Competing to Discover Compliance Violations: Self-Inspections and Enforcement Policies

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To enable cost-effective enforcement of environmental compliance standards, regulatory agencies encourage production firms to voluntarily discover and correct compliance violations. Although such self-regulation activities often bring desired benefits, they create nontrivial challenges. To study this tradeoff, we develop a model that captures the interactions between a regulator and a firm that unfold over time. Because constant monitoring is prohibitive, the regulator and the firm perform costly inspections to discover the compliance state of production. If the regulator detects noncompliance, the firm is required to pay penalty and restore compliance. To avoid penalty, the firm performs self-inspections to preemptively detect noncompliance and restore compliance without reporting the action to the regulator. We show that inefficiency caused by the firm’s private action is amplified if the regulator adopts a policy of requiring permanent restoration. Under such a policy, the firm’s self-inspections may leave the regulator and the environment worse off. By contrast, self-inspections always bring a net benefit to the regulator if repeated temporary restorations are allowed. We also find that, due to self-inspections, a paradoxical situation arises where the regulator prefers mandating permanent restoration despite having a small chance of enforcing it.

Key words: sustainability, environmental regulation, self-policing, inspections

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1. Introduction

To enable cost-effective enforcements of environmental standards, regulatory agencies encourage firms to engage in self-policing, i.e., voluntary detection, reporting, and correction of compliance violations that arise from the firms’ production activities. In the U.S., this movement started gaining momentum in the mid-1990s with publication of the “Audit Policy,” a set of guidelines from the U.S. Environmental Protection Agency (EPA) intended to provide firms with an incentive for self-policing. The goal of this initiative is to achieve compliance while relieving the agency’s monitoring burden: “Because government resources are limited, universal compliance cannot be achieved without active efforts by the regulated community to police themselves” (US EPA 2000). Reports indicate that the EPA has indeed curtailed its expenditure on compliance monitoring, but whether implementation of the Audit Policy has led to better environmental outcomes remains inconclusive (Pfaff and Sanchirico 2004, Stafford 2004, Knight 2014). Despite ongoing debates on
the policy and a new focus on technology-based solutions to monitoring, reliance on self-policing is expected to continue in the foreseeable future (US EPA 2014, Culleen and Glazer 2016).

Among all self-policing activities, self-discovery of compliance violations is critical since it is a prerequisite for follow-up actions such as voluntary disclosure and remediation. Self-discovery requires investment and effort. Most firms do not possess the capability to monitor compliance of their production minute-by-minute, and in many cases, random compliance violations go undetected unless an investigation is conducted to find out if harm is being done to the environment (Environmental Integrity Project 2004). An assessment of harm can be made, for example, through periodic sampling and measurements of emitted compounds, as the Organisation for Economic Co-operation and Development (OECD) recommends in its guideline for environmental self-monitoring in developing countries (OECD 2007).

Given that self-discovery is essential but requires efforts, it is important to understand how regulatory enforcement actions influence a firm’s decision to perform costly self-inspections. Typically, regulatory agencies conduct their own inspections of a firm’s production, mandating a correction and imposing a fine if an unreported violation is discovered. Clearly, these enforcement actions help limit the environmental damage. At the same time, they provide firms with an incentive to self-inspect their production because, by catching a violation before the regulator does, a firm has a chance to preemptively correct the problem and avoid the regulator’s scrutiny.\footnote{Alternatively, a firm may self-report discovery of a violation to the regulator in return for a penalty relief. In fact, inducing such an outcome is exactly what the Audit Policy is designed for; the policy promises substantial reduction in penalty for self-reported violations. It has been documented, however, that the majority of self-reported violations are minor infractions that do not involve actual environmental damage. This contrasts with more serious violations caught by the EPA when the Audit Policy was not in effect (Pfaff and Sanchirico 2004). This indicates that in many cases penalty relief is insufficient as an instrument for eliciting self-reporting. Based on this observation, in this paper we focus on situations where self-reporting does not occur in equilibrium.} Such opportunistic behavior by firms makes it difficult for the regulator to acquire information about occurrences of compliance violation, but it helps reduce the environmental harm.

That enforcement actions may lead firms to engage in private activities raises several questions. First, is there a way to infer the degree to which self-inspections are performed, and how much environmental harm is prevented as a result? Second, how do self-inspections interact with a regulator’s own inspections? Third, should an enforcement policy be designed to encourage or discourage self-inspections, and under what conditions?

In this paper, we aim to answer these questions by building and analyzing a model that captures the underlying process of self-inspections. While the role of self-inspections has been examined in the literature, none of the studies address nuanced interactions between a firm and a regulator that arise in a dynamic setting. As we demonstrate, such interactions have nontrivial implications on how an enforcement policy should be designed and managed. Our model features a firm and a regulator
who perform inspections over time in order to detect occurrences of random compliance violations. The regulator commits to an enforcement policy that specifies the penalty and a corrective measure required of the firm if a violation is discovered. We consider two types of corrective measures: temporary fix (TF) and permanent fix (PF). Both measures restore compliance, but only PF ensures that violations do not occur again in the future. Under a given enforcement policy, the firm and the regulator set their inspection rates to minimize their long-run disutilities. Strategic interactions between the two arise because their objectives are not aligned; whereas the regulator is concerned about the impact on the environment and on consumer welfare, the firm is interested in minimizing profit loss.

To gain insights, we focus on two enforcement policies that are opposites of each other: Continuation Policy (CP) and Stopping Policy (SP). Under CP, compliance violations occur repeatedly because the regulator allows the firm to restore compliance via TF. Under SP, on the other hand, the regulator requires the firm to apply PF after the earliest detection of a violation, thus eliminating the chance of future violations. The equilibrium outcomes under these two policies are determined by inspection rate choices made by the two parties, as well as the penalty set by the regulator.

Our analysis shows that, under both policies, the inspection rate choices can be complementary or substitutable depending on factors such as the degree of environmental harm and the cost of performing an inspection. Such an ambiguous relationship reflects the dual nature of inspections by the regulator, namely that the regulator’s inspections are used for both discovery of a violation and an incentive for the firm. We find that substitutability creates a situation where self-inspections worsen the environment.

The regulator may or may not benefit from self-inspections, depending on which enforcement policy is employed. We find that self-inspections always bring a net benefit to the regulator if she employs CP. By contrast, self-inspections may leave the regulator worse off if she employs SP; even if the regulator prefers requiring PF because she prioritizes environmental harm prevention, adopting such a policy may backfire. We identify the source of inefficiency that causes this outcome and show that its effects are amplified under SP. We also find that, due to self-inspections, a paradoxical situation arises where the regulator prefers mandating PF despite having a small chance of enforcing it. Finally, we show that a general enforcement policy that falls between CP and SP may emerge in equilibrium, with the key insights from CP and SP unchanged.

Both types of corrective measures are observed in practice. In 2016, the grocery store chain Trader Joe’s company was fined for coolant leaks from refrigerators in its stores. While the settlement plan includes managerial fixes, it does not require the company to upgrade equipment (US EPA 2016). By contrast, the Lehigh Hanson cement company has agreed to install an advanced wastewater treatment plant after its cement processing facility was found to have dumped millions of gallons of toxic wastewater into a nearby creek (Rogers 2015). These two examples correspond to TF and PF, respectively (see the definitions of TF and PF in §3.3).
The rest of the paper is organized as follows. After a brief literature review in §2, we set up the model and define the terms used throughout the paper in §3. We start with a benchmark case in §4, followed by the analyses of equilibria under CP and SP in §5. In §6 we extend the analyses of CP and SP to consider a generalized enforcement policy. Concluding remarks are found in §7.

2. Related Literature

Discussions on self-inspections and self-discovery of compliance violations are found mostly in the literature of regulatory and environmental economics. In this literature, there are debates on whether regulatory agencies’ enforcement actions encourage or discourage firms’ self-inspections and self-policing activities (see Short and Toffel (2008) and references therein). Efforts have been made to test the hypothesis that self-inspections and other voluntary actions by firms bring desired benefits, with mixed results; see Short and Toffel (2008), Toffel and Short (2011), Khanna and Widyawati (2011), and Evans et al. (2011). Game-theoretic analyses include Mishra, Newman, and Stinson (1997), Pfaff and Sanchirico (2000), Innes (2001), and Friesen (2006). Among them, Innes (2001) and Friesen (2006) view self-inspections (or self-auditing) mainly as a prerequisite for self-reporting, and as a result, discussions are centered around the idea that self-inspections can alleviate the monitoring burden of regulating agencies indirectly by enabling self-reporting. Mishra et al. (1997), Pfaff and Sanchirico (2000), and Friesen (2006) additionally consider direct benefits of self-inspections, i.e., voluntary corrections of compliance violations.

These papers describe how self-inspections fit into general enforcement policies that include many other aspects of self-regulation. Partly bounded by such a holistic approach, game-theoretic analyses are based on simplistic model assumptions that do not capture intricacies of self-inspections. For example, common in these models is a binary choice between self-inspections and no self-inspection, in a one-shot decision setting where a compliance violation is discovered with fixed probability. None of the papers mentioned above study self-inspections from a process perspective, and as a result, they do not address issues such as how probability of self-discovery is endogenously determined from the way inspections are performed over time. (Consequently, in some cases significant results are obtained only after invoking mixed equilibrium strategies; see Friesen (2006).) They also fail to establish a clear relationship between firms’ self-inspections and regulating agencies’ own inspections, an important step in evaluating the performance of an existing enforcement policy or designing a new policy. We fill this gap in the literature, offering new insights that can be obtained from considerations of operational details.

In this paper we develop a model based on analytical techniques commonly found in the operations management (OM) literature. Our model features a Markov process of repeated compliance failures and restorations, as well as random inspections performed competitively by a firm and a
regulator over time. Thus, we borrow elements from “inspection models” found in the reliability theory literature (Barlow and Proschan 1996, Nakagawa 2005) and combine them with a game-theoretic analysis to explain implications of compliance enforcement policies. Such an analytical framework was first developed by Kim (2015), followed by Wang et al. (2016). They focus on issues related to self-reporting but ignore self-inspections, as they assume that regulated firms have free access to information about compliance status. This is a simplification of reality, and we complement their findings by focusing on the role of self-inspections.

In recent years, OM researchers have been paying increased attention to topics in sustainability and social responsibility in the contexts of public policy and supply chain management. For recent works, see Örsdemir et al. (2016), Bondareva and Pinker (2017), Chen and Lee (2017), and Huang et al. (2017). A growing subset in this stream of research investigates issues related to audits or inspections. For example, Caro et al. (2015) study merits of different audit mechanisms that can be employed by multiple buyers dealing with a common supplier. Fang and Cho (2016) analyze a similar setting using a cooperative game approach. Chen et al. (2015) study how supply chain transparency influences an NGO’s auditing decisions. Plambeck and Taylor (2016) discuss suppliers’ efforts to evade audits by buyer firms, and strategies to mitigate the impact of such behaviors. Our paper adds to this area of research, with a focus on the subject of self-inspections that has been largely overlooked. Although our discussions on self-inspections are framed in a regulatory setting, the insights generated from our analysis are equally valid in supply chain management settings where external inspections or audits (e.g., those by NGOs) are used to incentivize firms to perform voluntary inspections and to take corrective measures. As such, our model has general applicability.

3. Model
3.1 Overview
We study interactions between a firm and a regulator that impact the environment over time. The firm (denoted by the subscript \( f \) and referred to as “he”) generates revenue through production at a facility. Production runs continuously without direct intervention by the firm (e.g., automated production or outsourced production at a facility managed by a third party). During normal times the facility is in compliance, i.e., the facility’s production adheres to an environmental compliance standard set by the regulator (denoted by the subscript \( r \) and referred to as “she”). Compliance at the facility does not last indefinitely, however; a compliance failure occurs at a random point in time, after which the facility remains in noncompliance until the state is discovered and a corrective measure is applied to restore compliance. Because the facility’s compliance status has no impact on the facility’s production capability, the firm generates revenue even while the facility stays in
noncompliance. Although noncompliant production does not directly impact the firm’s revenue, it harms the environment. To prevent the harm, the regulator takes enforcement actions and provides the firm with incentives to apply a corrective measure. These enforcement actions have limited effectiveness, however, because neither the firm nor the regulator has complete visibility to the facility’s compliance status; constant monitoring of the facility is prohibitive because it is costly to inspect the facility and discover its compliance status. As a result, inspections are performed only occasionally. The firm can apply a corrective measure only if he discovers noncompliance in an inspection. The regulator can also perform inspections of the facility, and in addition, she has the authority to order the firm to apply a corrective measure once noncompliance is discovered. Both the firm and the regulator are risk neutral.

3.2 Compliance Failure and Inspections
The facility is in compliance at time zero. Compliance failure occurs after the facility spends $U$ time units in compliance, after which the facility remains in noncompliance until a corrective measure is applied. We assume that $U$ is exponentially distributed with the compliance failure rate $\lambda$. Harm is done to the environment at rate $h$ per unit time while the facility stays in noncompliance. (For instance, pollutants enter into the environment at a constant rate while pollutant-blocking equipment used in production is malfunctioning.)

Both the regulator and the firm perform random inspections of the facility. We assume that inter-inspection times are exponentially distributed, with $Y_r \sim \exp(\nu_r)$ and $Y_f \sim \exp(\nu_f)$ each denoting the time interval between two successive inspections by the regulator and the firm, respectively. Exponential distributions are adopted in our model mainly to enable analytical tractability, but the key insights derived from our analysis depend minimally on this assumption. The inspection rates $\nu_r$ and $\nu_f$ are decision variables.

For simplicity, we assume that each inspection is completed in an instant and that an inspection uncovers the facility’s state with probability one, i.e., there is no false positive or false negative. Because of this assumption, noncompliance is detected if and only if an inspection is performed when the facility is in noncompliance. We assume that past noncompliance occurrences cannot be traced if they went undetected; hence, it is only the compliance violation “caught in the act” that prompts the regulator to take an enforcement action. The regulator and the firm incur fixed costs $\chi_r$ and $\chi_f$ each time they perform an inspection. We use the terms self-inspections and self-detections to describe inspections and detections by the firm, as opposed to those by the regulator. These terms reflect the idea that the firm has a direct tie to the facility through production activities whereas the regulator does not.
3.3 Corrective Measures
We consider two types of corrective measures: temporary fix (TF) and permanent fix (PF). TF and PF are applied immediately after an inspection is performed. The firm incurs fixed costs $c$ and $s$ (with $c < s$) once he applies TF and PF, respectively. TF restores the facility’s state to that at time zero, i.e., TF puts the facility back to compliance but exposes the facility to the risk of another compliance failure $U \sim \exp(\lambda)$ time units later. By contrast, PF eliminates the chance of future compliance failures, effectively resetting $\lambda$ to zero.\(^3\) To succinctly capture the idea that TF represents a quick, low-cost fix (e.g., swap of a defective component in pollution-blocking equipment), we assume that TF is completed in an instant. As a direct consequence, the firm can perfectly conceal a compliance failure from the regulator if he applies TF immediately after detecting noncompliance. By contrast, the firm cannot conceal a compliance failure by applying PF because PF requires a major overhaul at the facility that cannot escape the regulator’s attention (e.g., acquisition and installation of upgraded equipment which require regulatory approval). Once applied, PF stops further inspections by the regulator and the firm since compliance is restored permanently. After TF, on the other hand, inspections resume because compliance may fail again.

We assume that the regulator incurs cost $b \geq 0$ once compliance is restored via PF. This cost is best interpreted as consumer welfare loss (the term we use throughout the paper) since it captures the opportunity cost incurred due to PF. For example, PF may force the firm to raise the price of goods produced at the facility because upgraded pollution-blocking equipment requires a higher cost of maintenance, erasing the surplus that consumers may have enjoyed before PF. We assume that the regulator internalizes such an impact on consumers.

Because PF alters outlook of the future by resetting $\lambda$ to zero, unlike TF, one cannot preclude the possibility that the regulator and the firm are better off by having PF applied even when no compliance violation is found. Despite the possibility, we do not consider such a scenario; PF can be applied only after noncompliance is detected. For the regulator this is reasonable since, in practice, an enforcement order requires evidence of a violation. For the firm, we prove later (Lemma 1) that PF upon finding compliance does not emerge in equilibrium.

3.4 Enforcement Policies and Objectives
The regulator enforces compliance at the facility by engaging in three types of activities: (i) perform inspections of the facility, (ii) impose penalty on the firm upon noncompliance detection, and (iii) order the firm to restore compliance via either TF or PF upon noncompliance detection. The regulator’s inspections and penalty incentivize the firm to perform self-inspections, since the firm

\(^3\) The assumption of a zero failure rate can be replaced by the assumption of a reduced failure rate, i.e., resetting $\lambda$ to a smaller value, without altering key insights.
can avoid the penalty if he detects noncompliance before the regulator does and preemptively restores compliance. The penalty $\kappa$ is levied as soon as noncompliance is detected by the regulator. We assume that $\kappa$ cannot exceed $K$, the legal limit on the amount of penalty set for a violation (the same assumption is commonly found in the literature; see, for example, Livernois and McKenna 1999). Hence, the regulator has to ensure that the constraint $\kappa \leq K$ is satisfied when she optimally sets $\kappa$.

Because the regulator’s enforcement activities continue after TF but stop after PF, in general, an enforcement policy has the following “$N$ strikes and you are out” structure: order TF upon each of the first $N - 1$ noncompliance detections by the regulator, followed by a PF order at the $N^\text{th}$ detection. Hence, the time until PF consists of $N$ enforcement cycles, each of which starts with compliance restoration and ends with noncompliance detection by the regulator. (Note there is no delay in between cycles because restorations are assumed to be instantaneous.) The regulator assigns a fixed penalty and a fixed inspection rate to each enforcement cycle. Thus, an enforcement policy consists of three components: $N$, $\kappa = (\kappa_1, ..., \kappa_N)$, and $\nu_r = (\nu^r_1, ..., \nu^r_N)$. The regulator commits to an enforcement policy and announces it at time zero. In response, the firm sets his inspection rates $\nu_f = (\nu^f_1, ..., \nu^f_N)$ and decides which action he will take after each self-inspection. In general, the firm can apply TF or PF, or he may take no action. After time zero, the regulator and the firm start performing inspections at predetermined rates.

Two special cases of the general policy described above are $N = \infty$ and $N = 1$, which we call Continuation Policy (CP) and Stopping Policy (SP), respectively. Under CP, the regulator orders TF whenever she detects noncompliance. Under SP, the regulator orders PF at her first detection of noncompliance. Hence, CP and SP represent two opposite extremes. We focus on these two policies in the main part of our analysis (§5) because they generate key insights that are extended to the general policy with no restriction on $N$. The general policy is discussed in §6.

Under all policies, the future is exponentially discounted at rate $\alpha$, which is common to the regulator and the firm. The time-discounted total expected disutilities for the regulator and the firm, evaluated at time zero, are denoted respectively as $V$ and $\Psi$. The firm’s disutility $\Psi$ includes self-inspection costs ($\chi_f$ per inspection), compliance restoration costs ($c$ upon TF and $s$ upon PF), and penalties ($\kappa$ per noncompliance detection by the regulator). The regulator’s disutility $V$ includes her inspection costs ($\chi_r$ per inspection), environmental harm ($h$ per unit time summed over

4 This is not restrictive because, due to memorylessness arising from exponential distributions of random intervals $U$, $Y_r$, and $Y_f$, outlook of the future within each enforcement cycle does not change with time. Hence, optimal choices of $\nu_r$ and $\nu_f$ will remain constant within each cycle.

5 Although we frame the regulator’s enforcement actions as “TF order” vs. “PF order,” they can be recast as “TF/PF choice by the firm” vs. “PF order.” This is because the firm has no incentive to voluntarily apply PF given a choice between TF and PF; see Lemma A4 in Appendix A.
all noncompliance durations), and consumer welfare loss (b upon PF). We use the notations I, H, and B to denote the regulator’s inspection costs, environmental harm, and consumer welfare loss, respectively, evaluated at time zero as time-discounted total expected values. Hence, \( V = I + H + B \). The regulator sets \( N, \kappa, \) and \( \nu_r \) at time zero to minimize \( V \), and the firm sets \( \nu_f \) to minimize \( \Psi \). The precise expressions for \( V \) and \( \Psi \) depend on the enforcement policy employed by the regulator, and we evaluate them later as we discuss each policy.

Note that the above specification of \( V \) is based on the assumption that the regulator does not internalize the firm’s costs. This is a reasonable approximation of reality where the regulatory agencies focus on cost-effective protection of consumers and the environment, delegating the burden of inspections and compliance restorations to firms whenever possible. (It is unlikely, for example, that an agency will allocate part of its budget to reimbursing self-inspection costs incurred by the firm.)

Finally, throughout the paper we assume that the parameter values satisfy the following technical conditions that together rule out trivial cases:

**Assumption 1** (i) \( h > \chi_r \frac{(\alpha + \lambda)^2}{\lambda} \); (ii) \( b < h - \chi_r \frac{\alpha + \lambda}{\alpha} \); (iii) \( s > (c + K) \frac{\alpha + \lambda}{\alpha} \); (iv) \( \chi_f < \frac{K^2 \lambda}{4(c + K)(\alpha + \lambda)} \).

All of these conditions make intuitive sense. They ensure that the degree of environmental harm is significant (relatively large \( h \)), consumer welfare loss is not too severe (relatively small \( b \)), PF restoration is expensive (relatively large \( s \)), and performing a self-inspection is not too burdensome to the firm (relatively small \( \chi_f \)).

4. **Benchmark: No Self-Inspections by Firm**

To highlight the impact of self-inspections, we first consider a benchmark case in which no self-inspection exists, i.e., \( \nu_f^n = 0 \) for all \( n \in \{1, ..., N\} \). Only the regulator inspects the facility, at random intervals \( Y_r^n \sim \exp(\nu_r^n) \), in order to detect compliance violations that occur after the facility spends \( U \sim \exp(\lambda) \) time units in compliance. The firm complies with the regulator’s TF and PF orders whenever noncompliance is detected by the regulator. In this scenario the penalty \( \kappa \) plays no role as an incentive because the firm responds only to the regulator’s compliance restoration order. Hence, without loss of generality, we set \( \kappa^n = 0 \) for all \( n \in \{1, ..., N\} \).

Let \( V_+^n \) and \( V_-^n \) be the regulator’s expected disutilities (cost-to-go) in the \( n^{th} \) enforcement cycle when the facility is in compliance (\(+\)) and in noncompliance (\(-\)), respectively. Because of stationarity arising from exponentially-distributed time intervals, \( V_+^n \) and \( V_-^n \) depend only on \( n \) and the facility’s state. If the facility is in compliance, it stays in compliance until one of two events occurs: compliance failure or the regulator’s inspection. If compliance failure occurs first (with conditional probability \( \Pr(U < Y_r^n) = \lambda/(\lambda + \nu_r^n) \)), the facility’s state transitions to noncompliance.
If the inspection occurs first (with \( \Pr(U > Y^n_r) = \nu^n_r/(\lambda + \nu^n_r) \)), the facility remains in compliance but the regulator incurs the inspection cost \( \chi_r \). Therefore:

\[
V^n_+ = \int_0^\infty e^{-\alpha t} \left( V^n_- - \frac{\nu^n_r}{\lambda + \nu^n_r} + \frac{\nu^n_r}{\lambda + \nu^n_r} \right) \frac{\nu^n_r}{\lambda + \nu^n_r} d\tau = \frac{V^n_+ \lambda + (\chi_r + V^n_+ \nu^n_r)}{\alpha + \lambda + \nu^n_r}.
\]

On the other hand, if the facility is in noncompliance, it remains in that state until the regulator incurs cost \( \chi_r \) to inspect the facility at \( Y^n_r \sim \exp(\nu^n_r) \) time units later, triggering compliance restoration. Until then, harm is done to the environment at rate \( h \). Therefore,

\[
V^n_- = \int_0^\infty \left( \int_0^\tau e^{-\alpha t} h dt + e^{-\alpha \tau} (\chi_r + V^n_+\nu^n_r) \right) \int_0^\infty e^{-\alpha t} \frac{\nu^n_r}{\lambda + \nu^n_r} d\tau = \frac{h + (\chi_r + V^n_+\nu^n_r)}{\alpha + \nu^n_r}.
\]

Combining the two equations yields

\[
V^n_+ = \chi_r \nu^n_r(\alpha + \lambda + \nu^n_r) + (h + V^n_+\nu^n_r)\lambda.
\]

(1)

Note \( V^{N+1}_+ = b \), which represents the cost incurred by applying PF at the end of the \( N \)th cycle. The optimal enforcement policy is identified by solving the problem \( \min_{\nu^n_r \geq 0} V^n_+ \) recursively for given \( N \) and then finding the value of \( N \) that minimizes \( V_+^1 \), the regulator’s disutility at time zero. Since all activities stop after the \( N \)th cycle, the regulator’s choice of \( N \) amounts to an optimal stopping time problem. As the next proposition proves, the optimal solution exhibits a simple structure.

**Proposition 1** Without the firm’s self-inspections, the regulator employs either CP (\( N = \infty \)) or SP (\( N = 1 \)). The regulator employs CP and performs inspections at rate \( \nu_r = -(\alpha + \lambda) + \sqrt{\frac{\lambda h \chi_r}{\nu_r}} > 0 \) if \( b > -\chi_r \frac{\alpha + \lambda}{\alpha} + \frac{2 \alpha}{\alpha} \sqrt{h \chi_r \lambda} \). Otherwise, the regulator employs SP and performs inspections at rate \( \nu_r = -\alpha + \sqrt{\frac{\lambda}{\nu_r}} (h - (\chi_r + b)\alpha) \).

The regulator optimally chooses one of two extremes, CP (\( N = \infty \)) or SP (\( N = 1 \)), because any benefit that the regulator may enjoy by delaying PF accumulates if she keeps delaying PF. Such monotonicity results from built-in time symmetry; in the absence of the firm’s self-inspections, the regulator has an identical outlook of when she will end each enforcement cycle with her noncompliance detection and what consequences she will bear by renewing the cycle. This, combined with time discounting, gives rise to monotonicity. Proposition 1 specifies the necessary and sufficient condition under which CP is preferred to SP, and vice versa. The condition formalizes the idea that perpetual delay of PF under CP is preferred if consumer welfare loss, incurred upon PF, outweighs environmental harm and inspection costs. Otherwise, the regulator applies PF at the earliest opportunity by employing SP.
5. Equilibria Under Continuation Policy and Stopping Policy

The results from the last section guide us to focus on two special cases of the general enforcement policy, namely CP and SP. In this section, we restrict our attention to CP and SP and study the new dynamics that arise due to the interaction between the regulator and the firm through self-inspections. This restriction helps us isolate the net effects of self-inspections in comparison to the benchmark with no self-inspection. We start our analysis by specifying the firm’s best response under the two policies.

5.1 Firm’s Best Response

Under CP and SP, the penalty $\kappa$ and inspection rates $\nu_r$ and $\nu_f$ stay constant once they are optimally set at time zero. This is because under the two policies, either identical enforcement cycles are infinitely repeated or there is only one cycle. Therefore, it suffices to determine single values of $\kappa$, $\nu_r$, and $\nu_f$ that minimize disutilities of the regulator and the firm evaluated at time zero.

We use the subscripts $C$ and $S$ to denote the disutility functions under CP and SP, respectively: $V_C$ and $\Psi_C$ for CP and $V_S$ and $\Psi_S$ for SP.

To characterize the firm’s best response, suppose that the regulator has announced $\kappa \in [0, K]$ and $\nu_r > 0$ at time zero. The firm responds to this announcement by optimally setting $\nu_f \geq 0$ and deciding on the course of action he will take after each self-inspection. The next lemma narrows the set of actions that the firm takes after completing an inspection.

Lemma 1 Under Assumption 1, the firm facing CP or SP with $\kappa \in [0, K]$ and $\nu_r > 0$ (a) never applies PF voluntarily and (b) sets $\nu_f > 0$ only if he privately restores compliance via TF upon self-detecting noncompliance.

Given the results in Lemma 1, we see that equilibrium outcomes are determined by simple action rules. Under CP, compliance is restored via TF whenever noncompliance is detected by either the regulator or the firm. Under SP, compliance is restored via PF only if the regulator detects noncompliance; if the firm detects noncompliance, he applies TF. The firm has no incentive to apply PF voluntarily because he finds it more economical to apply a “quick fix” and bear the risk of continued compliance failures than to apply PF and incur a large cost associated with it.$^6$

Based on these action rules, we can derive the expressions for disutility functions following the steps similar to the ones presented in the last section. The firm’s disutility functions under CP and SP are:

$$
\Psi_C = \frac{\chi_f \nu_f}{\alpha} + \frac{(c + \kappa)\nu_r + \alpha \nu_f) \lambda}{\alpha(\alpha + \lambda + \nu_r + \nu_f)} \quad \text{and} \quad \Psi_S = \frac{\chi_f \nu_f(\alpha + \lambda + \nu_r + \nu_f) + (s + \kappa)\nu_r + \alpha \nu_f) \lambda}{(\alpha + \lambda)(\alpha + \nu_r + \alpha \nu_f)}.
$$

$^6$This is a direct consequence of the condition $s > (c + K) \frac{\alpha + \lambda}{\alpha}$ in Assumption 1, which is satisfied in reasonable situations where PF is substantially more expensive than TF (large $s/c$) and/or compliance failures occur relatively infrequently (small $\lambda/\alpha$).
Similarly, the regulator’s disutility functions are:

\[ V_C = \frac{\chi_r \nu_r}{\alpha} + \frac{h \lambda}{\alpha(\alpha + \lambda + \nu_r + \nu_f)} \quad \text{and} \quad V_S = \chi_r \nu_r \frac{\alpha + \lambda + \nu_r + \nu_f}{(\alpha + \lambda)(\alpha + \nu_r + \alpha \nu_f)}. \]  

(3)

Given \( \kappa \in [0, K] \) and \( \nu_r > 0 \), the firm optimally sets his inspection rate \( \nu_f \) to minimize \( \Psi_C \) and \( \Psi_S \) in (2). The following lemma characterizes the optimal solutions. Define

\[ \sigma_C = \frac{\chi_f(c + \kappa)(\alpha + \lambda)}{\kappa^2 \lambda} \quad \text{and} \quad \sigma_S = \frac{\chi_f(s + \kappa)(\alpha + \lambda)}{(s + \kappa - (\chi_f + c)(\alpha + \lambda)/\alpha)^2 \lambda}. \]  

(4)

**Lemma 2** Given \( \kappa \in [0, K] \) and \( \nu_r > 0 \), there exist \( \beta_i, \beta_i^o, \) and \( \beta_i^s \), \( i \in \{C, S\} \), with \( 0 < \beta_i < \beta_i^o < \beta_i^s \) such that \( \Psi_i \) is minimized at \( \nu_f = R_i(\nu_r, \kappa) > 0 \) if \( \sigma_i < \frac{1}{\lambda} \) and \( \beta_i, \nu_r < \beta_i^s \) while it is minimized at \( \nu_f = 0 \) otherwise. The function \( R_i(\nu_r, \kappa) \) satisfies \( \frac{\partial}{\partial \nu_r} R_i(\nu_r, \kappa) > 0 \) for \( \nu_r \in (\beta_i, \beta_i^o) \) and \( \frac{\partial}{\partial \nu_r} R_i(\nu_r, \kappa) < 0 \) for \( \nu_r \in (\beta_i^o, \beta_i^s) \).

The expressions for the firm’s best response functions, \( R_C(\nu_r, \kappa) \) under CP and \( R_S(\nu_r, \kappa) \) under SP, are found in Lemma A1 and Lemma A2 in Appendix A. As the lemma reveals, the firm’s best responses under CP and SP exhibit similar but nontrivial behaviors. First, the firm does not perform self-inspections (i.e., he sets \( \nu_f = 0 \)) if he expects that the regulator’s inspections will be performed at low or high frequencies (\( \nu_r \leq \beta_i^s \) or \( \nu_r \geq \beta_i^s \)). It is only when the regulator’s inspection rate is intermediate (\( \beta_i < \nu_r < \beta_i^s \)) that the firm has an incentive to set \( \nu_f > 0 \) and perform self-inspections. Second, when the firm does perform self-inspections, his optimal response \( R_i(\nu_r, \kappa) \) is non-monotonic in \( \nu_r \). In particular, \( \nu_r \) and \( \nu_f \) move in the same direction if \( \nu_r \) is sufficiently small, but the opposite is true if \( \nu_r \) exceeds a certain threshold.

These observations suggest that the rates of inspections chosen by the firm and the regulator can be complementary in some cases but substitutable in others. Such divergent behaviors arise from the dual role played by the regulator’s inspections. Namely, the regulator performs inspections not only to detect noncompliance and order compliance restoration, but also to provide the firm with an incentive to do the same on her behalf. While the former describes the search role of the regulator’s inspections, the latter describes the threat role: a promise of penalty \( \kappa \) incentivizes the firm to perform self-inspections in order to preemptively restore compliance and avoid the regulator’s detection. Such a threat is credible only if the regulator backs it up with frequent enough inspections. However, the threat becomes less effective if inspections are too frequent, as a threat turns into an actual detection. In such a situation, the firm has a diminished incentive to perform costly self-inspections because he faces a smaller chance of successful preemption. The non-monotonic response by the firm reflects this tension: the firm’s optimal choice of \( \nu_f \) increases with \( \nu_r \) if the regulator’s inspections are used primarily as a threat, but the relationship is reversed if
they are used more for the search purpose. That the inspection rates can have both complementary and substitutable relationships is key to understanding certain equilibrium features, as we discuss below.

5.2 Equilibrium Under Continuation Policy

Under CP, all noncompliance detections are followed by compliance restoration via TF. At time zero the regulator optimally sets the penalty $\kappa$ and her inspection rate $\nu_r$ to minimize her disutility $V_C$, anticipating the firm’s optimal choice of $\nu_f$ as specified in Lemma 2. (As we noted above, the optimal choices made at time zero remain unchanged due to infinitely repeated identical cycles.)

Substituting the result from Lemma 2 in $V_C$ evaluated in (3), we obtain the regulator’s objective function

$$V_C(\nu_r, \kappa) = \begin{cases} \frac{\chi r \nu_r}{\alpha} + \frac{h}{\alpha} \sqrt{\frac{\chi f \lambda}{h_{\nu_r - \kappa}} - c(\alpha + \lambda)} & \text{if } \sigma_C < \frac{1}{4} \text{ and } \beta_C < \nu_r < \beta_C, \\ \frac{\chi r \nu_r}{\alpha} + \frac{h\alpha}{\alpha(\alpha + \lambda + \nu_r)} & \text{otherwise}, \end{cases}$$

(5)

where $\sigma_C, \beta_C$, and $\beta_C$ are defined in (4) and Lemma A1 in Appendix A. The first and second lines of (5) correspond to the firm’s best responses $\nu_f = R_C(\nu_r, \kappa) > 0$ and $\nu_f = 0$, respectively.

The subgame perfect equilibrium under CP, denoted by the superscript $^*$, is found by identifying the values of $\kappa$ and $\nu_r$ that minimize $V_C(\nu_r, \kappa)$. Because of the cutoffs that exist at the boundaries $\nu_r = \beta_C$ and $\nu_r = \beta_C$, this function exhibits kinks, and furthermore, it may be bimodal in $\nu_r$ under some parameter value combinations. Although these features significantly complicate the analysis, sharp predictions can be made about the equilibrium. We first characterize the equilibrium and discuss some of its properties.

**Proposition 2** At the equilibrium under CP, the regulator sets $\kappa^* = K$ and $\nu_r^* = \frac{c(\alpha + \lambda)}{K} + \left(\frac{h^2 \chi f \lambda}{4K^2 \chi_r^2}\right)^{1/3}$ to induce $\nu_f^* = -\left(\frac{c(\alpha + \lambda)}{K} + \left(\frac{h^2 \chi f \lambda}{4K^2 \chi_r^2}\right)^{1/3}\right) > 0$. At this equilibrium, $I_C^*$, $H_C^*$, and $V_C^*$ increase in $h$ and in $\chi_f$.

The proposition confirms the intuition that the regulator finds it optimal to utilize the penalty $\kappa$ to the fullest extent, i.e., in equilibrium, she sets $\kappa$ to its maximum value $K$. The regulator benefits from such a choice because, although penalty and inspections together provide an incentive for self-inspections, only the latter is costly to the regulator; while she bears the cost $\chi_r$ for each inspection, she can freely impose a penalty. As a result, the regulator can save costs while maintaining the same level of incentive by lowering the frequency of inspections and substituting them with a higher penalty.

The proposition also examines how key determinants of the inspection rate choices impact equilibrium outcomes. The impacts of $\chi_f$, in particular, highlight the benefits of self-inspections. If the
firm can perform self-inspections more efficiently (smaller $\chi_f$), the regulator enjoys both environmental improvement and savings in inspection costs (smaller $H_C$ and $I_C$). As a result, the regulator is better off (lower disutility $V_C = I_C^r + H_C$).

Although these observations seem to suggest that the firm’s self-inspections will bring unambiguous benefits, caution is needed in making such a statement because Proposition 2 does not address how the presence of self-inspections impacts the regulator and the environment. To answer the question: “Do the regulator and the environment benefit from self-inspections?” one should compare the equilibrium described in Proposition 2 with the benchmark case in Proposition 1, i.e., with vs. without self-inspections.

This comparison is summarized as follows.

**Proposition 3** Let $h_0 \equiv \frac{4\chi_l(c+K)^2(\alpha + \lambda)^2}{K^2\lambda h}$, $\delta_0(h) \equiv \frac{K}{4} \sqrt{\frac{\chi h}{h}} \left(\frac{\chi}{h}\right)^3$, $\delta_1(h) \equiv \frac{K}{4} \sqrt{\frac{\chi h}{h}} \left(\frac{\chi}{h}\right)^3$, and $\delta_2(h) \equiv \frac{K}{4} \sqrt{\frac{\chi h}{h}}$, which satisfy $\delta(h) < \delta_0(h) < \delta_2(h)$ if $h < h_0$ and $\delta_2(h) < \delta_0(h) < \delta_1(h)$ if $h > h_0$. Under CP, $\nu^*_f > 0$ in equilibrium only if $V_C^* < V_C^0$, or equivalently, $\chi_f < \delta_0(h)$. At such an equilibrium:

(a) $I_C^r < I_C^0$ and $H_C^r < H_C^0$ if $\chi_f < \min\{\delta_1(h), \delta_2(h)\}$;

(b) $I_C^r > I_C^0$ and $H_C^r < H_C^0$ if $h < h_0$ and $\delta_1(h) < \chi_f < \delta_0(h)$;

(c) $I_C^r < I_C^0$ and $H_C^r > H_C^0$ if $h > h_0$ and $\delta_2(h) < \chi_f < \delta_0(h)$.

Proposition 3 proves that the presence of self-inspections under CP indeed benefits the regulator by lowering her disutility ($V_C^* < V_C^0$). However, they have mixed impacts on the two components of the disutility. As the proposition reveals, there are situations where self-inspections increase the regulator’s inspection costs ($I_C^r > I_C^0$; part (b)) or the environmental harm ($H_C^r > H_C^0$; part (c)). In other words, the regulator may have to spend more on her inspections or face greater environmental harm because of self-inspections.

These negative outcomes arise because of two drawbacks that self-inspections bring. First, the firm’s self-detections interfere with the regulator’s noncompliance detections. Although TF following a self-detection benefits the environment, it introduces efficiency loss for the regulator since preemptive compliance restoration reduces the chance of detection by the regulator. Proposition 3(b) describes a situation where such interference is so severe that the regulator overinvests in her inspections in order to compensate for lost efficiency; this case is depicted by region (B) of the $(h, \chi_f)$ map in Figure 1. This happens when the environmental consequence is small (small $h$), which leads the regulator to perform sporadic inspections that leave the firm with ample opportunities to preempt the regulator.

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7 In addition, the proposition identifies a necessary condition $\chi_f < \delta_0(h)$ for ensuring $\nu^*_f > 0$ in equilibrium. Numerical examples show that this condition is also a close approximation of the sufficient condition for $\nu^*_f > 0$ under reasonable parameter value combinations. A sufficient condition cannot be identified analytically because of the kinks present in the regulator’s objective function.
Impact of self-inspections:
(A) lower $I$, lower $H$
(B) higher $I$, lower $H$
(C) lower $I$, higher $H$

Light gray region: lower $V$

Figure 1  Comparison of performance measures $I_C$, $H_C$, and $V_C$ under CP, at the equilibrium with self-inspections vs. at the benchmark without self-inspections. The three regions (A), (B), and (C) each map to cases (a), (b), and (c) in Proposition 3. In this example, $\chi_r = 0.1$, $c = 1$, $K = 2$, $\alpha = 1$, and $\lambda = 1$.

Second, not only do self-inspections reduce efficiency of noncompliance detections by the regulator, but they can also reduce overall efficiency of detections. Reduction in overall efficiency of detections arises because a dollar invested in the regulator’s inspections returns a sublinear increase in the combined rate of inspections by the regulator and the firm. The next result identifies when this happens and what it entails.

Corollary 1 $H_C^* > H_C^0$ if and only if $\nu^*_r$ and $\nu^*_f$ are substitutes.

According to Corollary 1, a reduction in overall efficiency of detections leaves the environment worse off, and this happens in an equilibrium where a marginal increase in the regulator’s inspection rate reduces an incentive for the firm’s self-inspections. Such an outcome is observed especially when the environmental consequence is severe (large $h$), which leads the regulator to perform intense inspections to detect noncompliance on her own, prioritizing search over threat (see the discussions in §5.1). Proposition 3(c) describes this situation, depicted by Region (C) in Figure 1.

Interestingly, the two sources of inefficiency brought by self-inspections—interference and substitution—manifest themselves in isolation under CP when they become prominent. This is evident from Figure 1, where no overlap exists between region (B) and region (C). From this and the earlier observation that self-inspections always lower the regulator’s disutility under CP, we infer that CP is effective in containing inefficiencies. What enables such containment is repeated interactions between the regulator and the firm; because threat of penalty multiplies with repetition, in effect the regulator has an extra lever that allows her to extract the most benefits out of the firm’s self-inspections. With this insight, we now study the opposite scenario where repeated interactions are minimized, i.e., equilibrium under SP.
5.3 Equilibrium Under Stopping Policy

Under SP, the regulator orders compliance restoration via PF in the first instance she detects noncompliance, stopping further compliance failures and inspections. The firm, on the other hand, privately restores compliance via TF whenever he self-detects noncompliance. The analysis of the equilibrium under SP proceeds in a manner similar to that under CP. The regulator’s objective function that incorporates the firm’s best response is

\[ V_S(\nu, \kappa) = \begin{cases} \frac{X_f \nu - h + b \nu - c(\alpha + \nu)}{\alpha} \sqrt{\frac{X_f}{\xi(\nu, \kappa)}} & \text{if } \sigma_S < \frac{1}{2} \text{ and } \beta_S < \nu < \beta_S, \\ \text{otherwise}, \end{cases} \]

where

\[ \xi(\nu, \kappa) = \frac{X_f (\alpha + \lambda)}{\alpha^2} \nu^2 + \left( s + \kappa + \frac{(\chi_f - c)(\alpha + \lambda)}{\alpha} \right) \nu - c(\alpha + \lambda) \]

and \( \sigma_S, \beta_S, \) and \( \beta_S \) are defined in (4) and Lemma A2 in Appendix A. The first and second lines of (6) correspond to the firm’s best responses \( \nu_f = R_S(\nu, \kappa) > 0 \) and \( \nu_f = 0 \), respectively.

The subgame perfect equilibrium under SP, denoted by the superscript \( \dagger \), is found by identifying the values of \( \kappa \) and \( \nu_f \) that minimize \( V_S(\nu, \kappa) \). The analysis of the equilibrium under SP is substantially more challenging than that under CP because no closed-form solution exists. As such, in our discussions we complement analytical results with numerical observations. The equilibrium is specified as follows.\(^8\)

**Proposition 4** At the equilibrium under SP, the regulator sets either \( \kappa = 0 \) or \( \kappa = K \) along with \( \nu_f \) that uniquely solves the equation \( Q_S(\nu, \kappa^\dagger) = 0 \) to induce \( \nu_f^\dagger = -\frac{2 + \lambda}{\alpha} (\alpha + \nu_f^\dagger) + \frac{\sqrt{\lambda}}{\lambda f} \xi(\nu_f^\dagger, \kappa^\dagger) > 0 \), where

\[ Q_S(\nu, \kappa) = 2 \chi_f \xi(\nu^3)^{3/2} - \sqrt{X_f^2 \lambda \left[ (\frac{4 X_f \nu - 2 b)}{\alpha} \xi(\nu, \kappa) - \frac{X_f \nu^2 - b \nu - h}{\alpha} \frac{\partial \xi(\nu, \kappa)}{\partial \nu} \right]} \].

The regulator sets \( \kappa^\dagger = 0 \) if \( s + K \leq \theta \) while he sets \( \kappa^\dagger = K \) if \( \theta \leq s \), where \( \theta \equiv \left( c - \frac{x_f}{\chi_f} \right) \frac{a + \lambda}{\alpha} + \left( \frac{c(\alpha + \lambda)}{2h} + \frac{X_f}{2 X_f} \left( \frac{3a - \alpha}{\alpha} + \frac{8 \lambda}{\chi_f^2} \right) \right) \left( -b + \sqrt{b^2 + 4 \frac{X_f}{\chi_f}} \right) \).

Proposition 4 identifies a feature of the SP equilibrium that represents a fundamental departure from the CP equilibrium. Namely, the penalty constraint \( \kappa \leq K \) does not necessarily bind at the equilibrium under SP; while in some cases the regulator imposes the maximum penalty \( \kappa = K \), there are also cases where the regulator finds it optimal to impose no penalty \( \kappa = 0 \). From the proposition, we see that the no-penalty equilibrium arises if the firm has a weak incentive for self-inspections because of relatively small cost of PF (small \( s \)) and limited maximum on penalty (small \( K \)). Although one might intuit that it is precisely this weak-incentive situation that justifies imposing the largest possible penalty, Proposition 4 states the opposite; the regulator is better off

\(^8\)Proof of Proposition 4 assumes an extra condition \( h > \chi_f \frac{(\alpha + h)^2}{\alpha} \left( \frac{c}{x - c(\alpha + h)^2} \right) \), which is used to enable analytical tractability. Although this condition does not follow directly from the conditions in Assumption 1, it is satisfied by most reasonable parameter values, including all values used in the numerical examples considered in this paper.
by minimizing the penalty to weaken the incentive even further. This suggests that self-inspections are not as valuable under SP as under CP. This hypothesis is supported by the next result.

**Proposition 5** At the SP equilibrium described in Proposition 4, \( \frac{dV^\dagger_S}{d\chi_f} < 0 \) if \( s + K \leq \theta \).

According to Proposition 5, more efficient self-inspections (smaller \( \chi_f \)) under SP may hurt the regulator by raising her disutility \( V^\dagger_S \). In general, \( V^\dagger_S \) changes non-monotonically in \( \chi_f \); this is demonstrated by an example in Figure 2.

More importantly, as illustrated in the figure (thick black curve vs. thick horizontal gray line), the regulator may be worse off with self-inspections than without them: \( V^\dagger_S > V^0_S \). Hence, there are situations under SP where no self-inspection is better than some self-inspections. This contrasts with CP, under which such a reversal never occurs.

These observations suggest that self-inspections create a new dynamic when they interact with the regulator’s PF order. To understand this, consider the effects of PF. In comparison to TF, PF is a more decisive enforcement lever for the regulator since it eliminates future compliance failures and the need for further inspections. In addition, a threat of PF provides the firm with a high-powered incentive for preemptive TF because the firm wishes to avoid the large cost of PF. Hence, a PF order presents an advantage over a TF order not only for its post-restoration harm prevention but also for its pre-restoration incentive for self-detections. This advantage indeed benefits the environment and the regulator in many instances. However, a PF order also comes with a caveat that it can be used only once. This all-or-nothing nature of PF can be detrimental because it creates a new form of inefficiency by amplifying the interference effect of self-inspections: stopping
Impact of self-inspections:
(A) lower $I$, lower $H$
(B) higher $I$, lower $H$
(C) lower $I$, higher $H$
(D) higher $I$, higher $H$

Figure 3    Comparisons of performance measures under SP, at the equilibrium with self-inspections vs. at the benchmark without self-inspection. In these examples, $\chi_r = 0.1$, $c = 1$, $K = 2$, $s = 9$, $\alpha = 0.5$, and $\lambda = 1$.

time delay. That is, once preempted by the firm’s TF, the regulator has to wait until the next compliance failure to occur before getting another opportunity to order PF. Such a delay prolongs the duration of noncompliance, potentially hurting both the environment and the regulator.

From this discussion, we see that self-inspections introduce a new tradeoff under SP. On one hand, preemptive TF following a self-detection shortens the duration of each noncompliance occurrence. On the other hand, preemptive TF interferes with the regulator’s ability to detect noncompliance and causes delay, increasing the number of noncompliance occurrences before PF is finally applied. Since the environmental harm is proportional to the cumulative duration of noncompliance—duration of each noncompliance occurrence times the number of occurrences—the former effect reduces the environmental harm while the latter effect increases the harm. Proposition 4 and Proposition 5 reveal that the latter effect can dominate the former effect, and as a result, the regulator and the environment may be worse off due to the firm’s self-inspections. (This tradeoff is further verified by numerical examples that show shortened duration of each noncompliance occurrence and more occurrences of noncompliance as $\chi_f$ becomes smaller, i.e., as the firm performs self-inspections more efficiently.)

Figure 3 shows the regions in the $(h, \chi_f)$ space where self-inspections under SP increase or decrease the regulator’s inspection costs and environmental harm for two different values of consumer welfare loss $b$.

The patterns displayed in Figure 3 are distinct from the pattern in Figure 1 under CP mainly because the new tradeoff under SP gives rise to additional cases that do not exist under CP. In particular, one may encounter a case with simultaneous increases in inspection costs and environmental harm ($I^1_S > I^0_S$ and $H^1_S > H^0_S$) under SP. Dark gray regions represent the instances where the regulator is worse off due to self-inspections ($V^1_S > V^0_S$). As we can infer from the figure, self-inspections hurt the regulator when the firm is inefficient at self-inspections (relatively large $\chi_f$),
Continuation Policy preferred
Stopping Policy preferred

In the presence of self-inspections, $V_C^* = V_S^\dagger$ at the boundary between the shaded and non-shaded regions. In the absence of self-inspections, $V_C^0 = V_S^0$ at the horizontal line. Arrows indicate the boundary shift due to self-inspections. In this example, $\chi_f = 0.1$, $c = 1$, $K = 2$, $s = 0$, $h = 10$, $\alpha = 0.5$, and $\lambda = 1$.

In summary, we identify a tradeoff that exists under SP which requires a careful assessment of the value of self-inspections by a firm. Although the regulator may prefer SP because of the advantage that PF brings, namely permanent harm prevention, self-inspections may diminish the value of PF. This is in contrast to CP, under which the regulator always benefits from self-inspections.

5.4 Comparison of Equilibria

We now compare SP with CP by examining the conditions under which one policy is preferred to the other by the regulator, i.e., $V_C^* < V_S^\dagger$ or $V_C^* > V_S^\dagger$. Intuitively, SP is preferred if the regulator assigns priority to prevention of environmental harm; by requiring PF at the earliest opportunity, the regulator can minimize the harm. Conversely, CP is preferred if the regulator is concerned more about protecting consumer welfare; consumers incur a loss after PF, so it is to be avoided.

An example in Figure 4 illustrates these preferences. (It is verified that the features in Figure 4 are replicated by other numerical examples.) In the figure, the $(\chi_f, b)$ space is divided into two regions in which either of the two policies is preferred by the regulator in the presence of self-inspections ($V_C^* > V_S^\dagger$ in the shaded region and $V_C^* < V_S^\dagger$ in the non-shaded region). The horizontal line represents the boundary between preferred benchmark policies, i.e., CP vs. SP when no self-inspection exists ($V_C^0 > V_S^0$ below the horizontal line and $V_C^0 < V_S^0$ above the line). As indicated by arrows, an addition of self-inspections shifts the boundary such that the area in which SP is preferred expands to the right and decreases for CP.
preferred is expanded. This implies that self-inspections enhance the value of SP relative to that of CP. In particular, this preference change happens when $b$ is in an intermediate range, i.e., when the regulator does not strongly favor either CP or SP.

To understand the reason behind this preference change, suppose that the regulator considers switching from CP to SP. By switching to SP, the regulator can potentially enjoy further reduction in the environmental harm, achieved through either the regulator’s PF order or the firm’s frequent preemptive TFs, the latter due to the high-powered incentive from the threat of PF. At the same time, switching to SP exposes the regulator to the risk of incurring consumer welfare loss, which follows immediately after PF is applied. Although this downside presents a tradeoff, its effect is minimized if harm reduction is achieved mostly through the firm’s preemptive TFs rather than the regulator’s PF. This is because preemptive TFs delay the timing of PF, and with the delay, the risk of incurring consumer welfare loss is mitigated. Hence, the firm’s self-inspections provide a protection against the risk of consumer welfare loss.

Therefore, switching from CP to SP may offer further reduction in the environmental harm without requiring a substantial increase in the risk of consumer welfare loss. Such a desirable outcome arises only if harm reduction is done mostly through preemptive TFs by the firm, while PF by the regulator is significantly delayed. This is precisely when the regulator’s inspections are used primarily as a threat, not for search. This intuition is confirmed by the example in Figure 4, which shows that switching happens more readily when $\chi_f$ is small, i.e., when the firm is efficient at self-inspections and therefore is responsive to the threat of PF.

Based on these observations, we conclude that self-inspections enhance the value of SP when a PF order, despite it being the distinguishing feature of SP, cannot be easily carried out due to interference. In such instances, the regulator prefers mandating PF (by employing SP) despite having a small chance of enforcing it. This seemingly paradoxical situation arises when the regulator attempts to achieve a balance between environmental harm reduction and consumer welfare protection by using the interference from self-inspections to her advantage.

6. Equilibrium Under General Policy
Our discussions thus far have been centered around CP and SP partly because they represent two opposite extremes that allow us to highlight contrasting results. We now turn our attention to the general enforcement policy, under which the regulator orders PF upon her $N$th noncompliance detection after allowing the firm to restore compliance via TF upon each of all previous detections. Recall that CP and SP are special cases of this policy with $N = \infty$ and $N = 1$, respectively. It was proved in Proposition 1 that CP and SP are the only two optimal outcomes in the benchmark case where no self-inspections exist. As we demonstrate below, the same is not true at the equilibrium
The firm solves a continuous-time stochastic dynamic program similar to the one presented in §4 for the benchmark case, determining the optimal value of his disutility $\Psi$ for the $n$th enforcement cycle in the backward order, starting with $n = N$. The structure of the optimal solution is similar to that of the best response under SP given in Lemma 2 and Lemma A2. The only difference is that the best response function $R_S(\nu, \kappa)$ is replaced by a sequence of best response functions $R(\nu_1, \kappa, c + \Psi^2), \ldots, R(\nu_N, \kappa, s)$ for each of the $N$ cycles, where $R(\nu_1, \kappa, c + \Psi^1)$ is identical to $R_S(\nu, \kappa)$ with terminal cost $s$ replaced by the continuation cost $c + \Psi^{n+1}$ for $n = 1, \ldots, N - 1$. After completing this step, we substitute the best responses into the regulator’s disutilities $V^n, \ldots, V^N$ to obtain the recursive formula

$$V^n = \frac{\chi_r \nu^r \alpha + \lambda + \nu^r + R(\nu_1, \kappa, c + \Psi^1) + (h + V^{n+1}) \lambda}{(\alpha + \lambda)(\alpha + \nu^r) + \alpha R(\nu_1, \kappa, c + \Psi^{n+1})},$$

which resembles (1) except for the addition of the firm’s best responses.

Given fixed $N$, we compute the optimal values of $(\kappa^1, \ldots, \kappa^N)$ and $(\nu_1^1, \ldots, \nu_N^N)$ by solving the problem $\min_{(\nu, \kappa)} V^n$ recursively, thereby determining the equilibrium. The results mirror that of Proposition 4 for SP; in equilibrium, the regulator sets the penalty to either $\kappa^n = 0$ or $\kappa^n = K$ and chooses her inspection rate $\nu^n$, which is specified as an implicit solution to an equation. Finally, we numerically search for the optimal $N$ that minimizes the regulator’s disutility at time zero. From the examples based on 1000+ parameter value combinations, we observe the following patterns:

(a) Optimal $N$ may assume any value between $N = 1$ and $N = \infty$, with $1 < N < \infty$ observed for small $\chi_f$ and intermediate $b$.

(b) If $N > 1$, the sequence of self-inspection rates $\{\nu_j^N\}_{j=1}^N$ tend to increase in $n$, i.e., $\nu_1^1 \leq \nu_2^1 \leq \ldots \leq \nu_N^N$. (Small fluctuations may exist for small $n$, but a sharp increasing trend is observed for large $n$.)

(c) If $N > 1$, the sequence of penalties $\{\kappa^n\}_{n=1}^N$ satisfies one of the following three outcomes:

(i) $\kappa^1 = \ldots = \kappa^N = 0$; (ii) $\kappa^1 = \ldots = \kappa^N = K$; (iii) $\kappa^1 = \ldots = \kappa^n = 0$ and $\kappa^{n+1} = \ldots = \kappa^N = K$

for some $n$ satisfying $1 < n < \infty$.

Under the general policy, the regulator can adjust $N$ to balance the environmental harm against consumer welfare loss. What we found out in the benchmark case, however, was that such an adjustment always leads to the extreme cases $N = 1$ or $N = \infty$ when no self-inspections exist, due

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9 The proofs of the preceding results are omitted but are available upon request.
to time symmetry built into the problem. That $1 < N < \infty$ may emerge in equilibrium, as stated in (a) above, suggests that an introduction of self-inspections breaks time symmetry. Indeed, this is corroborated by the result in (b), which shows that the firm tends to increase the rate of his self-inspections as he approaches the final enforcement cycle $N$. This is caused by the deadline effect; as time passes by and the final cycle nears, the firm attempts to delay termination by performing more frequent self-inspections in order to preempt the regulator’s detection. Such a reaction by the firm cannot be avoided as long as $N$ is committed upfront, and as a result, time symmetry no longer exists.

In addition, we see from (a) that $1 < N < \infty$ happens when $\chi_f$ is small and $b$ is in an intermediate range; recall from our discussions in §5.4 that this is exactly the parameter combinations under which the regulator utilizes self-inspections to achieve both environmental harm reduction and consumer welfare protection. Such a balance can be further fine-tuned by choosing $N$ in between the two extremes $N = 1$ and $N = \infty$. Finally, the result in (c) above shows that the regulator encourages more self-inspections, not less, as the final enforcement cycle approaches, since the penalty $\kappa$ never goes down towards the end. This implies that the regulator increases her reliance on self-inspections as time passes by, hoping to delay PF as she becomes more conscious about the looming chance of incurring consumer welfare loss.

Our numerical observations indicate that the key features of the equilibrium under the general policy are qualitatively similar to the ones that we found under SP, differing only by the degrees. (For example, we observe non-monotonic changes in the regulator’s disutility with respect to $\chi_f$, just as we did under SP.) Therefore, the qualitative insights from the last section continue to hold under more general conditions.

7. Conclusions
Motivated by the practice in environmental compliance enforcements, in this paper we conduct an in-depth analysis of the role of self-inspections, i.e., firms’ voluntary efforts to discover compliance violations that occur in their productions. We develop a model that features dynamic interactions between a regulator and a firm, who engage in a game of repeated inspections to discover random violations and apply a corrective measure. Our main focus is answering the question: “Do the regulator and the environment benefit from self-inspections?” As our analysis reveals, the answer depends on an enforcement policy employed by the regulator as well as the factors such as the degree of environmental harm and the cost of performing an inspection.

We find that inspection rate choices by the regulator and the firm exhibit a nontrivial relationship: they may be complements or substitutes. This reflects the dual role of the regulator’s inspections, as they are used not only for detecting noncompliance but also for providing the firm
with an incentive to engage in self-inspections. This “search vs. threat” dual role is key to understanding the equilibrium outcomes under the two enforcement policies we focus on, Continuation Policy (CP) and Stopping Policy (SP).

Under CP, the firm is allowed to restore compliance via temporary fix (TF) whenever noncompliance is detected by the regulator. Under this policy, the risk of a compliance failure persists after each restoration. We find that the firm’s self-inspections always bring a net benefit to the regulator under CP. However, self-inspections may raise the regulator’s inspection costs by hindering her ability to detect noncompliance (interference) or worsen the environmental harm by reducing the overall efficiency of detections (substitution). The effect of interference is amplified if the regulator employs SP, under which the firm is required to restore compliance via permanent fix (PF) and eliminate future compliance failures once noncompliance is detected by the regulator. Under this policy, self-inspections can be detrimental to the regulator. This happens because interference caused by self-inspections manifests itself as a new form of inefficiency under SP, namely a time delay in applying PF. Such a delay prolongs the duration of noncompliance, potentially hurting both the environment and the regulator.

By examining the conditions under which either CP or SP is preferred by the regulator, we find that self-inspections enhance the value of SP relative to that of CP. Interestingly, self-inspections give rise to a situation where the regulator prefers mandating PF (by employing SP) despite having a small chance of enforcing it. This happens when the regulator attempts to achieve a balance between environmental harm reduction and consumer welfare protection by using the interference from self-inspections to her advantage.

Finally, we consider the general “N strikes and you are out” enforcement policy, of which CP and SP are special cases ($N = \infty$ and $N = 1$). Under this policy the regulator keeps ordering TF until the $N^{th}$ noncompliance detection, when PF is triggered. It is numerically observed that the optimal policy may not be either CP or SP when self-inspections are present. By contrast, only CP or SP is optimal when there is no self-inspection. We find that the key insights obtained from the analyses of CP and SP continue to hold under the general policy.

Our findings suggest that self-inspections by the firm have nuanced implications to environmental compliance enforcements. As such, it is important to recognize the conditions under which self-inspections are beneficial or harmful to the environment and the regulator. The analysis presented in this paper points to such conditions and offers insights into the causes and remedies of potential inefficiencies. In addition, the model developed in this paper can be used as a basis for inferring the impact of private self-inspections on the environment, which may not be directly observable. Such inferences will provide a valuable tool for designing an effective enforcement policy.
References


**Appendix**

**A. Supplemental Results**

**Corollary A1** Without the firm’s self-inspections, $I_C^0 = \frac{-\chi r (\alpha + \lambda) + \sqrt{\pi \chi r - \sigma}}{\alpha}$ under CP and $I_S^0 = \frac{\chi r (\lambda - \alpha)}{\alpha + \lambda} + \frac{h - (b + 2 \chi r) \alpha}{\alpha + \lambda} \sqrt{\frac{\chi r \lambda}{h - (b + \chi r) \alpha}}$, $H_C^0 = \frac{\sqrt{\pi \chi r - \sigma}}{\alpha}$, and $V_C^0 = \frac{-\chi r (\alpha + \lambda) + \sqrt{\pi \chi r - \sigma}}{\alpha}$ under SP. Setting $I_C^0 = \frac{\chi r (\lambda - \alpha)}{\alpha + \lambda} + \frac{h - (b + 2 \chi r) \alpha}{\alpha + \lambda} \sqrt{\frac{\chi r \lambda}{h - (b + \chi r) \alpha}}$, $H_C^0 = \frac{\sqrt{\pi \chi r - \sigma}}{\alpha}$, and $V_C^0 = \frac{-\chi r (\alpha + \lambda) + \sqrt{\pi \chi r - \sigma}}{\alpha}$ under SP.

**Proof** Under CP ($N = \infty$), the regulator’s disutility at time zero $V_C = V_C^1$ is obtained by setting $V_C^0 = V_C$ in (1) for all $n \geq 1$ and rearranging the resulting equation, which yields $V_C = \frac{\chi r}{\alpha} \nu + \frac{b \lambda}{\alpha (\alpha + \lambda + \nu r)}$. From this expression we get $I_C = \frac{\chi r}{\alpha} \nu + \frac{b \lambda}{\alpha (\alpha + \lambda + \nu r)}$ and $H_C = \frac{b \lambda}{\alpha (\alpha + \lambda + \nu r)}$. Substituting the optimal inspection rate $\nu = \alpha + \lambda + \frac{\sqrt{\pi \chi r - \sigma}}{\alpha}$ obtained in Proposition 1 in $I_C$, $H_C$, and $V_C$ yields $I_C^0$, $H_C^0$, and $V_C^0$ found in the corollary. Under SP ($N = 1$), setting $V_C^2 = b$ in (1) yields $V_C = V_C^1 = \frac{\chi r \nu (\alpha + \lambda + \nu r)}{\alpha + \lambda} + \frac{h - b \nu r \lambda}{\alpha (\alpha + \lambda + \nu r)}$. From this expression we get $I_S = \frac{\chi r \nu (\alpha + \lambda + \nu r)}{\alpha + \lambda}$, $H_S = \frac{b \lambda}{(\alpha + \lambda) (\alpha + \nu r)}$, and $B_S = \frac{b \nu r (\alpha + \lambda) + \nu b \lambda}{(\alpha + \lambda) (\alpha + \nu r)}$. Substituting the optimal inspection rate $\nu = \alpha + \lambda + \frac{\sqrt{\pi \chi r - \sigma}}{\alpha}$ obtained in Proposition 1 in $I_S$, $H_S$, $B_S$, and $V_S$ yields $I_S^0$, $H_S^0$, $B_S^0$, and $V_S^0$ found in the corollary.

**Lemma A1** Given $\in [0, K]$ and $\nu_r > 0$, the firm under CP sets $\nu_f = \max \{0, R_C (\nu_r, \kappa)\}$ as described in Lemma 2 with $\beta_c = \alpha + \lambda + \frac{\kappa \lambda}{2 \chi_f} (1 - \sqrt{1 - 4 \sigma C})$, $\beta^\circ = \frac{c (\alpha + \lambda)}{\kappa}$, $\bar{\beta} = \frac{c (\alpha + \lambda)}{\kappa}$, and $\beta^{\circ} = \frac{c (\alpha + \lambda)}{\kappa} + \frac{\kappa \lambda}{4 \chi_f}$. Suppose $\sigma = \frac{\chi_f (c (\alpha + \lambda))}{\alpha + \lambda} < \frac{1}{4}$. Then $\beta$ and $\bar{\beta}$ defined in the lemma satisfy the following: $\beta < \beta^c < \beta^\circ < \beta^{\circ}$. The firm’s expected disutility $\psi (\nu_f) \equiv \Psi_C$ specified in (2) satisfies $\psi' (\nu_f) = \frac{\chi_f}{\alpha} - \frac{\nu_r - \kappa (\alpha + \lambda)}{\alpha (\alpha + \lambda + \nu_r \nu_f)}$ and $\psi'' (\nu_f) = \frac{2 \nu_r (\alpha + \lambda) \lambda}{(\alpha + \lambda + \nu_r \nu_f)^2}$ with $\psi' (0) = \frac{\chi_f}{\alpha} - \frac{\nu_r - \kappa (\alpha + \lambda)}{\alpha (\alpha + \lambda + \nu_r \nu_f)^2}$ and
\[ \psi'(\infty) = \frac{\gamma}{\alpha} > 0. \] Let \( \tilde{\nu}_f \) be the solution to the first-order condition \( \psi'(\nu_f) = 0 \). Substituting \( \tilde{\nu}_f \) in \( \psi''(\nu_f) \) yields \( \psi''(\tilde{\nu}_f) = \frac{2\gamma}{\alpha(\alpha + \lambda + \nu_f)} > 0 \), which implies that \( \psi(\nu_f) \) is quasiconvex. Hence, the minimizer of \( \psi(\nu_f) \) is \( \max\{0, R(\nu_f)\} \). Substituting \( \tilde{\nu}_f \) in \( R(\nu_f) \) yields \( R(\nu_f) \equiv \tilde{\nu}_f = -\frac{\sqrt{\hat{\kappa}}}{\chi_f}(\kappa \nu_f - c(\alpha + \lambda)) \). Note \( R(\nu_f) \) is maximized at \( \nu_f = \beta^o \), with \( R(\beta) = R(\tilde{\beta}) = -\frac{(c + \kappa)(\alpha + \lambda)}{\kappa} < 0 \) and \( R(\beta^o) = \frac{\alpha}{\chi_f} - \frac{(c + \kappa)(\alpha + \lambda)}{\kappa} = \frac{\alpha}{\chi_f}\left(\frac{1}{4} - \sigma\right) > 0 \). Moreover, \( R'(\nu_f) = -1 + \frac{2\sqrt{\gamma}}{(\chi_f/\lambda)(\kappa \nu_f - c(\alpha + \lambda))} \) and \( R''(\nu_f) < 0 \). Together, these results imply that \( R(\nu_f) \) is a concave function that starts and ends with negative numbers at \( \nu_f = \beta^o \) and \( \nu_f = \tilde{\beta}^o \), crossing zero twice, once from below and the other from above. Therefore, \( R(\nu_f) > 0 \) if and only if \( \beta < \nu_f < \tilde{\beta}^o \), where \( \beta \) and \( \tilde{\beta} \) are solutions to the equation \( R(\nu_f) = 0 \).

Substituting \( \nu_f = R(\nu_f) \) in \( \psi(\nu_f) \) yields the firm’s expected disutilities evaluated at the optimum as \[ \Psi_C(\nu_f) = \frac{c\lambda - f(\alpha + \lambda + \nu_f) + 2\sqrt{f(\lambda + (\kappa \nu_f - c(\alpha + \lambda))}}}{\alpha} \] for \( \beta < \nu_f < \tilde{\beta}^o \) and \( \Psi_C(\nu_f) = \frac{(c + \kappa)(\lambda \nu_f)}{\alpha(\alpha + \lambda + \nu_f)} \) otherwise.

**Lemma A2** Given \( \in [0, K] \) and \( \nu_f > 0 \), the firm under SP sets \( \nu_f = \max\{0, R_S(\nu, \kappa)\} \) as described in Lemma 2 with \( \beta^o = -\alpha + \frac{[(c + \kappa)(\alpha + \lambda) - f(\alpha + c)\gamma]}{2\chi_f} (1 - \sqrt{1 - 4\sigma}) \), \( \beta^o = \gamma \left(1 + \sqrt{1 - 4\sigma} - \frac{\alpha + \lambda}{\chi_f} \right) \), \( \tilde{\beta}^o = -\alpha + \frac{[(c + \kappa)(\alpha + \lambda) - f(\alpha + c)\gamma]}{2\chi_f} (1 + \sqrt{1 - 4\sigma}) \), and \( R_S(\nu, \kappa) = -\frac{(\alpha + \lambda)(\nu_f + \nu_f)}{\alpha^2} + \frac{\alpha}{\chi_f} + \frac{\alpha}{\chi_f} \) for \( \beta^o < \nu_f < \tilde{\beta}^o \) and \( \Psi_C(\nu_f) = \frac{(c + \kappa)(\lambda \nu_f)}{\alpha(\alpha + \lambda + \nu_f)} \) otherwise.

**Proof** Suppress the subscript \( S \) and let \( \pi \equiv \frac{(c + \kappa)(\alpha + \lambda) - f(\alpha + c)\gamma}{\alpha^2} \) and \( \chi_f < \frac{\gamma}{\kappa(\alpha + \lambda)} \) < \( K \) in Assumption 1. Then \( \beta \) and \( \tilde{\beta} \) defined in the lemma can be written as \( \beta \equiv -\alpha + \frac{\frac{\chi_f}{c + K}(\alpha + \lambda)}{2\chi_f} (1 - \sqrt{1 - 4\sigma}) \) and \( \tilde{\beta} \equiv -\alpha + \frac{\chi_f}{2\chi_f} (1 + \sqrt{1 - 4\sigma}) \). Let \( \beta \equiv -\left(\frac{\pi}{2\chi_f} + 1\right) \alpha + \alpha \sqrt{\left(\frac{\pi}{2\chi_f} + 1\right)^2 + \frac{\chi_f}{\pi}} \), \( \beta^o \equiv -\left(\frac{\pi}{2\chi_f} + 1\right) \alpha + \sqrt{\alpha(\alpha + \lambda)\left(\left(\frac{\pi}{2\chi_f} + 1\right)^2 + \frac{\chi_f}{\pi}\right)} \), \( \tilde{\beta} \equiv -\alpha + \frac{\chi_f}{2\chi_f} \), and \( \tilde{\beta} \equiv -\alpha + \frac{\chi_f}{2\chi_f} \). Suppose \( \sigma \equiv \frac{\chi_f(\chi_f + c + \pi \alpha)}{\pi^2} \frac{\alpha}{\chi_f} < \frac{1}{4} \). Then it can be shown that \( \beta < \beta^o < \beta < \tilde{\beta} < \tilde{\beta}^o \). The firm’s expected disutility \( \psi(\nu_f) \equiv \Psi_S \) specified in (2) satisfies the following: \( \psi'(\nu_f) = \frac{\chi_f(\alpha + \lambda + \nu_f)(\alpha + \lambda + \nu_f)}{(\alpha + \lambda)(\alpha + \lambda + \nu_f)} \) of with \( \psi'(0) = \frac{c \psi(0)}{(\alpha + \lambda)(\alpha + \lambda + \nu_f)} > 0 \) and \( \psi'(\infty) = \frac{\chi_f}{\alpha} > 0 \), where \( \zeta(\nu_f) \equiv \left(\frac{\chi_f(\alpha + \lambda + \nu_f)}{(\alpha + \lambda)(\alpha + \lambda + \nu_f)} - \pi \alpha \right) > 0 \). Substituting this condition in \( \psi'(\nu_f) \), we see that \( \psi'(\nu_f) > 0 \) for all \( \nu_f > 0 \). Hence, \( \psi(\nu_f) \) is minimized at \( \nu_f = 0 \) in this case. Suppose \( \nu_f = 0 \). Substituting this condition in \( \psi''(\nu_f) \), we see that \( \psi''(\nu_f) > 0 \) for all \( \nu_f > 0 \); concavity of \( \psi(\nu_f) \) together with \( \psi'(0) < 0 \) and \( \psi'(\infty) > 0 \) implies that \( \psi(\nu_f) \) is minimized at a unique interior point in this case. Thus, the firm optimally sets \( \nu_f > 0 \) if and only if \( \psi'(0) < 0 \) or equivalently \( \zeta(\nu_f) < 0 \). Since \( \zeta(\nu_f) \) is a convex function with \( \zeta(0) > 0 \) and \( \zeta(\infty) > 0 \), it satisfies \( \zeta(\nu_f) < 0 \) if and only if \( \zeta(\nu_f) < 0 \) and \( \beta < \nu_f < \tilde{\beta} \), where \( \beta \) and \( \tilde{\beta} \) are the two solutions to the quadratic equation \( \zeta(\nu_f) = 0 \). Since \( \zeta(\tilde{\beta}) = \frac{\chi_f^2}{\chi_f} (\pi - \frac{1}{4}) > 0 \), \( \zeta(\nu_f) < 0 \) for \( \nu_f < \tilde{\beta} \). Hence, \( \psi(\nu_f) \) is minimized at \( \nu_f > 0 \) if and only if \( \beta < \nu_f < \tilde{\beta} \); otherwise,
ψ(νf) is minimized at νf = 0. The positive minimizer is identified by the first-order condition ψ′(νf) = 0, which can be written as νf = R(νr) ≡ −(α + λ)(1 + νr/α) + \sqrt{\lambda/\xi(νr)} > 0 where ξ(νr) ≡ (α + λ)[(π + χf(2 + \pi r/α)√r)]\pi r - c]. Note ξ(νr) is a convex function that starts from a negative number at νr = 0 and approaches ∞ as νr → ∞, crossing zero exactly once at νr = β from below and ξ′(νr) > 0 for all νr > β for which ξ(νr) > 0. Since R′(νr) = −\frac{α+λ}{α} + \frac{ξ(νr)}{2} \sqrt{\frac{\lambda}{ξ(νr)}}, these imply limνr→β R(νr) < 0, limνr→∞ R(νr) = limνr→∞ −\frac{(α+λ)+\sqrt{λ(α+λ)}}{α} νr < 0, limνr→β R′(νr) = ∞ > 0, and limνr→∞ R′(νr) = −\frac{(α+λ)+\sqrt{λ(α+λ)}}{α} < 0. Moreover, by setting R′(νr) = 0 we see that R(νr) peaks at a unique value νr = β∗, where R(β∗) = \frac{πλ(α+λ)}{2χfα} \left(−1 + \sqrt{1 + \frac{α}{α+λ}(1−4σ)}\right) > 0. Together, these results imply that R′(νr) > 0 for νr ∈ (β, β∗) and R′(νr) < 0 for νr ∈ (β∗, β). Substituting νf = R(νr) and νf = 0 in ψ(νf) yields the firm’s expected utilities evaluated at the optimum as ΨS(νr) = \frac{c−χf[α+λ+(1+2λ)/α]νr}{α} + 2s√χfλξ(νr)} for β < νr < β and ΨS0(νr) = \frac{(χf+c+λ)λνr}{α(α+νr)} otherwise.

Lemma A3 Under Assumption 1, σS < \frac{1}{2} for all κ ∈ [0, K].

Proof. Recall σS = \frac{χf(νr)α+λ}{(s−(χf+α)(α+λ)/α)^2} from (4). Since σS decreases in κ, σS < \frac{1}{2} if \frac{χf(νr)α+λ}{(s−(χf+α)(α+λ)/α)^2} < \frac{1}{4}, which can be rewritten as χf < w1(s) ≡ (\frac{\sqrt{α}(α+λ)−\sqrt{α}λ}{λ}) from (4). Moreover, conditions s > (c + K)\frac{α+λ}{α} and χf < \frac{K^2λ}{4(c+K)(α+λ)} from Assumption 1 together imply χf < w2(s) ≡ \frac{8α−c(α+λ)}{4σα(α+λ)^2}. Thus, σS < \frac{1}{4} if w2(s) < w1(s), which we now prove. Both w1(s) and w2(s) are defined for s > s ≡ c\frac{α+λ}{α}, and they do not intersect in the defined region. (Letting w1(s) = w2(s) yields three solutions, s = s, s = s, and s = s, none of which satisfy s > s.) Additionally, they have the following properties: (i) lims→∞ w1(s) = lims→∞ w2(s) = 0; (ii) lims→∞ w′1(s) = lims→∞ w′2(s) = 0; (iii) lims→∞ w′1(s) = \frac{α(\sqrt{α}−\sqrt{α}λ)}{λ(α+λ)} > 0; (iv) lims→∞ w′2(s) = \frac{α(\sqrt{α}−\sqrt{α}λ)}{λ(α+λ)} > 0; (v) w′1(s) > 0; (vi) w′2(s) > 0. Let λ(s) ≡ \frac{α(\sqrt{α}+\sqrt{α}λ)}{λ^2 α^2}. This function satisfies limλ→0 λ(λ) = \frac{1}{2}, limλ→∞ λ(λ) = 1, and λ′(λ) > 0, which together imply λ(λ) > \frac{1}{2}, which in turn implies lims→∞ w′1(s) > lims→∞ w′2(s). Since w1(s) and w2(s) are both convex functions with slope zero at s = s, they are increasing in s > s. Moreover, since w1(s) and w2(s) increase without intersecting one another with lims→∞ w1(s) = lims→∞ w2(s) = 0 and lims→∞ w′1(s) > lims→∞ w′2(s), it follows that w1(s) > w2(s) for all s > s, as we set out to prove.

Lemma A4 Under Assumption 1, the firm prefers CP to SP.

Proof. Suppose σC < \frac{1}{2} and σS < \frac{1}{2}, which, according to Lemma 2, ensure that the firm sets νf > 0 for some νr > 0. Assume that penalties under CP and SP are κc ≤ K and κs ≤ K. Comparing the firm’s expected utilities ΨC and ΨS from (2), we find ΨC < ΨS if ΨC < s − c + κs − κc. Evaluating ΨC at the optimum νf = R_C(νr) > 0 given in Lemma 2, which is defined for
\( \nu_r \in (\beta, \overline{\beta}) \) where \( \beta_C^o = \frac{c(a+\lambda)}{\kappa c} + \frac{\kappa \alpha}{4x_f}, \) and \( \overline{\beta}_C = - (\alpha + \lambda) + \frac{\kappa \alpha}{2x_f} (1 + \sqrt{1 - 4\sigma C}), \) yields \( \Psi_C(\nu_r) = \frac{c(a+\lambda)}{\kappa c} \frac{x_f}{\alpha} + \frac{c(a+\lambda)}{\kappa c} + \frac{\kappa \alpha}{x_f} \) since \( \frac{c(a+\lambda)}{\kappa c} \). It is straightforward to prove that \( \Psi'_C(\nu_r) > 0 \) if \( \beta < \beta < \nu_r < \overline{\beta} < \beta \), where \( \beta \equiv \frac{c(a+\lambda)}{\kappa c} \) and \( \overline{\beta} \equiv \frac{c(a+\lambda)}{\kappa c} + \frac{\kappa \alpha}{x_f} \). Hence, \( \Psi_C(\nu_r) < \Psi_C(\overline{\beta}) \) for all \( \nu_r \in (\beta, \overline{\beta}) \) in which the firm sets \( \nu_r > 0 \). Because of the condition \( s > (c+K) \frac{\alpha}{\kappa} \) from Assumption 1, we have \( \Psi_C(\overline{\beta}) = \frac{(c+\kappa)(\kappa x_f c a+\lambda)}{\alpha x_f} \frac{\alpha}{\kappa} \leq \frac{(c+K)\beta}{\alpha+\lambda} < s - c - K \leq s - c + \kappa x - \kappa c, \) which establishes \( \Psi_C(\nu_r) < s - c + \kappa x - \kappa c \) for all \( \nu_r \in (\beta, \overline{\beta}) \). This implies \( \Psi_C < \Psi_S \) at the optimum, i.e.,

\[ \text{the firm prefers CP to SP}. \]

**B. Proofs**

**Proof (Proposition 1)** Let \( \phi_1 \equiv \frac{-\chi_r(a+\lambda)+2\sqrt{\lambda \chi_r}}{\lambda} \) and \( \phi_2 \equiv \frac{-\chi_r(a+\lambda)}{\lambda}. \) Under the conditions \( h > \frac{\chi_r(a+\lambda)}{\lambda} \) and \( b < \frac{\chi_r(a+\lambda)}{\lambda} \) in Assumption 1, \( \phi_1 < \phi_2 \) and \( b < \phi_2 \); hence, either \( b < \phi_1 \) or \( \phi_1 < b < \phi_2 \). Moreover, the function \( \Pi(y) \equiv \frac{\chi_r(a+\lambda) y^{1+2\sqrt{\lambda \chi_r}}}{\alpha+a+\lambda} \) is concave increasing-decreasing with a peak occurring at \( y = \phi_2 \), satisfying \( \Pi(0) > 0 \), \( \Pi(\phi_1) = \phi_1 \) and \( \Pi(\phi_2) = \frac{h\alpha a+\lambda}{\alpha+\lambda} < \phi_2 \), where the last inequality follows from \( h > \frac{\chi_r(a+\lambda)}{\lambda} \). It then follows that \( \Pi'(y) > 0 \) if \( y < \phi_2 \) with \( \Pi(y) > y \) for \( y < \phi_1 \), \( \Pi(\phi_1) = \phi_1 \), and \( \Pi(y) < y \) for \( y > \phi_1 \).

Fix \( N \geq 1 \) and rewrite (1) as \( V_n^N = \frac{V_N^n \alpha(a+\lambda)+\alpha+\lambda+1}{(a+\lambda)(a+\lambda+1)} \) for \( n = 1, ..., N \) with the boundary condition \( V_{N+1}^N = b \). Note \( V_{N+1}^N \) on the right-hand side of the recursive formula is independent of \( \nu_r \) because of memorylessness. Differentiating \( V_n^N \) with respect to \( \nu_r \), we get \( \frac{dV_n^N}{d\nu_r} = \frac{\lambda(a+\lambda)}{(a+\lambda)(a+\lambda+1)} \left( \frac{\chi_r(a+\lambda)+2\sqrt{\lambda \chi_r}}{\alpha+a+\lambda} \right) \frac{dV_n^N}{d\nu_r} \right)^2 \). Noting \( \lim_{\nu_r \to -\infty} \frac{dV_n^N}{d\nu_r} \frac{d^2V_n^N}{d\nu_r^2} = \frac{V_n^N}{\alpha+a+\lambda} > 0, \) we find that \( V_n^N \) is minimized at \( \nu_r = -\alpha + \frac{\chi_r(a+\lambda)+2\sqrt{\lambda \chi_r}}{\alpha+a+\lambda} (h-(\chi_r+V_{N+1}^N)\alpha) > 0 \) if \( V_{N+1}^N \) while it is minimized at \( \nu_r = 0 \) if \( V_{N+1}^N \geq \phi_2 \). Substituting these values in \( V_n^N \), we find that at the optimum \( \tilde{V}_n^N = \Pi(\tilde{V}_{N+1}^N) \) if \( \tilde{V}_{N+1}^N < \phi_2 \) and \( \tilde{V}_n^N = \Pi(\phi_2) = \frac{h\alpha a+\lambda}{\alpha+\lambda} \) if \( \tilde{V}_{N+1}^N > \phi_2 \). Therefore, \( \tilde{V}_n^N = \Pi(\min(\phi_2, \tilde{V}_{N+1}^N)) \). Since \( \Pi(y) \geq y \) for \( y < \phi_1 \) and \( \Pi(y) < y \) for \( y > \phi_1 \), as we found above, the mapping \( \tilde{V}_n^N = \Pi(\min(\phi_2, \tilde{V}_{N+1}^N)) \) implies \( \tilde{V}_n^N \geq \tilde{V}_{N+1}^N \) if \( \tilde{V}_{N+1}^N < \phi_1 \) and \( \tilde{V}_n^N < \tilde{V}_{N+1}^N \) if \( \tilde{V}_{N+1}^N > \phi_1 \).

Consider a sequence \( \{\tilde{V}_n^N\}_{n=1}^{N+1} \). Suppose there exists \( i \leq N \) such that \( \tilde{V}_n^N \geq \phi_1 \geq \tilde{V}_{i+1}^N \). The first inequality \( \tilde{V}_n^N \geq \phi_1 \) implies \( \tilde{V}_n^N = \Pi(\min(\phi_2, \tilde{V}_{N+1}^N)) \leq \phi_1 \). On the other hand, the second inequality \( \phi_1 < \tilde{V}_{i+1}^N \) implies \( \Pi(\phi_1) = \phi_1 = \Pi(\min(\phi_2, \tilde{V}_{N+1}^N)) \) since \( \Pi'(y) > 0 \) for \( y < \min(\phi_2, \tilde{V}_{N+1}^N) \leq \phi_2 \), as we proved above. The resulting inequalities \( \Pi(\min(\phi_2, \tilde{V}_{i+1}^N)) \leq \phi_1 \) and \( \phi_1 < \Pi(\min(\phi_2, \tilde{V}_{i+1}^N)) \) contradict one another; hence, no \( i \leq N \) satisfies \( \tilde{V}_n^N \geq \phi_1 < \tilde{V}_{i+1}^N \). By a similar argument, no \( i \leq N \) satisfies \( \tilde{V}_{i+1}^N < \phi_1 \leq \tilde{V}_{i}^N \). From these observations we conclude that the sequence \( \{\tilde{V}_n^N\}_{n=1}^{N+1} \) satisfies either \( \tilde{V}_n^N \leq \phi_1 \) or \( \tilde{V}_N^N > \phi_1 \) for all \( n = 1, ..., N+1 \). Combining this with the earlier result \( \tilde{V}_n^N \geq \tilde{V}_{i+1}^N \) if \( \tilde{V}_{N+1}^N \leq \phi_1 \) and \( \tilde{V}_N^N < \tilde{V}_{i+1}^N \) if \( \tilde{V}_{N+1}^N > \phi_1 \), we see that either \( b = \tilde{V}_{N+1}^N \leq \tilde{V}_N^N \leq \tilde{V}_{i+1}^N \leq \tilde{V}_i^N \leq \phi_1 \) or \( \phi_1 < \tilde{V}_N^N \leq \tilde{V}_{i+1}^N \leq \tilde{V}_i^N \leq \tilde{V}_{i+1}^N = b \).

Due to time symmetry, the minimization problem in the \( n \)th enforcement cycle of \( N \) total cycles is identical to that in the \( (n-j) \)th cycle of \( N-j \) total cycles. Therefore, \( \tilde{V}_{N-j}^N = \tilde{V}_N^N \) for \( n = 1, ..., N+1 \).
and \( j = 1, \ldots, n - 1 \). Then the above conclusion that either \( b = \hat{V}^{N+1}_N \leq \hat{V}^N_N \leq \cdots \leq \hat{V}^2_N \leq \hat{V}^1_N \leq \phi_1 \) or \( \phi_1 < \hat{V}^1_N < \hat{V}^2_N < \cdots < \hat{V}^N_N < \hat{V}^{N+1}_N = b \) implies either \( b \leq \hat{V}^1_1 \leq \hat{V}^2_1 \leq \cdots \leq \hat{V}^1_{N-1} \leq \hat{V}^1_N \leq \phi_1 \) or \( \phi_1 < \hat{V}^1_N < \hat{V}^2_N < \cdots < \hat{V}^1_{N-1} < \hat{V}^1_N < b \) for any \( N \geq 1 \). In other words, \( \hat{V}^1_N \) nondecreases in \( N \) if \( b \leq \phi_1 \) and \( \hat{V}^1_N \) decreases in \( N \) if \( \phi_1 < b < \phi_2 \). Therefore, \( \hat{V}^1_N \) is minimized by setting \( N = 1 \) if \( b \leq \phi_1 \) and by setting \( N = \infty \) if \( \phi_1 < b < \phi_2 \). Earlier we found that \( V^*_N \) is minimized at \( \nu^N_r = -\alpha + \sqrt{\frac{2}{\chi_r} (h - (\nu^N_r + 1)\lambda \alpha)} > 0 \) if \( \hat{V}^{N+1}_N < \phi_2 \). If \( b \leq \phi_1 \), then \( N = 1 \) at the optimum and therefore \( V^*_1 \) is minimized at \( \nu^1_r = -\alpha + \sqrt{\frac{2}{\chi_r} (h - (\nu^1_r + b)\lambda \alpha)} \) since \( \hat{V}^1_1 = b < \phi_2 \). If \( \phi_1 < b < \phi_2 \), on the other hand, \( N = \infty \) at the optimum and the optimal \( \nu_r \) in each cycle is \( \nu_r = -\alpha + \sqrt{\frac{2}{\chi_r} (h - (\nu_r + 1)\lambda \alpha)} = -\alpha + \lambda \sqrt{\frac{h}{\chi_r}} \). This is because \( \Pi(y) \) is a contraction mapping that ensures that the sequence \( \hat{V}^1_1, \ldots, \hat{V}^N_N \) converges to \( \phi_1 < \phi_2 \) as \( N \to \infty \), given that \( \Pi'(y) = \frac{\lambda}{\alpha + \lambda} \left( 1 - \frac{\chi_r \alpha}{\sqrt{\chi_r \lambda (h - (\nu_r + y)\alpha)}} \right) > 0 \) and \( \Pi'(y) < 1 \) for all \( y \neq \phi_2 \). \( \square \)

**Proof (Lemma 1)** The firm’s action set consists of no-action (denoted by “NA”), TF, and PF. The firm takes one of these actions as soon as he discovers the facility’s state, i.e., (i) at time zero or (ii) immediately after his inspection or (iii) immediately after the regulator’s noncompliance detection. Recall from §3.3 that the firm may apply PF upon finding compliance but the regulator cannot. If the regulator finds noncompliance in her inspection, the firm is required to restore compliance via either TF or PF. For generality, assume that the firm can reset his inspection rate \( \nu_f \) immediately after taking an action. He does not revise the inspection rate set at time zero if he finds compliance, because the future looks identical to that at time zero due to stationarity. On the other hand, the firm may revise the inspection rate if he chooses NA upon detecting noncompliance, since compliance is not restored.

Suppose that the firm never starts inspections by setting \( \nu_f = 0 \) at time zero, when the firm has visibility to the facility’s compliance state. Before setting \( \nu_f = 0 \), the firm can choose between two options: (i) PF, which stops further inspections by the regulator; (ii) NA, which lets the regulator perform inspections at rate \( \nu_r > 0 \). With the first option, the firm incurs terminal cost \( s \) at time zero. With the second option, the firm incurs \( c + \kappa \) each time the regulator detects noncompliance and orders TF under CP while he incurs \( s + \kappa \) the next time the regulator detects noncompliance and orders PF under SP. Following the steps similar to the ones presented in §4, we can show that the firm’s costs-to-go after choosing NA are \( \Psi^+_{C_0} = \frac{(c+\kappa)\lambda \nu_r}{\alpha + \lambda + \nu_r} \) under CP and \( \Psi^+_{S_0} = \frac{\kappa}{\alpha + \lambda + \nu_r} \) under SP (superscript “+” denotes the compliance state at time zero). Since \( \kappa \leq K \) and \( s > (c + K) \frac{\lambda + \kappa}{\alpha} \) (see Assumption 1), \( \Psi^+_{C_0} \leq \frac{(c+K)\lambda \nu_r}{\alpha + \lambda + \nu_r} < s \frac{\lambda}{\alpha + \lambda + \nu_r} < s \) under CP and \( \Psi^+_{S_0} \leq s \frac{\lambda + \kappa}{\alpha + \lambda + \nu_r} < s \frac{\lambda}{\alpha + \lambda + \nu_r} < s \) under SP. These results imply that the firm never finds it optimal to choose PF at time zero, since the cost \( s \) associated with that action is higher than the costs-to-go he incurs by choosing NA and setting \( \nu_f = 0 \).
Next, suppose that the firm performs inspections by setting \( \nu_f > 0 \) at time zero. If the firm detects noncompliance, he may choose one among NA, TF, and PF. If the firm finds compliance, on the other hand, he only chooses NA because (i) TF cannot be applied when the facility is in compliance and (ii) if it were optimal for the firm to choose PF upon finding compliance, he would have chosen the same action at time zero and never have started inspections. These eliminations leave three possible combinations of the firm’s actions upon his inspection, namely NA upon finding compliance and (i) TF or PF upon noncompliance detection. Among them, NA/NA is clearly suboptimal because the firm could have set \( \nu_f = 0 \) at time zero and saved inspection costs without altering outcomes. Therefore, only two possibilities remain regarding the firm’s action upon his inspection: NA upon finding compliance and either TF or PF upon noncompliance detection. We now show that the firm never chooses NA/PF, leaving NA/TF to be the only option for the firm.

Suppose that the firm is subject to a TF order by the regulator under CP. If, upon his own noncompliance detection, the firm chooses NA and stops further inspections by setting \( \nu_f = 0 \), from then on the firm applies TF only after noncompliance is detected by the regulator. We can show that the firm’s cost-to-go after this choice is \( \Psi_{C0}^- = (s+\kappa)(\alpha+\lambda+\nu_f) \). Using the condition \( s > (c+K) \frac{\alpha+\lambda}{\alpha} \) from Assumption 1, we see that \( \Psi_{C0}^- \leq (s+K)(\alpha+\lambda+\nu_f) \frac{\nu_f}{\alpha+\lambda+\nu_f} < s \). This implies that, under CP, the firm prefers choosing NA and stopping further inspections to choosing PF, which costs him \( s \). Hence, he never chooses PF upon his noncompliance detection under CP. Next, suppose that the firm is subject to a TF order by the regulator under SP. If the firm chooses PF upon his noncompliance detection, the firm chooses NA and stops further inspections by setting \( \nu_f = 0 \) at time zero. If, on the other hand, the firm chooses TF upon detecting noncompliance under both CP and SP.

In all cases, we found that the firm never chooses PF voluntarily. Moreover, if he sets \( \nu_f > 0 \) at time zero he maintains the same inspection rate afterwards, choosing NA upon finding compliance while choosing TF upon detecting noncompliance under both CP and SP.

**Proof (Lemma 2)** The results follow directly from Lemmas A1 and A2 in Appendix A.

**Proof (Proposition 2)** Rewrite \( \sigma_C = \frac{\nu_f(\alpha+\lambda)}{\alpha+\lambda} \), \( \beta_C' = -(\alpha+\lambda) + \frac{\alpha}{2\nu_f} (1 - \sqrt{1 - 4\sigma_C}) \), and \( \beta_C = -(\alpha+\lambda) + \frac{\alpha}{2\nu_f} (1 + \sqrt{1 - 4\sigma_C}) \) from (4) and Lemma A1 as \( \sigma_\kappa, \beta_\kappa' \) and \( \beta_\kappa \). The following
can be proved: \( \frac{\partial}{\partial \nu} \sigma_k < 0 \), \( \frac{\partial}{\partial \nu} \beta_k < 0 \), and \( \frac{\partial}{\partial \nu} \sigma_k > 0 \). Since \( \sigma_k < \frac{1}{4} \) by the condition \( \chi_f < \frac{K^2 \lambda}{4(c+\lambda)(\alpha+\lambda)} \) in Assumption 1 and \( \sigma_k < \sigma_k \) for all \( \kappa < K \), there exists \( \kappa \) in the vicinity of \( K \) that satisfies \( \sigma_k < \frac{1}{4} \); consider such \( \kappa \). Then the regulator’s objective function given in (5) is equal to 
\[
V_{\kappa}(\nu_f) \equiv \frac{\chi_f}{\alpha} + \frac{h}{\alpha} \sqrt{\frac{\chi_f}{\nu_f-c(\alpha+\lambda)}} \text{iff } \beta_k < \nu_f < \beta_k, \text{ the interval in which } \nu_f > 0. \text{ Since } \frac{\partial}{\partial \nu} V_{\kappa}(\nu_f) < 0, \text{ we see that } V_{\kappa}(\nu_f) \text{ is minimized by setting } \kappa = K \text{ for any given } \nu_f \in (\beta_k, \beta_k). \text{ Hence, the constraint } \kappa \leq K \text{ binds in equilibrium if the minimizer of } V_{\kappa}(\nu_f) \text{ exists in the interval } (\beta_k, \beta_k). \text{ Consider } V_{\kappa}(\nu_f) \text{ in the expanded interval } \left(\frac{c(\alpha+\lambda)}{K}, \infty\right). \text{ Since } V_{\kappa}(\nu_f) = \frac{\chi_f}{\alpha} - \frac{h}{\alpha} \sqrt{\frac{\chi_f}{K \nu_f-c(\alpha+\lambda)}} \text{ is minimized at } \nu_f = 0, \text{ we have } V_{\kappa}(\nu_f) = \frac{\chi_f}{\alpha} - \frac{h}{\alpha} \sqrt{\frac{\chi_f}{K \nu_f-c(\alpha+\lambda)}} \text{ is minimized at } \nu_f = 0, \text{ which yields } \nu^*_f = \frac{c(\alpha+\lambda)}{K} + \left(\frac{h^2 \chi_f \lambda}{4 K \nu_f}\right)^{1/3}. \text{ At this value, we find } \nu^*_f = R_C(\nu^*_f) = \left(\frac{c(\alpha+\lambda)}{K \nu_f-c(\alpha+\lambda)}\right)^{1/3} + \left(\frac{h^2 \chi_f \lambda}{4 K \nu_f}\right)^{1/3}, \text{ and } H^*_C = \frac{h \lambda}{\alpha(c+\lambda)+\nu^*_f} = \frac{1}{\alpha} \left(\frac{h^2 \chi_f \lambda}{4 K \nu_f}\right)^{1/3}, \text{ where } V_C^*_C = H^*_C. \text{ The comparative statics results in the proposition follow directly from these expressions.} \]

**Proof (Proposition 3)** Let \( V_0(\nu_f) \equiv \frac{\chi_f}{\alpha} + \frac{h}{\alpha} \sqrt{\frac{\chi_f}{K \nu_f-c(\alpha+\lambda)}} \) and \( V_+ (\nu_f) \equiv \frac{\chi_f}{\alpha} + \frac{h}{\alpha} \sqrt{\frac{\chi_f}{\nu_f-c(\alpha+\lambda)}} \). Both are unimodal functions, the first minimized at \( \nu^*_f = - \frac{\chi_f}{\alpha} + \frac{h}{\alpha} \sqrt{\frac{\chi_f}{K \nu_f-c(\alpha+\lambda)}} \) and the second at \( \nu^*_f = \frac{c(\alpha+\lambda)}{K \nu_f-c(\alpha+\lambda)} \). Recall from (5) that the regulator’s objective function with \( \kappa \) set to \( K \) is 
\[
V^*_C(\nu_f, K) = V_+ (\nu_f) \text{ for } \beta < \nu_f < \beta \text{ while } V^*_C(\nu_f, K) = V^*_0(\nu_f) \text{ for } \nu_f \leq \beta \text{ or } \nu_f \geq \beta, \text{ where } \beta = - (\alpha + \lambda) + \frac{K^2 \lambda}{2 K \nu_f} (1 - \sqrt{1 - 4 \sigma}) \text{ and } \beta = - (\alpha + \lambda) + \frac{K^2 \lambda}{2 K \nu_f} (1 + \sqrt{1 - 4 \sigma}) \text{ with } \sigma = \frac{\chi_f (c+K)(\alpha+\lambda)}{K \lambda}. \text{ Then the conditions } \beta < \nu^*_f < \beta \text{ is satisfied, since the firm sets } \nu_f = 0 \text{ otherwise (see Lemma 2). Suppose } \beta < \nu^*_f < \beta \text{ and } \nu^*_f \notin (\beta, \beta). \text{ In this case } V^*_C(\nu_f, K) \text{ is bimodal in } \nu_f \text{ and therefore the equilibrium with } \nu^*_f > 0 \text{ exists iff } V^*_C < V^*_0. \text{ Hence, } V^*_C < V^*_0 \text{ follows from the existence of the equilibrium with } \nu^*_f > 0 \text{ in this case. Next, suppose } \beta < \nu^*_f < \beta \text{ and } \beta < \nu^*_f < \beta. \text{ In this case } V^*_C(\nu_f, K) \text{ is unimodal in } \nu_f \text{ and its minimum coincides with the minimum of } V_+ (\nu_f). \text{ Let } z \equiv \frac{K}{(c+k)(\alpha+\lambda)} \sqrt{\frac{h \lambda}{\chi_f}}. \text{ Then the conditions } \beta < \nu^*_f < \beta \text{ and } \beta < \nu^*_f < \beta \text{ can be rewritten as } z < z < z. \]
and $\eta < z < \bar{\eta}$, respectively, where $\eta \equiv \frac{(1-\sqrt{1-4\sigma})^{3/2}}{2\sigma}$, $\bar{\eta} \equiv \frac{(1+\sqrt{1-4\sigma})^{3/2}}{2\sigma}$, $\bar{\eta} \equiv \frac{1-\sqrt{1-4\sigma}}{2\sigma}$, and $\bar{\eta} \equiv \frac{1+\sqrt{1-4\sigma}}{2\sigma}$. Clearly, $\eta < \eta < \bar{\eta} < \bar{\eta}$, and therefore, $\eta < z < \bar{\eta}$ implies $\eta < z < \bar{\eta}$. The following results can be proved: $\frac{\delta \sigma}{\delta \eta} > 0$, $\frac{\delta \sigma}{\delta \bar{\eta}} < 0$, $\frac{\delta \sigma}{\delta \bar{\eta}} < 0 < \delta \sigma \bar{\eta} < 0$ with $\lim_{\sigma \to 0} \eta = 0$, $\lim_{\sigma \to 0} \eta = 1$, $\lim_{\sigma \to 0} \bar{\eta} = \lim_{\sigma \to 0} \bar{\bar{\eta}} = \infty$, and $\lim_{\sigma \to 1/4} \eta = \lim_{\sigma \to 1/4} \eta = \lim_{\sigma \to 1/4} \eta = 2$. Note $\eta < z < \bar{\eta}$ requires $z > 1$, since $\eta$ has a lower bound equal to one; if $z \leq 1$, then $z \leq \bar{\eta}$ and therefore $\nu^*_r \notin (\bar{\beta}, \bar{\bar{\beta}})$. Moreover, noting that $\eta$ increases in $\sigma$ while $\bar{\eta}$ decreases in $\sigma$ until the two quantities converge to $\eta = \bar{\eta} = 2$ at $\sigma = \frac{1}{4}$, we can rewrite the condition $\eta < z < \bar{\eta}$ as $\sigma < \frac{\zeta}{z^2}$. Rewriting $V_C^*$ using the identities $\chi_f = \sigma \bar{\chi} \sqrt{\frac{\chi_f}{\lambda}}$ and $\frac{\chi_f (\alpha + \lambda)}{\alpha} = -\frac{\chi_f (\alpha + \lambda)}{\alpha} + \frac{\chi_f (\alpha + \lambda)}{\alpha} \frac{1}{3}$ and applying the condition $\sigma < \frac{\zeta}{z^2}$, we find $V_C^* = \frac{\chi_f (\alpha + \lambda)}{\alpha} + \frac{3}{\alpha} \left( \frac{h^2 \chi_f \chi_f}{4K} \right)^{1/3}$ where $L(z) = \frac{1}{z^2} + 3 \left( \frac{z^3}{4z^2} \right)$ defined for $z > 1$ is a quasiconcave function that peaks at $z = 2$ with the maximum value $L(2) = 2$. Hence, $V_C^* < \frac{\chi_f (\alpha + \lambda)}{\alpha} + \frac{\chi_f (\alpha + \lambda)}{\alpha} L(z) < \frac{\chi_f (\alpha + \lambda)}{\alpha} + \frac{\chi_f (\alpha + \lambda)}{\alpha} L(z) = \frac{\chi_f (\alpha + \lambda)}{\alpha} + \frac{\chi_f (\alpha + \lambda)}{\alpha} = V_C^*$. As we set out to prove. Rewriting $V_C^* < V_C^*$ using the expressions above, we find $V_C^* < V_C^*$ iff $\chi_f < \delta_0(h)$ where $\delta_0(h)$ is defined in the proposition.

In the proofs of Proposition 1 and Proposition 2, it is shown that $I_C^* = \frac{\chi_f (\alpha + \lambda)}{\alpha} + \frac{\chi_f (\alpha + \lambda)}{\alpha} L(z)$, $I_C^* = \frac{\chi_f (\alpha + \lambda)}{\alpha} + \frac{\chi_f (\alpha + \lambda)}{\alpha} L(z)$, and $H_C^* = \frac{\chi_f (\alpha + \lambda)}{\alpha} + \frac{\chi_f (\alpha + \lambda)}{\alpha} L(z)$. From these expressions it immediately follows that $I_C^* < I_C^*$ iff $\chi_f < \delta_1(h)$ and that $H_C^* < H_C^*$ iff $\chi_f < \delta_2(h)$, where $\delta_1(h)$ and $\delta_2(h)$ are defined in the proposition. Since $V_C^* = I_C^* + H_C^*$ and $V_C^* = I_C^* + H_C^*$, the earlier result $V_C^* < V_C^*$ implies that a case with $I_C^* > I_C^*$ and $H_C^* > H_C^*$ never arises, which also implies that a case with $\chi_f > \max \{\delta_1(h)\}$ never arises. This leaves three possibilities: (a) $I_C^* < I_C^*$ and $H_C^* < H_C^*$; (b) $I_C^* > I_C^*$ and $H_C^* < H_C^*$; (c) $I_C^* < I_C^*$ and $H_C^* > H_C^*$. Combining the observations (i) $I_C^* < I_C^*$ iff $\chi_f < \delta_1(h)$, (ii) $H_C^* < H_C^*$ iff $\chi_f < \delta_2(h)$, and (iii) $\chi_f < \delta_0(h)$, which follows from existence of the equilibrium with $\nu^*_r > 0$, we see that Case (a) arises if $\chi_f < \min \{\delta_0(h), \delta_1(h), \delta_2(h)\}$. Case (b) arises if $\delta_1(h) < \chi_f < \min \{\delta_0(h), \delta_2(h)\}$, and Case (c) arises if $\delta_0(h) < \chi_f < \min \{\delta_0(h), \delta_1(h)\}$. Since $\delta_1(h) < \delta_0(h) < \delta_2(h)$ if $h < h_0$ and $\delta_1(h) < \delta_0(h) < \delta_1(h) = \delta_1(h)$ if $h > h_0$, as stated in the proposition, we conclude that Case (a) arises if $\chi_f < \min \{\delta_1(h), \delta_2(h)\}$, Case (b) arises if $h < h_0$ and $\delta_1(h) < \chi_f < \delta_0(h)$, and Case (c) arises if $h > h_0$ and $\delta_2(h) < \chi_f < \delta_0(h)$. 

Proof (Corollary 1) From Lemma 2 and Lemma A1 we see that, at the equilibrium, the firm’s best response function $\nu^*_f = R_C(\nu^*_r) > 0$ satisfies $R'_{\nu}(\nu^*_r) < 0$ iff $\nu^*_r > \frac{c(\alpha + \lambda)}{K} + \frac{\chi_f}{\chi_f}$. Since $\nu^*_r = \frac{c(\alpha + \lambda)}{K} + \frac{\chi_f (\alpha + \lambda)}{4K \chi_f}$, as proved in Proposition 2, this condition can be rewritten as $\chi_f < \delta_2(h)$ where $\delta_2(h)$ is defined in Proposition 3. According to the same proposition, $\chi_f > \delta_2(h)$ iff $H_C^* > H_C^*$, and therefore, $H_C^* > H_C^*$ iff $R'(\nu^*_r) < 0$. 

Proof (Proposition 4) Assume $h > c(\frac{\alpha + \lambda}{\alpha})^2$, $\bar{\beta} = \frac{\alpha + \lambda}{\alpha}$, $\bar{\beta}$, and $\xi(\nu^*_r, \kappa)$ from (4), Lemma 2, and (7) as $\sigma$, $\bar{\beta}$, $\bar{\beta}$, and $\xi(\nu^*_r, \kappa)$. It is proved in Lemma A3 that $\sigma < \frac{1}{4}$.
for all $\kappa \in [0, K]$, which ensures that the interval $(\beta_\kappa, \beta_\kappa^*)$ in which the firm sets $\nu_f > 0$ is nonempty (see Lemma 2). Assume $\kappa$ is fixed and let $\pi \equiv \frac{(s+\kappa)\alpha}{\alpha+s+\lambda} - \chi_f - c$, which satisfies $\pi > 0$ by the conditions $s > (c+K)\frac{\alpha+\lambda}{\alpha}$ and $\chi_f < \frac{K^2\lambda}{4(c+K)(\alpha+\lambda)} < K$ in Assumption 1. With this notation, $\beta_\kappa$, $\beta_\kappa^*$, and $\overline{\beta}_\kappa$ from Lemma A2 can be written as $\beta_\kappa = -\alpha + \frac{\pi\lambda(1-\sqrt{1-4\pi\kappa})}{2\chi_f}$, $\beta_\kappa^* = -\left(\frac{\pi}{2\chi_f} + 1\right)\alpha + \sqrt{\alpha(\alpha+\lambda)}\left(\frac{\pi}{2\chi_f} + 1\right)^2 + \frac{\alpha}{\chi_f}$, and $\overline{\beta}_\kappa = -\alpha + \frac{\pi\lambda(1+\sqrt{1-4\pi\kappa})}{2\chi_f}$. In addition, let $\beta \equiv -\left(\frac{\pi}{2\chi_f} + 1\right)\alpha + \alpha\sqrt{\left(\frac{\pi}{2\chi_f} + 1\right)^2 + \frac{\alpha}{\chi_f}} < \beta_\kappa$, which satisfies $\xi_\kappa(\beta) = 0$ where $\xi_\kappa(\nu_f) = (\alpha+\lambda)(\pi + \chi_f(2 + \frac{c}{\alpha})\frac{\nu_f}{\nu} - c)$. Since $\xi'_\kappa(\nu_f) = \frac{\alpha+\lambda}{\alpha}\left(\frac{2\chi_f\nu_f}{\alpha} + \pi + 2\chi_f\right) > 0$ and $\xi''_\kappa(\nu_f) = 2\chi_f\frac{\alpha+\lambda}{\alpha} > 0$, we have $\xi_\kappa(\nu_f) > 0$ for all $\nu_f > \beta$. The regulator’s objective function (6) is equal to $V_\kappa(\nu_f) \equiv \frac{\chi_f\nu_f - \xi'\nu_f}{\sqrt{\chi_f\xi''\nu_f}}$ with $\epsilon(\nu_f) \equiv \frac{\nu_f}{\alpha}\nu_f^2 - bv_r - h$ if $\beta < \nu_f < \overline{\beta}_\kappa$, where $\nu_f > 0$. Differentiating $V_\kappa(\nu_f)$, we find that the first-order condition is equivalent to $Q_\kappa(\nu_f) = 0$ where $Q_\kappa(\nu_f) \equiv 2\chi_f\nu_f - \sqrt{\chi_f\alpha}(\beta\xi'_\kappa(\beta) < 0$, which follows from the observation that $(\beta\xi'_\kappa(\beta) < 0$ if $\beta < \sqrt{\frac{2\alpha}{\pi\lambda}}$, which is true because $\beta = \left(\frac{\pi}{2\chi_f} + 1\right)\alpha\left(-1 + \sqrt{1 + \frac{c}{\chi_f}}\left(\frac{\pi}{2\chi_f} + 1\right)^2\right) < \frac{\alpha c}{\pi + 2\chi_f} < \frac{2\alpha}{\pi\lambda};$ the first and second inequalities are proved based on $\sqrt{1+x} < 1 + \frac{x}{2}$ and the assumption $h > \chi_f(\frac{\alpha+\lambda}{\alpha})^2 \left(\frac{\alpha+\lambda}{\alpha}\frac{c}{\alpha}\right)^2 = \chi_f\alpha\left(\frac{\alpha+\lambda}{\alpha}\frac{c}{\alpha}\right)^2$. Moreover, $\lim_{\nu_f \to \infty} Q_\kappa(\nu_f) = \infty$ and $Q''_\kappa(\nu_f) = \frac{3\chi_f}{2\alpha^2}\xi_\kappa(\nu_f)^{-1/2}\left[(\alpha\xi'_\kappa(\nu_f) - 2\sqrt{\chi_f\alpha}\xi''_\kappa(\nu_f) + 4\chi_f\alpha\xi'_\kappa(\nu_f)\right] > 0$. Since $Q_\kappa(\nu_f)$ is a convex function that starts from a negative number at the lower bound $\nu_f \to \beta$ and diverges to $\infty > 0$ as $\nu_f \to \infty$, it crosses zero exactly once at $\nu_f = \tilde{\nu}_f > \beta$ from below. The regulator’s objective function attains minimum at this point for fixed $\kappa$; let $\tilde{V}_\kappa \equiv V_\kappa(\nu_f) = \frac{\chi_f\nu_f - \epsilon(\nu_f)}{\sqrt{\chi_f\xi''(\nu_f)}}$. Since $\xi_\kappa(\nu_f)$ increases in $\nu_f$ for fixed $\nu_f$, and hence in $s + \kappa$, using the envelope theorem we see that $\tilde{V}_\kappa$ decreases in $s + \kappa$ if $\epsilon(\nu_f) < 0$, or equivalently, $\tilde{V}_f < \frac{\alpha\nu_f}{2\chi_f}\left(b + \sqrt{b^2 + 4\frac{b\nu_f}{\alpha}}\right) \equiv \tilde{V}_f$ where $\tilde{V}_f$ satisfies $\epsilon(\tilde{V}_f) = 0$. Because $\nu_f > \tilde{V}_f$, if $Q_\kappa(\nu_f) > 0$, as we proved above, this implies that $\tilde{V}_\kappa$ decreases in $s + \kappa$ if $Q_\kappa(\tilde{V}_f) = 2\chi_f\xi_\kappa(\tilde{V}_f)\left(\sqrt{\chi_f\xi''}\tilde{V}_f - \sqrt{\frac{\chi_f^2}{\lambda}\left(b^2 + 4\frac{b\nu_f}{\alpha}\right)}\right) > 0$, or equivalently $\xi_\kappa(\tilde{V}_f) > \frac{\chi_f^2}{\lambda^2}\left(b^2 + 4\frac{b\nu_f}{\alpha}\right)$, which can be rewritten as $s + \kappa > \theta$ where $\theta$ is defined in the proposition. Therefore, $\tilde{V}_\kappa$ increases in $s + \kappa$ if $s + \kappa < \theta$ while it decreases in $s + \kappa$ if $s + \kappa > \theta$. Hence, for fixed $s$, $\tilde{V}_\kappa$ is minimized at $\kappa = 0$ if $s + K \leq \theta$ and at $\kappa = K$ if $s \geq \theta$, which guarantees $s + \kappa \geq \theta$ for all $\kappa \leq K$. 

**Proof (Proposition 5)** According to Proposition 4, the regulator’s objective function (6) evaluated at the equilibrium with $s + K \leq \theta$ is $V^* = \frac{\chi_f\nu_f^* - \epsilon(\nu_f^*)}{\sqrt{\chi_f\xi''(\nu_f^*)}}$ where $\epsilon(\nu_f^*) = \frac{\nu_f^*}{\alpha}\nu_f^* - 2bv_r - h$ and $\xi(\nu_f) = \frac{\chi_f(\alpha+\lambda)}{\alpha}\nu_f^* + \left(s + (\chi_f-c)(\alpha+\lambda)\right)\nu_f - c(\alpha+\lambda)$ since $\kappa^* = 0$. Note $\epsilon(\nu_f^*) > 0$ iff $\nu_f^* > \frac{\alpha}{2\chi_f}\left(b + \sqrt{b^2 + 4\frac{b\nu_f^*}{\alpha}}\right) \equiv \tilde{V}_f$ and $\frac{\partial}{\partial \nu_f}\left(\frac{\chi_f^2}{\lambda}\right) > 0$ iff $\nu_f^* > \frac{c(\alpha+\lambda)}{s-c(\alpha+\lambda)/\alpha} \equiv \tilde{V}_f$. By the condition
\( h > \chi_r \frac{(\alpha + \lambda)^2}{\alpha} \left( \frac{\epsilon}{\pi - \alpha \epsilon} \right)^2 \) assumed in the proof of Proposition 4, we have \( \tilde{\nu}_r < \nu_r \). Following the proof of Proposition 4, we see that the he condition \( s + K \leq \theta \) implies \( \epsilon(\nu_r^*) > 0 \) and therefore \( \nu_r^* > \tilde{\nu}_r \), which in turn implies \( \nu_r^* > \nu_r \) because \( \tilde{\nu}_r < \nu_r \), and therefore \( \frac{\partial}{\partial \chi_f} \left( \frac{\chi_f}{\xi(\nu_r^*)} \right) > 0 \). It then follows from the envelope theorem that \( V^* \) decreases in \( \chi_f \). \( \square \)