unit production cost with respect to a percentage change in quality) by

$$
\epsilon(\theta) \equiv \frac{\theta c'(\theta)}{c(\theta)}.
$$

(1)

We make the following assumptions about the production cost curve:

**Assumption 1.** The unit production cost $c(\theta)$ and its elasticity $\epsilon(\theta)$ have the following properties:

(i) $c'(\theta) > 0$; $c''(\theta) > 0$; $\lim_{\theta \to 0} c(\theta) = \lim_{\theta \to 0} c'(\theta) = 0$; $\lim_{\theta \to \infty} c(\theta) = \lim_{\theta \to \infty} c'(\theta) = \infty$.

(ii) $c'(\theta) \geq 0$; $\lim_{\theta \to \infty} \epsilon(\theta) > 1$.

The first part of the assumption states that $c(\theta)$ is convex increasing and that zero quality can be had at no cost while attaining infinite quality is infinitely costly. Combined with the fact that the selling price linearly increases in $\theta$, these assumptions on the cost of quality capture the plausible scenario of decreasing marginal returns on quality, and hence, an infinite amount of investment in quality is never optimal. The assumptions on elasticity in the second part are made to facilitate the analysis, but they are not restrictive.\(^3\)

While many representative cost functions satisfy all of the conditions listed in Assumption 1, throughout the paper we pay special attention to the isoelastic unit cost function $c(\theta) = c_0 \theta^n$, where $n > 1$ and the constant $c_0 > 0$ is the unit cost coefficient. This function brings both versatility (i.e.,

\[^3\text{In fact, the last condition } \lim_{\theta \to \infty} \epsilon(\theta) > 1 \text{ follows from other assumptions; by l'Hopital's rule, } \lim_{\theta \to \infty} \epsilon(\theta) = \lim_{\theta \to \infty} \frac{c''(\theta)}{c'(\theta)} = 1 + \frac{\theta c''(\theta)}{c'(\theta)} > 1.\]
varying degrees of quality cost are represented by \( n \) and simplicity, enabled by the fact that its elasticity is independent of \( \theta \), a decision variable: \( \epsilon (\theta) = n \). For similar reasons many authors in the literature have adopted this function in their analyses, especially the quadratic function \( (n = 2) \); see Moorthy & Png (1992) and Netessine & Taylor (2007), for example.

Because the unit cost function is convex in \( \theta \) while price is linearly increasing in \( \theta \), an excessive quality level will reduce the profit margin to zero or negative. To rule out such a scenario, we restrict attention to the range of \( \theta \) that guarantees a positive margin for the supply chain. This range is defined by the upper bound \( \theta > 0 \), which is the unique solution to the equation \( c(\theta)/\theta = v_0 \). (It can be shown that such a solution exists under Assumption 1.) Any quality of value \( \theta > \bar{\theta} \) leads to a negative margin. In Lemma A1 found in the Appendix we provide a number of additional properties of \( c(\theta) \) in the range \( \theta \in (0, \bar{\theta}) \) that we use throughout the paper.

The total market size during the selling season is \( D \), a positive random variable with mean \( \mu = E[D] \), distribution function \( F \), density \( f \), and support on \( (0, \bar{D}) \), where \( \bar{D} \) may be infinite. Let \( \overline{F} \) denote the complement of \( F \). We assume that the demand distribution has an increasing generalized failure rate (IGFR, see Lariviere 2006), i.e., \( g'(x) > 0 \) where \( g(x) \equiv xf(x)/\overline{F}(x) \). For inventory level given by \( q \), we define

\[
S(q) \equiv E[\min\{D, q\}] = \int_0^q yf(y)dy + \int_q^{\bar{D}} xf(y)dy
\]

(2)
as the expected sales or the sales function. We denote the elasticity of the sales function by \( \eta(q) \), i.e.,

\[
\eta(q) \equiv \frac{qS'(q)}{S(q)} = \frac{q\overline{F}(q)}{\int_0^q F(y)dy}.
\]

(3)

This function plays a central role in our analysis, and a number of its useful properties are derived in Lemma A2 found in the Appendix. We also note that for inventory level \( q \), the in-stock probability is given by \( F(q) \) and the stockout probability is given by \( \overline{F}(q) = 1 - F(q) \).

4 Benchmark: Analysis Without Newsvendor Considerations

In this section, we analyze the case without newsvendor considerations, i.e., when demand and supply can be perfectly matched. This could be because there is zero or small enough lead-time
between production and selling, so that the quantity can be decided after demand realization. Alternatively, this could be because demand is deterministic, so that the manufacturer knows exactly the correct quantity to be produced. We will adhere to the latter explanation for our analysis, but the results are identical if we adhere to the former explanation. We first present the analysis for a centralized supply chain, followed by the analysis for a decentralized supply chain.

4.1 Centralized Distribution Channel Without Newsvendor Considerations

In this case, the manufacturer has to make two decisions: the amount to produce and the quality level. If demand is perfectly known to be equal to \( D = \mu \), the firm produces exactly the amount that satisfies demand, i.e., the firm produces the amount \( q^c_b = \mu \), where the superscript \( c \) denotes “centralized” and the subscript \( b \) denotes the “benchmark” case of no newsvendor considerations. The manufacturer’s profit function is \( \Pi^c_{M,b} = (v_0 - c(\theta)) \mu \), with the quality level, \( \theta \), being the decision variable. At the optimal quality level, we have \( v_0 = c'(\theta^c_b) \). Notably, with deterministic demand, the optimal quality and inventory are independent of one another;

For the cost function \( c(\theta) = c_0 \theta^n \), the optimal quality is given by \( \theta^c_b = (v_0/(nc_0))^{1/(n-1)} \).

4.2 Decentralized Distribution Channel Without Newsvendor Considerations

In this case, there is a manufacturer and a retailer and we assume that they interact through a wholesale price contract. The manufacturer moves first and decides the quality level and the wholesale price, and the retailer decides the quantity to be ordered given the quality level and the wholesale price. Solving by backward induction, consider the retailer’s problem of deciding the quantity to order. Since demand is perfectly known to be equal to \( D = \mu \), the retailer orders exactly the amount that satisfies demand, i.e., \( q^d_b = \mu \), where the superscript \( d \) denotes “decentralized” and the subscript \( b \) denotes the “benchmark” case of no newsvendor considerations. The retailer’s profit function is then given by \( \Pi^d_{R,b} = (v_0 - w) \mu \), where \( w \) is the wholesale price.

Next, the manufacturer has to decide the quality and the wholesale price. The manufacturer’s profit function is given by \( \Pi^d_{M,b} = (w - c(\theta)) \mu \). Note that, in our model, the manufacturer’s profit increases monotonically in the wholesale price and the retailer’s profit decreases monotonically in the wholesale price. Therefore, the manufacturer increases the wholesale price up to the point where the retailer does not reject the manufacturer’s offer; in other words, the manufacturer increases
the wholesale price to ensure that the retailer’s reservation profit (i.e., the outside option) is just met. We assume that the reservation profit of the retailer is zero. Therefore, for quality level $\theta$, the optimal wholesale price is given by the condition $(v_0\theta - w^d_b) \mu = 0$, which gives $w^d_b = v_0\theta$. Substituting the above in the manufacturer’s profit function, we obtain $\Pi^d_{M,b} = (v_0\theta - c(\theta)) \mu$, with the quality level, $\theta$, being the decision variable. Note that $\Pi^d_{M,b}$ is now identical to $\Pi^c_{M,b}$. Therefore, at the optimum, $\theta^d_b = \theta^c_b$.

The above analysis shows that, in our model (specifically, due to the assumption of homogeneous consumer valuations in the model), without newsvendor considerations the optimal outcomes are identical in a centralized supply chain and a decentralized supply chain with a wholesale price contract. We will see later that when newsvendor considerations are present, the wholesale price contract in a decentralized supply chain cannot, in general, achieve the centralized supply chain outcome for either quality or quantity. In other words, newsvendor dynamics introduce inefficiency in the supply chain with respect to both quality and quantity.

5 Analysis With Newsvendor Considerations

In this section, we present the analysis of the model when newsvendor considerations are present, which is the focus of the paper.

5.1 Centralized Distribution Channel With Newsvendor Considerations

In this case, the manufacturer both designs the product and sells to consumers, i.e., the manufacturer has decision rights for both quality and inventory. Due to uncertainty in the market size $D$, and a significant lead-time after production, demand-supply mismatch may occur, leading to one of two possible scenarios. If $q < D$, the firm sells out during the selling season. If $q > D$, all demand is satisfied during the selling season, and $q - D$ units are scrapped for zero salvage value at the end of the season. Hence, anticipating these events, the firm determines the quality $\theta$ and quantity $q$. Its expected profit is equal to

$$\Pi^c_{M} (\theta, q) = v_0\theta S(q) - c(\theta)q,$$

(4)
where the superscript \( c \) denotes a centralized supply chain and \( S(\cdot) \) is the sales function defined in equation (2).

**Analysis with general cost function** \( c(\theta) \)

For the general cost function \( c(\theta) \), we identify the unique optimal quality-inventory pair in the following proposition.

**Proposition 1.** The centralized supply chain’s optimal inventory-quality pair \((q^c, \theta^c)\) is the unique solution to the system of equations

\[
\bar{F}(q) = \frac{c(\theta)}{v_0 \theta}, \tag{5}
\]

\[
\frac{S(q)}{q} = \frac{c'(\theta)}{v_0}. \tag{6}
\]

Moreover, \( q^c \) and \( \theta^c \) are substitutes.

Notice that equation (5) determines the optimal inventory level for a given level of quality \( \theta \) via the usual newsvendor fractile solution. Equation (6), on the other hand, determines the optimal quality level for a given level of inventory \( q \). This equation states that at the optimum, the ratio \( c'(\theta)/v_0 \) — the relative marginal increase in the unit cost against the marginal increase in price due to higher quality — should exactly match the ratio \( S(q)/q \), i.e., the sales conversion rate (the fraction of inventory eventually sold). Together, these equations specify which combination of quality and inventory levels should be chosen by the centralized firm to maximize profit.

While the two equations (5) and (6) together provide a linkage between the optimal choices of quality and inventory levels, a cleaner relationship emerges after we combine the two. Observe that dividing (5) by (6) yields

\[
\epsilon(q^c) \eta(\theta^c) = 1, \tag{7}
\]

where \( \epsilon(\cdot) \) and \( \eta(\cdot) \) are defined in equations (1) and (3), respectively. In other words, the sales function elasticity \( \eta(q) \) should be equal to the reciprocal of the unit cost function elasticity \( \epsilon(\theta) \) at the optimum.\(^4\) The succinct nature of this relationship is striking, and it suggests that the firm

\[\text{\footnotesize \(^4\)That the magnitudes of these two values are equal originates from the mirror symmetry built in the problem. Examining the firm profit function (4), we see that a unit increase in quality \( \theta \) leads to a linear increase in the}]

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that simultaneously determines quality and inventory does so while maintaining a constant level of joint elasticity (i.e., the product of $\eta(q)$ and $\epsilon(\theta)$) embedded in its profit function.

Another interesting observation from the equation (7) is that the firm always chooses the quality level at an elastic part of the unit cost curve, i.e., $\epsilon(\theta^c) > 1$. This is true because the sales function, by definition, is always inelastic: $\eta(q) < 1$ for any $q > 0$. To see this, observe that $\eta(q)$ defined in (3) can be written as

$$\eta(q) = \frac{qF(q)}{\int_0^q xf(x) \, dx + qF(q)},$$

which is clearly smaller than one. Written this way, we see that the sales function elasticity is equal to the fraction of expected sales that occur in the event of a stockout, i.e., $qF(q)$. Therefore, optimal quality is always set at a level where a percentage increase in quality leads to an increase in the unit cost by more than a percent.

Proposition 1 also states that optimal quality and inventory levels are substitutes. This is a direct consequence of our last finding that optimal quality is set at an elastic part of the unit cost function. To see this, consider adjusting the newsvendor fractile in the equation (5) by increasing the quality level $\theta$:

$$\frac{d}{d\theta} \left( \frac{c(\theta)}{v_0\theta} \right) = \frac{c(\theta)}{v_0\theta^2} [\epsilon(\theta) - 1].$$

Since $\epsilon(\theta^c) > 1$, this implies that the critical ratio appearing in the newsvendor formula $F(q) = 1 - c(\theta)/v_0\theta$ is decreasing in $\theta$. To match this change the firm has to reduce the in-stock probability $F(q)$, which is accomplished by lowering the inventory level $q$. Hence, higher quality results in lower inventory.\(^5\) Viewed this way, one can frame the firm’s joint optimization problem as a modified newsvendor model with endogenous determination of the optimal critical ratio.

\(^5\)Substitutability can also be understood from the following intuition. As quality improvement becomes more difficult due to higher unit cost sensitivity (i.e., higher elasticity $\epsilon(\theta)$), the firm compensates for the associated loss by producing more units to increase the sales, despite the fact that doing so brings diminishing percentage returns (i.e., reduced sales elasticity $\eta(q)$). The last part follows from the fact that $\eta(q)$ is a decreasing function, which is proved in Lemma A2.
Analysis with isoelastic cost function $c(\theta) = c_0 \theta^n$

We now switch to the cost function $c(\theta) = c_0 \theta^n$, for $n > 1$. The elasticity requirement $\epsilon(\theta^c) > 1$ is automatically satisfied by the isoelastic unit cost function $c(\theta) = c_0 \theta^n$ with $n > 1$. That is, this convex function is elastic everywhere. With this function, we can combine (5) and (7) to derive closed-form solutions for optimal inventory and quality levels:

$$q^c = \eta^{-1} \left( \frac{1}{n} \right),$$

$$\theta^c = \left( \frac{v_0}{c_0} F(q^c) \right)^{1/(n-1)}.$$

We make several interesting observations about these optimal values, as summarized below.

**Proposition 2.** In a centralized supply chain with the isoelastic unit cost function $c(\theta) = c_0 \theta^n$:

(i) The optimal inventory $q^c$ is independent of $v_0$ and $c_0$.

(ii) The optimal quality $\theta^c$ increases in the ratio $v_0/c_0$, and it does not exceed $\left( \frac{v_0}{c_0} \right)^{1/(n-1)}$.

(iii) The stockout probability $F(q^c)$ at the optimum does not exceed $1/n$.

(iv) If the demand distribution is symmetric or its median is greater than its mean, then $q^c > \mu$ for all $n \geq 2$.

Part (i) of Proposition 2 states that the optimal inventory level is determined solely by the elasticities of the sales and unit cost functions, if the latter is isoelastic; the coefficients $v_0$ and $c_0$ do not impact the inventory decision. On the other hand, these coefficients determine the optimal quality, as part (ii) shows. Thus, the relative scale of price and unit cost — represented by the ratio $v_0/c_0$ — impacts the quality choice but not the inventory choice. This is to be contrasted to the classical newsvendor model where this ratio appears in the firm’s optimal inventory decision.

Adding a quality choice has the effect of separating the roles played by scale coefficients and the elasticities, the former impacting the quality choice more and the latter the inventory choice more.

Part (iii) reaffirms substitutability between optimal quality and inventory. The upper bound $1/n$ on the stockout probability suggests that higher cost of improving quality (greater $n$) leads the firm to adjust the quality-inventory combination so as to compensate the limited pricing ability caused by quality loss with more inventory, thereby increasing product availability. In effect, the firm resorts to “flood the market” strategy when it is unable to offer a high-quality product.
Lastly, part (iv) reveals how the firm’s quality choice determines the newsvendor critical ratio. Stating this result a different way, the optimal critical ratio (in-stock probability) is always greater than \((n - 1)/n\). If \(n \geq 2\), this means the optimal critical ratio is always greater than \(1/2\); hence, it is optimal for a firm facing quadratic (or greater) quality costs to adjust the quality level such that the underage cost exceeds the overage cost. If the demand distribution is symmetric or right skewed\(^6\), as many commonly used demand distributions are, this implies it is optimal for the firm to produce more units than the expected demand.

Comparing the result in part (ii) of Proposition 2 with the result in Section 4.1, we see that the optimal quality with newsvendor considerations is less than the optimal quality without newsvendor considerations. This is because the benefit of quality (an increase in price) is earned on each unit sold, whereas the cost of quality, which is convex in quality level, is paid on each unit produced. With stochastic demand and newsvendor considerations, sales are less than production, hence quality has less value than with deterministic demand.

Moreover, if \(n \geq 2\) and the demand distribution is symmetric or right skewed, it is guaranteed that the optimal inventory with newsvendor considerations is greater than the optimal inventory without newsvendor considerations — that is, the product is designed such that positive safety stock is optimal.\(^7\) Thus, interestingly, while it is well known that negative safety stock is possible in the classical newsvendor model even with symmetric demand distributions (such as normal), if quality costs are sufficiently elastic, such a product would have been improperly designed: firm profit would be increased with a lower quality product and a higher inventory level. In other words, if the firm fails to take demand uncertainty into account when making product design decisions, then it selects a quality level that is too high and an inventory level that is too low.

### 5.2 Decentralized Distribution Channel With Newsvendor Considerations

In the preceding section we assumed that the quality and inventory decisions were made by the same, centralized firm. Many supply chains, however, separate the design and operational decisions, with one firm making the quality decision (a manufacturer) while a different firm makes the inventory decision (a retailer); examples include traditional apparel brands (e.g., Ralph Lauren, Hugo Boss).

---

\(^6\)More accurately, if the median is greater than the mean.

\(^7\)Indeed, even if \(n < 2\) and the distribution is left skewed, it still may be the case that positive safety stock is optimal, although this depends on the problem parameters and is not assured.
and Tommy Hilfiger) and department stores (e.g., Macy’s, Nordstrom, and Bloomingdale’s). In this section we consider such a decentralized supply chain and assume that the quality-setting manufacturer sells the products through an inventory-setting retailer via a wholesale price contract. Wholesale price contracting without quality choice has been thoroughly analyzed by Lariviere & Porteus (2001), and it is well-known that the wholesale price contract does not coordinate the supply chain. What is unknown, however, is how quality choice impacts the performance of this contract and how it distorts the inventory decision. In particular, we are interested in comparing the optimal decisions under the wholesale price contract against the results from the centralized supply chain case.

Given the two decisions made in this supply chain, there are multiple degrees of coordination that can occur; see Table 1. If the retailer chooses an inventory level that maximizes total supply chain profit conditional on the manufacturer’s chosen quality level, we say the supply chain achieves inventory coordination. If the manufacturer selects the quality that maximizes total supply chain profit conditional on the retailer’s inventory level, we say the supply chain achieves quality coordination. If both inventory and quality coordination are simultaneously achieved, the supply chain makes the decisions described in Proposition 1, and we say the supply chain achieves full coordination. If neither inventory nor quality coordination are achieved, we say the supply chain achieves no coordination.

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Table 1. Dimensions of supply chain coordination.

Here we make an important observation: note that even if coordination is achieved along one dimension, that does not necessarily imply that the supply chain makes the same decision as the centralized system along that dimension. For instance, the supply chain achieves quality coordination if the manufacturer selects a quality level that maximizes supply chain profit given the retailer’s inventory choice; however, the retailer may choose inventory that differs substantially from the centralized optimal level, in which case the quality that maximizes supply chain profit
will differ from the centralized optimal solution. Nevertheless, given the uncoordinated inventory decision, the supply chain could not achieve higher profits by adjusting product quality, so we say the supply chain achieves quality coordination (but not inventory coordination).

With a wholesale price contract, the sequence of events is as follows. First, the manufacturer chooses a quality level $\theta$, then the manufacturer chooses a wholesale price $w$. Given these values, the retailer then chooses the inventory level $q$. The retailer assumes all inventory risk. The manufacturer’s profit is $\Pi^d_M(w, \theta) = (w - c(\theta))q$, where $q$ is the retailer’s order quantity, the superscript $d$ stands for a decentralized supply chain, and the subscript $M$ stands for manufacturer. The retailer’s expected profit is the usual newsvendor function at procurement cost $w$, $\Pi^d_R(q) = v_0 \theta S(q) - wq$. The total supply chain profit is the sum of these two expressions, i.e., $\Pi^d_M + \Pi^d_R = p\theta S(q) - c(\theta)q$, which is identical to the profit function analyzed in the centralized case in the preceding section.

Analysis with general cost function $c(\theta)$

For the general cost function $c(\theta)$, we identify the unique equilibrium of this sequential game in the following proposition.

**Proposition 3.** A unique equilibrium $(q^d, \theta^d)$ exists that solves the system of equations

$$F(q)(1 - g(q)) = \frac{c(\theta)}{v_0 \theta}, \quad (8)$$

$$F(q) = \frac{c'(\theta)}{v_0}. \quad (9)$$

Moreover, $q^d$ and $\theta^d$ are substitutes.

The detailed proof is available in the Appendix. However, an insightful step in the proof is the following. The retailer solves for the optimal $q$ given $w$; this value is $q = F^{-1}(1 - w/(v_0 \theta))$. Using the change of variable $w = v_0 \theta F(q)$, we obtain the manufacturer’s profit as $\pi^d_M = v_0 \theta q F(q) - c(\theta)q$, where it has to decide $\theta$ and $q$. This profit function can be rewritten as

$$\pi^d_M = v_0 \theta \left(S(q) - \int_0^q x f(x) dx\right) - c(\theta)q. \quad (10)$$

Comparing this with $\pi^c_M$ in Equation (4), one can immediately see that the manufacturer’s return is lower in the decentralized case than in the centralized case, i.e., there is an inefficiency in the
The equilibrium conditions (8) and (9) are counterparts to (5) and (6) for the centralized supply chain case. Combining these equations yields

$$\epsilon(\theta^d)(1 - g(q^d)) = 1,$$

where \( g(x) = xf(x)/F(x) \) is the generalized failure rate of the demand distribution. This is analogous to (7), the only difference being that the sales function elasticity \( \eta(q) \) is replaced by \( 1 - g(q) \). As in the centralized setting, equation (11) establishes a relationship between the unit cost function elasticity \( \epsilon(\theta) \) and an elasticity originating from the demand distribution. In the centralized setting it was the elasticity of the total expected sales \( S(q) \), namely \( \eta(q) \). The fact that \( 1 - g(q) \) appears in (11) in place of \( \eta(q) \) suggests that the generalized failure rate \( g(q) \) is related to an elasticity, and indeed such an interpretation is offered by Lariviere & Porteus (2001): \( g(q) \) represents “the percentage decrease in the probability of a stockout from increasing the stocking quantity by 1%.” To be more precise, the quantity \( 1 - g(q) \) is equal to the elasticity of the expected sales conditional on the event that a stockout occurs, namely, \( qF(q) \). It is then intuitive that this elasticity is smaller than the elasticity of the total expected sales, i.e., \( 1 - g(q) < \eta(q) \) (this is proved in Lemma A2). Comparing (11) with (7), we see that any deviation in quality and inventory choices under decentralization should be done under the constraint that the same level of joint elasticity is maintained, i.e., the product of two elasticities should be equal to one in both centralized and decentralized supply chains.

From this comparison, we infer that any change in quality has a smaller impact on the retailer’s inventory decision than it has on the centralized firm’s decision. Proposition 3 also states that quality and inventory levels in equilibrium are substitutes, just as in the centralized setting. In order to clearly delineate how these effects interact under decentralization, we next consider the scenarios in which one of the two decision variables, quality and inventory, is set to exactly the same level as that in the centralized setting: \( q^c \) and \( \theta^c \). The outcomes of these scenarios, summarized below, allow us to isolate the impact of wholesale price contracting on one decision variable at a time, and determine whether wholesale price contracts are capable of either inventory or quality coordination.
Proposition 4. (i) With quality fixed at the system-optimal level $\theta^c$, the retailer is induced to choose the inventory level smaller than $q^c$.

(ii) In order to induce the retailer to choose the system-optimal inventory level $q^c$, the manufacturer must set the quality level smaller than $\theta^c$.

(iii) A wholesale price contract achieves neither inventory nor quality coordination.

Part (i) of the proposition states the well-known double marginalization effect on inventory choice; because each unit sold brings less profit to the retailer than to the centralized firm, the retailer orders a smaller quantity. More interestingly, part (ii) reveals that a similar double marginalization effect applies to quality choice as well. It shows that, ceteris paribus, decentralization with wholesale pricing leads the manufacturer to choose lower quality than the centralized firm would — the supply chain would be better off with higher quality. The reason behind this result is more subtle because, unlike the retailer who simply reacts to a given quality and wholesale price in its inventory decision, the manufacturer has to anticipate the retailer's best response when setting the quality level that influences the wholesale pricing decision. As expected, higher quality allows the manufacturer to charge a higher wholesale price because consumer willingness-to-pay increases. However, a higher wholesale price also leads the retailer to order less; as a result of this retailer response, the manufacturer's incentive to improve quality is muted. The net effect of these two opposing forces — higher wholesale margin against lower order quantity — is that the manufacturer in a decentralized supply chain chooses quality at a lower level than the centralized firm would. The combination of (i) and (ii) implies that a wholesale price contract cannot achieve full coordination — simultaneous coordination of product quality and inventory, as shown in Table 1. Part (iii) of the proposition further shows that a wholesale price contract achieves no coordination at all — given the equilibrium inventory level, supply chain profit would be greater with higher quality, and given the equilibrium quality level, supply chain profit would be greater with higher inventory. Hence, wholesale price contracts lie in the upper left cell of Table 1.

Proposition 4 isolates the impact of decentralization on one decision variable at a time, but does not compare the equilibrium inventory and quality in the decentralized system to the optimal values in the centralized supply chain. In general, any of the following combinations may arise: lower quality with lower inventory, higher quality with lower inventory, and lower quality with higher
inventory. The only combination that never arises under wholesale pricing is higher quality with higher inventory. This wide range of outcomes arises as a result of combining the two effects we have identified. That is, while decentralization leads to underinvestment in both quality and inventory (Proposition 4), substitutability between them (Proposition 3) leaves one variable moving in the opposite direction. Hence, it is possible, e.g., for double marginalization to induce a lower inventory level that, due to the substitution effect between inventory and quality, causes the equilibrium quality in the decentralized system to rise above the centralized optimal quality. Which combination emerges in equilibrium depends on how demand is distributed and how the unit cost function is shaped.

**Analysis with isoelastic cost function** \( c(\theta) = c_0 \theta^n \)

We now switch to the cost function \( c(\theta) = c_0 \theta^n \) for \( n > 1 \). The equilibrium inventory and quality levels with this function are:

\[
q^d = g^{-1} \left( 1 - \frac{1}{n} \right), \quad \theta^d = \left( \frac{v_0 n c_0}{F(q^d)} \right)^{\frac{1}{n-1}}.
\]

Comparing these values with their counterparts \( q^c \) and \( \theta^c \) for the centralized supply chain case, we make the following observation:

**Proposition 5.** Under the isoelastic unit cost function \( c(\theta) = c_0 \theta^n \), \( q^c > q^d \). Moreover, \( \theta^c > \theta^d \) if and only if

\[
\frac{F(q^c)}{F(q^d)} > \frac{1}{n}. \quad (12)
\]

According to the proposition, the equilibrium inventory level under wholesale pricing never exceeds the inventory level in the centralized setting \( (q^c > q^d) \) when the unit cost function has constant elasticity. Thus, the usual under-ordering due to double marginalization applies. The equilibrium quality level, by contrast, may or may not be greater than the centralized supply chain optimum, i.e., both \( \theta^c > \theta^d \) and \( \theta^c < \theta^d \) are possible. Thus, Proposition 5 shows that double marginalization’s impact on inventory underinvestment dominates when the unit cost function is isoelastic; the quality level absorbs the impact of the substitution effect between inventory and
quality, and as a result may be greater or smaller than the centralized optimal quality. Thus, in contrast to the known results from the quality decision literature (Jeuland & Shugan 1983, Economides 1999), decentralization may actually lead to increased product quality even with an extremely simple consumer valuation distribution (unlike, e.g., Shi et al. 2013). We emphasize that what drives this difference is demand uncertainty, which creates a link between quality and inventory choices through the newsvendor dynamics.

An examination of (12) reveals that lower quality is obtained if decentralization does not distort the inventory level too much. If the degree of distortion is sufficiently high, the manufacturer finds it optimal to over-invest in quality, in an attempt to compensate for the loss in sales with higher per-unit wholesale margin.

As the preceding discussions have made clear, supply chain coordination cannot be achieved under wholesale price contracting. While this is true even without quality choice, we find evidence that adding quality choice exacerbates the coordination challenge. This is supported by our findings in Proposition 4 which show that the efficiency loss due to double marginalization manifests itself in both quality and inventory decisions. As a consequence, it is impossible to attain coordination for either quality or inventory using a wholesale price contract.

6 Heterogeneous Consumer Valuations

In our main model, we assume that consumers are homogeneous in their valuation for quality, which leads to demand being inelastic in price. We now conduct an important robustness check by extending our analysis to a more standard model in which consumers have heterogeneous willingness to pay for quality, which leads to demand being elastic in price (Mussa & Rosen 1978, Moorthy 1984, Villas-Boas 1998).

Consider a specific consumer. Consumer utility is assumed to be $V = v\theta - p$, where $v$ is the valuation, $\theta$ is the quality level and $p$ is the price. Hence, consumers value higher quality and lower price. Consumer valuation, $v$, is heterogeneous and assumed to be uniformly distributed between $v_l$ and $v_h$, $v_l \leq v_h$. The multiplicative relationship between $v$ and $\theta$ captures the idea that higher valuation consumers are more responsive to quality changes.

Let $X$ be the potential market size for the manufacturer’s product. It is a random variable
with cdf $F$ and pdf $f$. Let $D$ be the realized demand. It is related to $X$ as $D = \alpha X$, where $\alpha$ denotes the fraction of consumer population that ends up purchasing the product. The fraction $\alpha$ is determined endogenously by the optimal retail price.

In a centralized supply chain the manufacturer decides quality, quantity and retail price. In a decentralized supply chain, the manufacturer first decides the quality and the wholesale price, and the retailer then decides the quantity and the retail price. In both cases, the realization of $X$ is after the retail price has been decided.

This model is not amenable to a full analytical treatment. Therefore, we resort to numerical analysis. Our primary goal of developing this model is to understand the robustness of the results from our main model when consumer valuation for quality is heterogeneous and demand is elastic in retail price.

For our simulation, we consider the following values of the different parameters. We assume $X$ to be distributed Normal with mean $\mu$ and standard deviation $\sigma$, where $\mu = 100$ and $\sigma \in \{0.1, 5, 10, 15, 20, 25, 30\}$. For the cost function, we use $c_0 \in \{2, 5\}$ and $n \in \{1.5, 2, 2.5, 3, 4\}$. For the valuation distribution $v$ that is uniformly distributed between $v_l$ and $v_h$, we use $v_l \in \{0, 2.5, 5, 7.5, 10\}$ and $v_h = 10$. Using these values gives us 350 combinations of parameter values. This leads to a comprehensive numerical analysis. The following results from the numerical analysis show that the main insights from the simpler focal model are reproduced in this model.

- Quality is always decreasing in the standard deviation of market size. This confirms one of our main results that newsvendor issues (which arise and become more prominent as demand uncertainty is higher) lead to quality distortion (specifically, lower quality).

- Compared to the centralized case, approximately 75% of the time quality goes up in the decentralized system; approximately 10% of the time it is the same (this usually happens when there is no demand uncertainty); and approximately 15% of the time quality goes down. Under uncertainty, a centralized decision maker reduces quality relative to the deterministic case because his return (which accrues from realized sales) is lower than his investment (which is incurred for all units produced). However, in a decentralized channel, the manufacturer always sells all units that he produces. Although the manufacturer is no longer getting as much profit on each unit (due to double marginalization), he does not have the “sales less than
production” problem that drives reduced quality in the centralized channel. In our sample, it turns out, the double marginalization effect is “weaker” than the “sales less than inventory” effect.

In addition, this analysis can help us to understand the impact of decentralization on consumer surplus. In all of the 350 cases that we consider, consumer surplus is lower in the decentralized system compared to the centralized system (despite higher quality in the decentralized system). There are two reasons for this. First, since inventory is lower in the decentralized system, fewer consumers obtain a unit of the product. Second, the average consumer surplus conditional on successfully obtaining a unit also typically decreases in the decentralized system due to a higher price. In other words, in the decentralized system, fewer consumers obtain a unit, and each consumer that does has lower surplus on average, ultimately implying that decentralization reduces consumer surplus.

7 Impact of Advanced Contracts on Quality

Our focal analysis in Section 5 showed that the inefficiency in a decentralized supply chain with the simple but popular wholesale price contract is due to the separation between the agent who decides quality and the agent who bears inventory risk. This raises the possibility that more advanced contracts between the manufacturer and the retailer can influence how the inventory risk and the marginal return are shared in the supply chain. Indeed, it can be shown that a number of advanced contracts coordinate the decentralized system, including two-part tariffs, revenue sharing, and buyback contracts.\(^8\) In this section, we analyze in detail the buyback contract (Pasternack 1985, Donohue et al. 2015), in which the manufacturer sells inventory to the retailer at a wholesale price and, after demand realization, buys back any unsold inventory at the buyback price (i.e., the manufacturer shares the inventory risk with the retailer). Using this contract, we illustrate both the form of the optimal coordinating mechanism and the behavior of quality and inventory as the

\(^8\)We note here that implementing a coordinating contract of any of these types requires that quality be fully finalized before the contract parameters are determined. If some component of quality is determined after contract parameters are fixed (e.g., as would be the case if quality is primarily determined by process control rather than product design, or if contracts are signed based on prototype or sample products and the final product quality is determined by the manufacturer after establishing a contract), then the manufacturer will in general have an incentive to shirk on quality by under-investing. In such cases, a more complicated contract, for instance one that involves contracting directly on manufacturing cost or product quality, may be required to achieve coordination.
contract moves from a suboptimal wholesale price contract to a coordinating contract. Throughout this section we restrict attention to the case of iso-elastic cost \( c(\theta) = c_0 \theta^n \) to focus our discussion.

### 7.1 Coordinating Buyback Contract

To begin, consider a buyback contract with parameters \( w \) (the wholesale price) and \( b \) (the buyback price). Under such a contract, the retailer purchases inventory from the manufacturer at price \( w \) at the beginning of the selling season, and manufacturer pays the retailer \( b \) for each unit that is unsold at the end of the selling season, i.e., the manufacturer “buys back” unsold inventory. Using the superscript \( k \) for buyback, the manufacturer’s expected profit is \( \Pi^k_M(w, b, \theta) = (w - c_0 \theta^n)q - b(q - S(q)) \) and the retailer’s expected profit is \( \Pi^k_R(q) = v_0 \theta S(q) - wq + b(q - S(q)). \) Total supply chain expected profit is \( \Pi^k_M(w, b, \theta) + \Pi^k_R(q) = v_0 \theta S(q) - c_0 \theta^n q. \)

The sequence of events is the following. First, the manufacturer determines product quality. Second, given this quality level, the manufacturer offers a contract \( (w, b) \), where the parameters are chosen to maximize the manufacturer’s profit. Third, the retailer determines the inventory level given the product’s quality and the contract parameters. As we shall see, buyback contracts are sufficiently flexible as to allow the manufacturer to extract all profit from the retailer. Hence, we introduce a new parameter, \( R \), which is the retailer’s reservation profit; the retailer is assumed to accept any contract which leaves it with at least a profit level of \( R \). Defining \( \alpha^k \equiv R/\Pi^k_M \) to be the retailer’s fraction of the total (centralized optimal) supply chain profit when it achieves its reservation level \( R \), we have the following proposition.

**Proposition 6.** In a decentralized supply chain with buyback contracts, the unique equilibrium is for the manufacturer and retailer to select \( \theta^k = \theta^c \) and \( q^k = q^c \), respectively (i.e., the centralized optimal quality and inventory are selected). In equilibrium, the manufacturer leaves the retailer with exactly its reservation profit \( R \), and the equilibrium wholesale price and buyback price are \( w^k = \alpha^k c_0 (\theta^c)^n + (1 - \alpha^k) v_0 \theta^c \) and \( b^k = (1 - \alpha^k) v_0 \theta^c \).

Under this contract, coordination is achieved by transferring the appropriate amount of risk via the buyback price to the manufacturer for a given wholesale price. Noting that the equilibrium
buyback price as a percentage of the wholesale price is

\[
\frac{b^k}{w^k} = \frac{1}{\alpha^k c_0 (1 - \alpha^k) v_0 (\theta c)^{\alpha - 1} + 1},
\]

several interesting observations can be made about the coordinating buyback contract. First, the buyback price as a fraction of the wholesale price is decreasing in the centralized optimal quality of the product. In other words, for low quality products, the manufacturer should offer to buy back unsold inventory at a large fraction of the wholesale price; for high quality products, the manufacturer should offer to buy back at a smaller percentage of the wholesale price. Second, the buyback price as a fraction of the wholesale price is decreasing in \(\alpha^k\), and hence \(R\), the retailer’s reservation profit. In other words, if the retailer demands a large reservation profit, the manufacturer should offer to buy back unsold inventory at a small fraction of the wholesale price; if the retailer demands a low reservation profit, the manufacturer should offer to buy back at a larger percentage of the wholesale price. Third, while the coordinating wholesale and buyback prices in a standard newsvendor model without a quality decision are independent of the distribution of demand (Pasternack 1985, Cachon 2003), when quality is endogenously determined by the manufacturer, these coordinating prices depend on the centralized optimal quality level and the retailer’s share of the total supply chain profit, both of which depend on the demand distribution. In particular, since demand uncertainty leads to reduced quality in a centralized system, greater uncertainty leads to a smaller buyback price and wholesale price, and a larger ratio \(b^k/w^k\). Thus, product quality, relative bargaining position (i.e., the retailer’s outside option), and the degree of demand uncertainty all play a role in determining the optimal division of risk amongst the members of the supply chain in a coordinating buyback contract.

### 7.2 Impact of Division of Risk on Quality

We saw in the previous section that under the optimal buyback contract, coordination is achieved by transferring the appropriate amount of risk via the buyback price to the manufacturer for a given wholesale price. In this section, we vary the buyback price below the optimal level. This transfers less than optimal risk to the manufacturer, and allows us to see how transferring different amounts of risk to the manufacturer impacts the quality and inventory levels in the supply chain.
To develop the framework, first note that an equivalent way to state the result in the previous section is that for a wholesale price \( w \), the buyback price given by

\[
b(w) = v_0\theta \left[ \frac{w - c_0\theta^n}{v_0\theta - c_0\theta^n} \right]
\]

will coordinate the supply chain (i.e., result in the centralized optimal quality and inventory decisions), yielding a fraction

\[
\alpha(w) = \frac{v_0\theta - w}{v_0\theta - c_0\theta^n}
\]

of the total supply chain profit to the retailer. Adjusting the fraction \( \alpha(w) \) by manipulating the wholesale price allows the manufacturer to give the retailer exactly its reservation profit, leaving all supply chain profit above this level with the manufacturer.

Now suppose that instead of offering the coordinating buyback contract given by some pair \((w, b(w))\) as described above, the manufacturer offers some \((w, \gamma b(w))\) where \( \gamma \in [0, 1] \). This might be the case, e.g., if the manufacturer is unwilling to assume the degree of risk implied by the optimal (greater) buyback price. Note that by definition this contract is not coordinating if \( \gamma < 1 \), and hence both the manufacturer and the retailer could be better off under a coordinating contract; however, our intent in analyzing this contract is not to derive a coordinating contract but rather to consider how quality and inventory change as different amounts of risk are transferred from the retailer to the manufacturer. To see this, we increase \( \gamma \) from zero to one, which in essence transitions the supply chain from the use of a wholesale price contract to a coordinating buyback contract, and illuminates the impact of risk sharing on quality and inventory in the supply chain.

The sequence of events under this suboptimal buyback contract is the same as under the optimal contract. The manufacturer first determines quality, then contract parameters are determined according to \((w, \gamma b(w))\), where \( w \) is chosen by the manufacturer to maximize its own profit and the corresponding buyback price is restricted to be

\[
\gamma b(w) = \gamma v_0\theta \left[ \frac{w - c_0\theta^n}{v_0\theta - c_0\theta^n} \right]
\]

for some exogenous \( \gamma \in [0, 1] \). In one extreme case, if \( \gamma = 0 \), the contract reduces to a wholesale price contract, while on the other extreme, if \( \gamma = 1 \), it is a coordinating buyback contract. Inter-
mediate values of $\gamma$ represent a partial, but insufficient for coordination, transfer of inventory risk to the manufacturer. The impact of $\gamma$ on quality, inventory, supply chain profit, and firm profit in a decentralized supply chain is presented in the Figure 1 below. (In the figure, the retailer’s reservation profit is zero, so when $\gamma = 1$ the manufacturer can extract all profit from the retailer. The parameter values used to generate the plots are the following: demand is assumed to be distributed Normal with a mean of 100 and a standard deviation of 20, $c_0 = 2, n = 3, v_0 = 10$.)

Figure 1 illustrates two important effects. First, it shows that as $\gamma$ is decreased away from 1 (i.e., less risk resides with the manufacturer), quality is higher. This confirms the point made in our earlier discussion that a key factor leading to overinvestment in quality in a decentralized supply
chain is the fact that under a wholesale price contract the manufacturer does not feel sufficient risk, i.e., it sells every unit it produces and so it invests more in quality than is optimal for the entire supply chain. Second, it illustrates precisely how a coordinating buyback contract works: by shifting risk from the retailer to the manufacturer, i.e., by increasing \( \gamma \), inventory rises due to the retailer's reduction in risk, and quality decreases due to the manufacturer's increase in risk.

Finally, a counterintuitive implication from Figure 1 is as the manufacturer assumes greater risk (i.e., \( \gamma \) approaches 1), the expected profit of the manufacturer increases while that of the retailer decreases. The reason for this is that as the manufacturer assumes more risk, the inventory ordered by the retailer increases and the wholesale price charged to the retailer increases.

8 Conclusions

The quality level of a product is a central product design decision. We consider a situation in which a firm designs and sells a seasonal product with demand uncertainty and a single ordering opportunity with significant lead time. In this situation, there can be a mismatch between supply and demand. This may lead to leftover inventory which distorts incentives to invest in enhancing product quality. A significant body of academic literature has addressed the question of setting the quality level for a product. However, the extant literature has overlooked operational aspects of the nature discussed above, even though these aspects can influence quality investment decisions in many real-world situations.

In this paper, we combine a quality decision model with a newsvendor model. We assume that the cost of enhancing product quality is convex increasing in quality. Using this model, we study the implications for both marketing decisions, such as the quality level of the product, and operational decisions, such as how much inventory to produce/order. Furthermore, we consider both a centralized supply chain in which the manufacturer makes both these decisions, and a decentralized supply chain in which the manufacturer decides the quality and the retailer decides the amount of inventory to be ordered with the two parties interacting through a wholesale price contract.

We find that due to newsvendor considerations (i.e., supply and demand mismatch) the quality level of a product goes down. The reason is that the cost of enhancing quality is incurred for every
unit produced but revenue is obtained only on units sold. Therefore, under the possibility that supply exceeds demand, the firm stands to lose money. As marginal production cost is increasing with quality, by reducing the quality of the product, the firm can limit the downside from leftover inventory. We also find that optimal quality and optimal inventory are strategic substitutes. Therefore, there are situations in which the optimal inventory with newsvendor considerations is higher than the optimal inventory without newsvendor considerations, while the optimal quality is lower.

Furthermore, interesting comparisons can be drawn between the quality and inventory levels in centralized and decentralized supply chains. Note that, in our model, without newsvendor considerations the quality and inventory levels are the same in both centralized and decentralized supply chains. While newsvendor considerations lead to quality reduction in both supply chains, the quality reduction in decentralized supply chains is lesser. This is because of two reasons. First, due to reduced margin per unit, the retailer in a decentralized supply chain orders less inventory as compared a manufacturer selling directly in a centralized supply chain; since inventory and quality are substitutes, the manufacturer has the incentive to choose a higher quality level in the decentralized case. Second, a reason for the manufacturer to reduce quality under newsvendor considerations was that due to demand uncertainty it was facing the “sales less than units produced” problem, which led to underinvestment in quality. However, in a decentralized case, the manufacturer does not face inventory risk directly because it sells all the units it produces at the wholesale price to the retailer, and the retailer essentially faces the demand uncertainty. In other words, the manufacturer is shielded from demand uncertainty in a decentralized supply chain which leads to a reduced incentive to decrease quality, even though the manufacturer makes a smaller per unit margin than in the centralized case.

We extend our analysis to incorporate heterogeneous consumer preferences for quality. The main results from our focal model continue to hold in this model as well. In addition, we examine consumer surplus and find that it decreases in the decentralized channel, despite the fact that quality is higher. This is because, compared to the centralized channel, fewer consumers obtain the product, and those that do pay a higher price on average.

Finally, we consider a buyback contract in which the manufacturer sells inventory to the retailer at a wholesale price, but promises to buy back unsold units after demand realization. In other words,
this contract places some inventory risk with the manufacturer, and how much risk is placed with the manufacturer can be varied by varying the buyback price. We find that as the buyback price is increased from zero, i.e., the manufacturer assumes more risk, the quality in the channel decreases. In the case of a sufficiently large buyback price, indicating a sufficiently large transfer of risk to the manufacturer, the buyback contract can fully coordinate the channel.

We do not directly address the question of channel choice by a manufacturer when newsvendor considerations are present. In our model, the answer to this is straightforward — if the manufacturer can use a centralized channel, or can use an advanced contract such as a buyback contract, it should do so. Nevertheless, our research is a first step towards understanding how inventory issues influence product quality choice and coordination in a supply chain. Further research can study richer problems of this flavor. For instance, we assume that the retailer has no impact on the quality of the product. However, retailers often impact the final consumer-perceived quality of a product by offering value-added services, and future research could explore this. Another interesting avenue for future research could be to understand the impact of supply chain management strategies such as quick response (Iyer & Bergen 1997, Cachon & Swinney 2009) on the manufacturer’s incentives to invest in a quality product.

To conclude, we show that quality decisions are significantly influenced by newsvendor dynamics. Furthermore, in a decentralized channel quality and inventory decisions depend on the sharing of inventory risk between the manufacturer and the retailer.
References


Appendix

A  Supporting Results

**Lemma A1.** Under Assumption 1, (i) $c(\theta)/\theta < v_0$ for all $\theta \in (0, \bar{\theta})$ and (ii) $\lim_{\theta \to \bar{\theta}} c'(\theta) > v_0$.

**Proof.** To prove (i), we show that the supremum of the ratio $c(\theta)/\theta$ in the interval $\theta \in (0, \bar{\theta})$ occurs at the boundary $\theta = \bar{\theta}$, where $c(\bar{\theta})/\theta = v_0$ by definition. To see this, observe $\lim_{\theta \to 0} \frac{c(\theta)}{\theta} = \lim_{\theta \to \bar{\theta}} c'(\theta) = 0$ and $\lim_{\theta \to \infty} \frac{c(\theta)}{\theta} = \lim_{\theta \to \infty} c'(\theta) = \infty$ where we used l'Hopital’s rule along with Assumption 1, and $\frac{\frac{c(\theta)}{\theta}}{\frac{c(\theta)}{\theta}} = \frac{\frac{c(\theta)}{\theta}}{\frac{c(\theta)}{\theta}} [\epsilon(\theta) - 1]$. Hence, the ratio $c(\theta)/\theta$ starts at zero, is increasing at sufficiently large $\theta$ where $\epsilon(\theta) > 1$ (see Assumption 1), and approaches infinity as $\theta \to \infty$. The ratio may be increasing or decreasing in the vicinity of $\theta = 0$. Together, they imply that the supremum of $c(\theta)/\theta$ in the interval $\theta \in (0, \bar{\theta})$ occurs at the boundary $\theta = \bar{\theta}$. It then follows that $c(\theta)/\theta < v_0$ for all $\theta < \bar{\theta}$. To prove (ii), recall that the ratio $c(\theta)/\theta$ increases if and only if $\epsilon(\theta) > 1$ and that it is increasing at $\theta = \bar{\theta}$. This implies $\lim_{\theta \to \bar{\theta}} \epsilon(\theta) = \lim_{\theta \to \bar{\theta}} \frac{\theta c'(\theta)}{c(\theta)} = \frac{1}{v_0} \lim_{\theta \to \bar{\theta}} c'(\theta) > 1$, or equivalently, $\lim_{\theta \to \bar{\theta}} c'(\theta) > v_0$. □

**Lemma A2.** For $x \in (0, \bar{D})$, the sales function elasticity $\eta(x)$ defined in (3) has the following properties: (i) $\lim_{x \to 0} \eta(x) = 1$; (ii) $\lim_{x \to \bar{D}} \eta(x) = 0$; (iii) $\eta'(x) < 0$; (iv) $F(x) < \eta(x) < 1$; (v) $\eta(x) + g(x) > 1$; (vi) $\frac{xf(x)'}{g(x)} / (x \eta(x)) = 1 - \eta(x) - g(x)$.

**Proof.** Note that $\eta(x)$ can be written as $\eta(x) = \frac{x F(x)}{S(x)} = 1 - \frac{\int_0^x yf(y)dy}{\int_0^x F(y)dy}$ since $S(x) = E \left[ \min \{D, x \} \right] = \int_0^x F(y)dy = \int_0^x yf(y)dy + xF(x)$. By l'Hopital’s rule,

$$
\lim_{x \to 0} \eta(x) = 1 - \lim_{x \to 0} \frac{\int_0^x yf(y)dy}{\int_0^x F(y)dy} = 1 - \lim_{x \to 0} \frac{xf(x)}{F(x)} = 1 - \lim_{x \to 0} g(x) = 1,
$$

$$
\lim_{x \to \bar{D}} \eta(x) = 1 - \frac{\int_0^\bar{D} yf(y)dy}{\int_0^\bar{D} F(y)dy} = 1 - \frac{\mu}{\mu} = 0.
$$

Differentiating $\eta(x)$ yields $\eta'(x) = -\frac{F(x)}{S(x)^2} \left( g(x) S(x) - \int_0^x yf(y)dy \right)$. Observe that

$$
\lim_{x \to 0} \left( g(x) S(x) - \int_0^x yf(y)dy \right) = 0,
$$

$$
\frac{d}{dx} \left( g(x) S(x) - \int_0^x yf(y)dy \right) = g'(x) S(x) + g(x) F(x) - xf(x) = g'(x) S(x) > 0,
$$

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which together imply \( g(x) S(x) - \int_0^x y f(y) \, dy > 0 \). Therefore, \( \eta'(x) < 0 \). Combined with 
\[
\lim_{x \to 0} \eta(x) = 1 \text{ and } \lim_{x \to \bar{D}} \eta(x) = 0, \text{ this implies } \eta(x) < 1.
\]
Moreover, \( \eta(x) = \frac{x \mathcal{F}(x)}{\int_0^x \mathcal{F}(y) \, dy} > \frac{x \mathcal{F}(x)}{\int_0^x \mathcal{F}(y) \, dy} = \mathcal{F}(x) \) since \( \mathcal{F}(y) < 1 \) for \( y < \bar{D} \). Finally, rewriting \( \eta'(x) \) yields \( \eta'(x) = -\frac{\mathcal{F}(x)}{S(x)} (g(x) - \int_0^x y f(y) \, dy) = -\frac{\eta_x}{x} [\eta(x) + g(x) - 1] \). Since \( \eta'(x) < 0 \), this implies \( \eta(x) + g(x) > 1 \). \( \Box \)

**B Proofs**

**Proof of Proposition 1**

**Proof.** Differentiating \( \Pi^M = v_0 \theta S(q) - c(\theta) q \) with respect to \( q \) and \( \theta \) yields 
\[
\frac{\partial \Pi^M}{\partial q} = v_0 \theta \quad \text{and} \quad \frac{\partial \Pi^M}{\partial \theta} = v_0 S(q) - c'(\theta) q.
\]
In Assumption 1 and Lemma A1 we established \( \lim_{\theta \to 0} c'(\theta) = 0 \), \( c(\theta) / \theta < v_0 \) for all \( \theta \in (0, \bar{D}) \), and \( \lim_{\theta \to \bar{D}} c'(\theta) > v_0 \). Using these results together with \( \lim_{q \to 0} \mathcal{F}(q) = 1 \) and \( \lim_{q \to \bar{D}} \mathcal{F}(q) = 0 \), we then have the following limiting values: (i) \( \lim_{q \to 0} \frac{\partial \Pi^M}{\partial q} = v_0 \theta - c(\theta) > 0 \); (ii) \( \lim_{q \to \bar{D}} \frac{\partial \Pi^M}{\partial q} = -c(\theta) < 0 \); (iii) \( \lim_{\theta \to 0} \frac{\partial \Pi^M}{\partial \theta} = v_0 S(q) > 0 \); (iv) \( \lim_{\theta \to \bar{D}} \frac{\partial \Pi^M}{\partial \theta} = v_0 S(q) - q \lim_{\theta \to \bar{D}} c'(\theta) < v_0 [\mathbb{E} \{ \min \{ D, q \} \} - q] \leq 0 \). These inequalities imply that a maximizer exists in the interior. Let \( \widehat{(q, \theta)} \) be the maximizer. These values should satisfy the first-order conditions \( \frac{\partial \Pi}{\partial q} = 0 \) and \( \frac{\partial \Pi}{\partial \theta} = 0 \) or

\[
v_0 \theta \mathcal{F}(q) = c(\theta) \quad \text{and} \quad v_0 S(q) = c'(\theta) q.
\]
Dividing the first equation by the second equation yields \( \eta(q) \epsilon(\theta) = 1 \). It is shown in Lemma A2 that \( \eta(x) < 1 \) for all \( x \in (0, \bar{D}) \); hence, \( \epsilon(\theta) > 1 \). The Hessian \( H \) of \( \Pi^M \) has the following components (subscripts 1 and 2 correspond to \( q \) and \( \theta \), respectively): 
\[
H_{11} = -v_0 \theta f(q), \quad H_{12} = H_{21} = v_0 \mathcal{F}(q) - c'(\theta), \quad \text{and} \quad H_{22} = -c''(\theta) q.
\]
Evaluating these at the maximizer \( \widehat{(q, \theta)} \) yields

\[
\widehat{H} = \begin{bmatrix}
-c(\theta) f(\widehat{q}) / \mathcal{F}(\widehat{q}) & -c(\theta) [\epsilon(\theta) - 1] / \theta \\
-c(\theta) [\epsilon(\theta) - 1] / \theta & -c''(\theta) \widehat{q}
\end{bmatrix}.
\]
Its determinant is then \( \det \widehat{H} = c(\theta) c''(\theta) g(\widehat{q}) - \frac{c(\theta)^2}{\theta^2} [\epsilon(\theta) - 1]^2 \). Note that, since \( c'(\theta) = \frac{c(\theta) \epsilon(\theta)}{\theta} \),
\[
c''(\theta) = \frac{\theta c'(\theta) c'(\theta) + \theta c'(\theta) \epsilon(\theta) - c(\theta) \epsilon(\theta)}{\theta^2} = \frac{\theta c(\theta) c'(\theta) + c(\theta) \epsilon(\theta)^2 - c(\theta) \epsilon(\theta)}{\theta^2} \geq \frac{c(\theta) \epsilon(\theta)}{\theta^2} [\epsilon(\theta) - 1],
\]
where we used the condition \( c'(\theta) \geq 0 \) given in Assumption 1. Using this inequality, we get

\[
\det \hat{H} = c(\hat{\theta})c''(\hat{\theta})g(\hat{q}) - \frac{c(\hat{\theta})^2}{\hat{\theta}^2} [c(\hat{\theta}) - 1]^2 \geq \frac{c(\hat{\theta})^2}{\hat{\theta}^2} \epsilon(\hat{\theta})[c(\hat{\theta}) - 1] \left( g(\hat{q}) - \frac{c(\hat{\theta}) - 1}{\epsilon(\hat{\theta})} \right)
\]

\[
= \frac{c(\hat{\theta})^2}{\hat{\theta}^2} \epsilon(\hat{\theta})[c(\hat{\theta}) - 1] [g(\hat{q}) + \eta(\hat{q}) - 1] > 0,
\]

where the second inequality follows from \( \epsilon(\hat{\theta}) > 1 \) we found above and the inequality \( g(x) + \eta(x) > 1 \) proved in Lemma A2. Since \( \hat{H}_{11} < 0, \hat{H}_{22} < 0, \) and \( \det \hat{H} > 0, \) the maximizer is unique. Finally, to prove that \( \hat{q} \) and \( \hat{\theta} \) are substitutes, observe that, since \( \epsilon(\hat{\theta}) > 1, \) the cross partial \( \hat{H}_{12} \) satisfies

\[
\hat{H}_{12} = -\frac{c(\hat{\theta})}{\hat{\theta}} [\epsilon(\hat{\theta}) - 1] < 0.
\]

This implies that \( \frac{\partial \hat{q}}{\partial \hat{\theta}} = -\frac{\hat{H}_{12}}{\hat{H}_{11}} < 0, \) i.e., \( \hat{q} \) and \( \hat{\theta} \) are substitutes. \( \square \)

**Proof of Proposition 2**

**Proof.** In Lemma A2 we proved \( \overline{F}(x) < \eta(x) \). Hence, \( \overline{F}(q^c) < \eta(q^c) = 1/n. \) Moreover, \( \frac{c_v}{w_0} (\theta^c)^{n-1} = \overline{F}(q^c) < \eta(q^c) = \frac{1}{n} \) implies \( \theta^c < \left( \frac{v_0}{w_0} \right)^{1/(n-1)}. \) (iv) follows directly from (iii). \( \square \)

**Proof of Proposition 3**

**Proof.** We proceed with backward induction by considering the retailer’s problem first. Given \( w \) and \( \theta, \) the retailer chooses \( q. \) Differentiating \( \Pi_R^d = v_0 \theta S(q) - wq \) with respect to \( q \) yields

\[
\frac{\partial \Pi_R^d}{\partial q} = v_0 \theta F(q) - w \quad \text{and} \quad \frac{\partial^2 \Pi_R^d}{\partial q^2} = -v_0 \theta f(q) < 0.
\]

Observe \( \lim_{q \to 0} \frac{\partial \Pi_R^d}{\partial q} = v_0 \theta - w \geq 0 \) and \( \lim_{q \to \overline{q}} \frac{\partial \Pi_R^d}{\partial q} = -w < 0. \) Therefore, the retailer chooses \( q = F^{-1} \left( 1 - \frac{w}{v_0 \theta} \right). \) The resulting manufacturer profit is \( \Pi_M^d = (w - c(\theta)) F^{-1} \left( 1 - \frac{w}{v_0 \theta} \right), \) which can be rewritten as \( \Pi_M^d = v_0 \theta q F(q) - c(\theta) q \) by changing the manufacturer’s decision variable from \( w \) to \( q \) using the one-to-one mapping \( w = v_0 \theta \overline{F}(q). \) Differentiating \( \Pi_M^d \) with respect to \( q \) and \( \theta \) yields \( \frac{\partial \Pi_M^d}{\partial q} = v_0 \theta F(q) (1 - g(q)) - c(\theta) \) and \( \frac{\partial \Pi_M^d}{\partial \theta} = v_0 q F(q) - c'(\theta) q. \) In Assumption 1 and Lemma A1 we established \( \lim_{q \to 0} c'(\theta) = 0, c(\theta) / \theta < v_0 \) for all \( \theta \in (0, \overline{\theta}), \) and \( \lim_{q \to \overline{q}} c'(\theta) > v_0. \) Using these results together with \( \lim_{q \to 0} \overline{F}(q) = 1, \lim_{q \to 0} g(q) = 0, \) and \( \lim_{q \to \overline{q}} \overline{F}(q) = 0, \) we then have the following limiting values: (i) \( \lim_{q \to 0} \frac{\partial \Pi_M^d}{\partial q} = v_0 \theta - c(\theta) > 0; \) (ii) \( \lim_{q \to \overline{q}} \frac{\partial \Pi_M^d}{\partial q} = -c(\theta) < 0; \) (iii) \( \lim_{\theta \to 0} \frac{\partial \Pi_M^d}{\partial \theta} = v_0 q \overline{F}(q) > 0; \) (iv) \( \lim_{\theta \to \overline{\theta}} \frac{\partial \Pi_M^d}{\partial \theta} = v_0 q \overline{F}(q) - q \lim_{q \to \overline{q}} c'(\theta) < v_0 q (\overline{F}(q) - 1) = -v_0 q \overline{F}(q) < 0. \) The inequalities imply that a maximizer exists in the interior. Let \( (\hat{q}, \hat{\theta}) \) be the maximizer. These values
Lemma A2. Since \( y \) yields (i) and (ii). The equilibrium inventory level with fixed quality \( \hat{q}_c \) requires \( c(\hat{q}) = 1 \) and \( v_0 F(\hat{q}) = c'(\hat{q}) \).

Dividing the first equation by the second equation yields \( [1 - g(\hat{q})] \epsilon(\hat{q}) = 1 \). The first equation requires \( g(\hat{q}) < 1 \), and therefore \( \epsilon(\hat{q}) > 1 \). The Hessian \( H \) of \( \Pi^d_M \) has the following components (subscripts 1 and 2 correspond to \( q \) and \( \theta \), respectively): \( H_{11} = -v_0 \theta \left[ f(q) (1 - g(q)) + \bar{F}(q) g'(q) \right], \)
\( H_{12} = v_0 \bar{F}(q) (1 - g(q)) - c'(\theta), H_{22} = -c''(\theta) q \). Evaluating these at the maximizer \((\hat{q}, \hat{\theta})\) yields
\[
\hat{H} = \begin{bmatrix}
-c(\hat{q}) f(\hat{q}) / \bar{F}(\hat{q}) - \theta c'(\hat{\theta}) g'(\hat{q}) & -c'(\hat{\theta}) g(\hat{q}) \\
-\theta c'(\hat{\theta}) g(\hat{q}) & -c''(\hat{\theta}) \hat{q}
\end{bmatrix}.
\]

Its determinant is then \( \det \hat{H} = \left[ c(\hat{\theta}) g(\hat{q}) + \theta c'(\hat{\theta}) \hat{q} g'(\hat{q}) \right] c''(\hat{\theta}) - c'(\hat{\theta})^2 g(\hat{q})^2 \). In the proof of Proposition 1 we showed \( c''(\theta) \geq \frac{c(\theta) \epsilon(\theta)}{\theta^2} [\epsilon(\theta) - 1] \). Rewriting the expression above using the relation \( c'(\theta) = c(\theta) \epsilon(\theta) / \theta \) and applying this inequality along with the IGFR condition \( g'(q) > 0 \),
\[
\det \hat{H} = \left[ c(\hat{\theta}) g(\hat{q}) + \theta c'(\hat{\theta}) \hat{q} g'(\hat{q}) \right] c''(\hat{\theta}) - c'(\hat{\theta})^2 g(\hat{q})^2 \\
> c(\hat{\theta}) g(\hat{q}) \frac{c(\hat{\theta}) \epsilon(\hat{\theta})}{\theta^2} [\epsilon(\hat{\theta}) - 1] - \frac{c(\hat{\theta})^2 \epsilon(\hat{\theta})^2}{\theta^2} g(\hat{q})^2 = \frac{c(\hat{\theta})^2 \epsilon(\hat{\theta})^2}{\theta^2} \left( 1 - \frac{1}{\epsilon(\hat{\theta})} - g(\hat{q}) \right) g(\hat{q}) = 0,
\]
where we used the equilibrium condition \([1 - g(\hat{q})] \epsilon(\hat{q}) = 1\) to prove the last equality. Since \( \hat{H}_{11} < 0, \hat{H}_{22} < 0, \) and \( \det \hat{H} > 0 \), the maximizer is unique. In addition, \( \hat{H}_{12} < 0 \), implying that \( \frac{\partial \hat{q}}{\partial \theta} = -\frac{\hat{H}_{12}}{\hat{H}_{11}} < 0 \), i.e., \( \hat{q} \) and \( \hat{\theta} \) are substitutes.

\[ \Box \]

Proof of Proposition 4

Proof. (i) and (ii). The equilibrium inventory level with fixed quality \( \theta^c \), denoted by \( \hat{\theta} \), is characterized by the equation \( \bar{F}(\hat{q}) (1 - g(\hat{q})) = c(\hat{q}) \) (see (8)). Comparing this with (5), we see that \( \frac{\bar{T}(\hat{q})}{\bar{F}(\hat{q})} = 1 - g(\hat{q}) < 1 \), which implies \( \hat{q} < q^c \). The equilibrium quality with fixed inventory \( q^c \), denoted by \( \hat{\theta} \), is characterized by the equation \( v_0 \bar{F}(q^c) = c'(\hat{\theta}) \) (see (9)). Comparing this with (6), we see that \( c'(\hat{\theta}) = \frac{c(\hat{\theta})}{\hat{q}^c} = \frac{c'(\hat{\theta})}{\hat{q}^c} = c'(\theta^c) \eta(q^c) \). The equilibrium quality with fixed inventory \( q^c \), denoted by \( \hat{\theta} \), is characterized by the equation \( v_0 \bar{F}(q^c) = c'(\hat{\theta}) \eta(q^c) \). Comparing this with (7), we see that \( c'(\hat{\theta}) = c'(\theta^c) \eta(q^c) < c'(\theta^c) \), where we used (7) and the fact \( \eta(x) < 1 \) from Lemma A2. Since \( c''(\theta) > 0 \), the inequality \( c'(\hat{\theta}) < c'(\theta^c) \) implies \( \hat{\theta} < \theta^c \).
(iii) Inventory coordination is achieved if \( \bar{F}(q) = c(\theta)/v_0\theta \). However, per equation (8), in the decentralized supply chain \( \bar{F}(q) = c(\theta)/v_0\theta(1 - g(q)) > c(\theta)/v_0\theta \), hence for any \( \theta \), the induced inventory level in the decentralized supply chain is too small. Similarly, quality coordination is achieved if \( S(q) = \frac{c'(\theta)}{v_0} \). Since \( S(q) > q\bar{F}(q) \), this implies \( \bar{F}(q) < \frac{c'(\theta)}{v_0} \); however, per equation (9), in the decentralized supply chain \( \bar{F}(q) = \frac{c'(\theta)}{v_0} \). Since \( c'(\theta) > 0 \), this implies that quality is too low, for any inventory level, in the decentralized supply chain.

**Proof of Proposition 5**

*Proof.* From (7) and (11) we have \( \eta(q^c) = 1 - g(q^d) = 1/n \). In Lemma A2 we showed that \( \eta(x) > 1 - g(x) \). Since both sides of this inequality are decreasing, the condition \( \eta(q^c) = 1 - g(q^d) \) implies \( q^c > q^d \). In addition, \( \theta^c > \theta^d \) can be written as \( \left( \frac{v_0 c_0 \bar{F}(q^c)}{n} \right)^{1/(n-1)} > \left( \frac{v_0 c_0 \bar{F}(q^d)}{n} \right)^{1/(n-1)} \), which is equivalent to (12).

**Proof of Proposition 6**

*Proof.* We solve the game by backwards induction. In Stage 3, the retailer determines inventory to maximize its profit for the given buyback contract and quality level. Knowing this, in Stage 2 the manufacturer chooses a buyback contract to maximize its profit, given a fixed quality level. Let \( \Pi(\theta, q) = v_0\theta S(q) - c_0\theta^n q \) be the total supply chain profit at a given quality and inventory level. Consider, for some general \( \alpha \in (0, 1) \), a contract with parameters \( b = v_0\theta(1 - \alpha) \) and \( w = \alpha c_0\theta^n + v_0\theta(1 - \alpha) \). Given this contract, retailer profit during the inventory determination stage (Stage 3) is

\[
\Pi_M^b(q) = \alpha [v_0\theta S(q) - c_0\theta^n q] = \alpha \Pi(\theta, q),
\]

while manufacturer profit in Stage 2 is

\[
\Pi_M^b(\theta) = (1 - \alpha) [v_0\theta S(q) - c_0\theta^n q] = (1 - \alpha) \Pi(\theta, q).
\]

The retailer’s optimal inventory level \( q(\theta) \) (determined in Stage 3) is the same as the centralized optimal inventory level as a function of quality, and maximizes the total supply chain profit \( \Pi(\theta, q) \) and the manufacturer’s profit for a given quality level. By adjusting \( \alpha \) such that at optimality the
retailer is left with precisely its reservation profit, i.e., by setting $\alpha = \Pi(\theta, q(\theta))$, the manufacturer’s profit becomes $\Pi^b_M(\theta) = \Pi(\theta, q(\theta)) - R$. Thus, this is clearly an optimal contract for the manufacturer, as it allows the manufacturer to take all supply chain profit above the reservation level $R$ and induces the retailer to maximize that profit via its inventory decision. In Stage 1, the manufacturer chooses a quality level to maximize its total profit, which is equivalent to maximizing $\Pi(\theta, q(\theta))$, i.e., the total supply chain profit. Hence, the manufacturer chooses the centralized optimal quality, and the retailer chooses the centralized optimal inventory.