Econometrics of Fair Values

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Abstract

Properties of many important valuation rules can be quantified, examined and compared in a unified framework to assist policy decisions. Valuation rules can be viewed as econometric estimators. Which valuation rule has minimum mean squared error (relative to the unobserved economic value of bundles of resources) is a matter of econometrics, not of theory or principle; it depends on the relative magnitudes of the parameters—price volatility and measurement errors—in the economy, industry or firm. In general, no valuation rule, fair or not, dominates the others.

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Econometrics of Fair Values
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This note summarizes a framework and results developed in the past four decades of research to characterize various valuation rules as alternative econometric estimators of economic value. Two key determinants of the properties of these estimators are the degree of price instability, and the magnitude of price measurement errors. The framework can help choose valuation rules or estimators on the basis of their objective properties in the relevant economic environments, not opinions.

In accounting, few topics generate more impassioned debate than rules of valuation. They directly affect accounting numbers used in investment decisions, stewardship, management of enterprise resources, and contract enforcement. Reliability, relevance, bias, timeliness, and representational faithfulness are some of the oft-mentioned qualitative criteria for evaluation and comparison of valuation rules.

Judgments of individuals, even experts, about the qualitative properties of the valuation rules differ (see Joyce, Libby and Sunder, 1982), and there is no systematic way of assessing or reconciling them. Without a framework for quantified comparison, valuation debates remain largely unresolved, sometimes leading to misguided recommendations.

An econometric approach can help us analyze valuation rules by considering each of them as a member of a larger class (called exchange valuation rules). Under this approach, valuation methods are viewed as estimators of unobserved parameters of interest. Econometric approach can help transform what has been essentially a qualitative debate into quantitative analysis, so researchers can contribute constructively to social policy by adducing evidence on falsifiable propositions.
In September 2006, the Financial Accounting Standards Board issued SFAS 157, “Fair Value Measurements” to take effect in 2007. Fair value is defined as the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction (not forced liquidation or distress sale) between market participants at the measurement date. Fair values are to be determined from the perspective of a market participant using the best-use framework, and without using any entity-specific assumptions (even if the acquirer has different plans).

Labels Matter

Before addressing the econometrics of fair values, a few words on semantics seem appropriate. Labels matter, because language can do harm. What, for example, is common to the following three proposals?

- Unified Budget Act (Lyndon B. Johnson, 1964)
- Patriot Act (George W. Bush, 2002)
- Fair Values (FASB, 2006)

President Johnson wanted to use the Social Security Trust Fund surpluses to finance increased spending on Great Society programs and the Vietnam War. He sent legislation labeled Unified Budget Act to Congress, forcing his opponents to have to argue for a non-unified budget.

After the 9/11 attacks, President Bush wanted to place limits on certain civil liberties in order to fight the war on terror. He sent legislation labeled Patriot Act to Congress, forcing those who worried about civil liberties to appear to be arguing against patriotism.
Now, the FASB had decided that financial reports should use current valuation. They have chosen the exit (as opposed to entry) version of this valuation rule; both have been analyzed and debated over the past century in some detail. Paton (1922), Sweeny (1936), MacNeal (1939), Alexander et al. (1950), Chambers (1966), Edwards and Bell (1961) and Sterling (1971) are but a small sampling of distinguished contributions to this literature. Yet, the FASB has decided that this old bottle of wine needs a new label—fair values.

Fairness is a personal judgment, not a valuation rule. Affixing a new loaded label on a well-researched and well-discussed method of valuation, may amount to playing the old game of policy rhetoric: using clever labels to put the opponents of your proposal on defensive even before the debate starts. Who would want to defend the use of unfair values in accounting? It is perhaps best to put the “fair” aside, and discuss current values of which generations of accountants and researchers have thought and written about. Econometrics can help us bring an element of quantified rationality to the debate about valuation rules.

**Econometrics of Valuation**

Great achievements of econometrics arise from our ability and willingness to: (1) postulate an underlying structure and unknown parameters of the problem at hand; (2) characterize the properties of alternative estimators (e.g., OLS, GLS, 2SLS, etc.) as a function of the underlying environment; (3) choose an estimator appropriate to the postulated environment; (4) use data to estimate the unknown parameters, holding the structure constant; (5) examine propositions about the underlying parameter on the basis of estimates; and (6) use alternative datasets to examine the propriety of the assumed
structure. When the assumed structure is found not to be appropriate, we assume a
different structure.

We can use a similar strategy for examining the properties of valuation rules in
various environments. This strategy will not get rid of judgments entirely, but will help
move debates among valuation rules from the domain of opinion towards data. As a start
on analyzing valuation rules as econometric estimators, let us postulate a structure,
subject to subsequent correction on the basis of data and observations.

Postulated Structure\(^2\)

There are many \((N)\) resources in the economy. We represent these resources by a
vector of relative weights \((\omega)\). Each firm is represented as a randomly drawn
(multinomial) bundle of these resources—a vector of relative weights \((w)\). Current values
of resources are subject to change over time, and the relative (percentage) changes have a
given vector of means \((\mu)\) and matrix of covariances \((\Sigma)\). Historical costs of resources
in the bundles are known. Relative changes in current values of the resources are
observed with an (unbiased) error term \((\varepsilon)\) which has a given covariance matrix \((\Delta)\).

If the error term is not zero, it means that the measured current values of
individual as well as baskets of resources can deviate from the true but unobserved
current values of those resources. Econometric analysis can be used to derive the
properties of valuation rules as alternative estimators of the true value of a basket of
resources (which are functions of observations), on the basis of the statistical proximity
of the two.

\(^2\) For further specification of the technical details of the postulated structure, see Sunder (1978) and Lim
and Sunder (1991). Notation is introduced here only for the convenience of referring to the key results as a
function of the postulated structural parameters in the later part of this note.
Two Sources of Error in Valuation

The difference between the valuation of a basket of resources (estimate) and its unobserved true value is the valuation error. It can be decomposed into two parts. First, values change over time but the valuation rules may either ignore or incorporate them less than perfectly. Errors of valuation from this source can be labeled price movement errors. Second, current values used to revalue the resource bundles are prone to errors due to imperfection and incompleteness of markets from which current values are gathered. These can be labeled price measurement errors.

Metric and Magnitude of Errors of Valuation Rules

The actual valuation error for a given firm depends on the realized price changes and on the composition of the bundle of resources it controls. Following the standard econometric practice, we can take the expectation of this error (to get the bias), and of squared error (to get the mean squared error) with respect to the postulated probability distributions of price changes and compositions of resource bundles. Let us focus on the mean squared error of estimators as the metric for assessing how well various valuation rules capture the true unobserved value of bundles of assets. This metric is frequently used in econometrics, and has the advantage of allowing the two components of the error term mentioned above to be decomposable.

The magnitudes of mean squared error (MSE) associated with various valuation rules depend on the structural parameters postulated above: vector of relative weights of various goods in the economy \( \omega \), vector of expectations of price changes for individual goods in the economy \( \mu \), covariance matrix of price changes for individual goods in the economy \( \Sigma \),
economy $\Sigma$, and the covariance matrix of measurement errors in price changes for individual goods in the economy $\Delta$.

Valuation rules differ in how each rule adjusts historical to current values. The space of valuation rules, even their linear subset, is huge. For the sake of simplicity, we limit the present discussion to three—two polar and one intermediate—elements of the linear subset of valuation rules—historical, general price level and current valuation.

Historical valuation lies at one extreme of the spectrum of valuation rules (see left extreme of the three panels in Figure 1); it ignores price changes from the time of resource acquisition to the time of valuation, and therefore suffers from price movement errors. However, since it does not depend on error-prone current values, this valuation is free of the second kind of error that arises from measurement. The magnitude of MSE depends on the parameters of the economy: $\mu$, the mean of the vector of relative price changes, and $\Sigma$, the covariance matrix of the vector of relative price changes. Greater the “magnitude” of these two parameters, greater is the price movement error associated with historical valuation.

At the other end of the spectrum (the right extreme of panels of Figure 1), current valuation takes into account the changes in prices of each resource individually, and is therefore free of price movement errors. It does have price measurement errors arising from assessment of current values, and its MSE depends on parameters of the economy. If we assume that the relative changes in current values are measured without bias (i.e., $E \epsilon = 0$), the MSE arising from the mean of measurement errors is zero. The only remaining source of error is $\Delta$, the covariance matrix of the vector of measurement
errors in relative price changes. Greater the “magnitude” of this covariance matrix, greater is the measurement error associated with current valuation.

General price-level valuation (GPL) uses a single price index to adjust the historical values towards current values (see the intermediate point in the panels of Figure 1). The single price index reduces the price movement error associated with the historical estimator but does not eliminate it. The use of a single price index also introduces some measurement error, although it is not as large as the error associated with the current value estimator. The magnitudes of these two kinds of errors, and their sum associated with GPL estimator depends on the values of the parameters $\mu$, $\Sigma$, $A$ and $\omega$.

The behavior of error associated with valuation rules can be seen in Figure 1 which is a schematic (not drawn to scale) representation of how the two kinds of error and their sum might vary from one valuation rule to another. Each of these three valuation rules can be described by the number of price indexes used to adjust historical numbers. The historical (0-price index) valuation rule is to the left, the general price level (1-index) valuation rule is in the middle, and the current (N-index) valuation rule is to the right.

Panel 1 shows the behavior of price movement error. It is highest for historical, zero for current, and an intermediate value for GPL. The actual magnitudes of the historical and GPL movement errors depend on parameters $\mu$, $\Sigma$, and $\omega$. In general, as valuation rules use a more disaggregated set of price indexes, their price movement error tends to decline.

Panel 2 shows the behavior of price measurement error. It is zero for historical, highest for current, and an intermediate value for GPL. The actual magnitudes of the
current and GPL movement errors depend on parameters $A$ and $\omega$. In general, as valuation rules use a more disaggregated set of price indexes, their price measurement error tends to rise.

Panel 3 shows the behavior of the total valuation error which is the sum of the above two components. In the example drawn in schematic Figure 1, the total error for GPL valuation is shown to be the lowest of the three valuation rules. However, this is not true in general. Depending on the values of the parameters of the economy, the lowest MSE could be associated with any of the three estimators or valuation rules.

If the price volatility is high and measurement errors are small, MSE of the current value estimator would be the lowest. With low price volatility and high measurement errors, GPL, and even historical estimator, would have the lowest MSE. In general, we should not expect that the MSE minimizing estimator will be any one of the three explicitly considered above. Instead, it is most likely that the lowest MSE estimator would be one of the very large number of estimators which use an intermediate number (between 1 and N) and configuration of specific price indexes to adjust historical to current values.

**Testable Implications**

These theoretical results about the properties of valuation rules as econometric estimators have several testable implications. First, current valuation should be more informative for firms and industries whose (i) assets have a larger mean rate of price changes; (ii) assets have greater variability of price changes, (iii) assets are traded in relatively perfect and complete markets (i.e., current values have smaller measurement errors). If, for example, real estate, mineral deposits, films, software, patents tend to be
traded in less perfect markets, and therefore have larger measurement errors, current valuation in such industries would have less advantage (or even be disadvantageous) relative to historical valuation.

Second, these results also suggest that the relative informativeness of valuation rules is not a matter of general accounting theory. Depending on the parameters of the economy, industry and the firm involved, any valuation rule could be better than the others. In contrast, a large literature in accounting theory tries to establish the general dominance of one valuation rule over the others.

Although efficient valuation rules would vary across assets, firms and industries, accounting empirical literature on informativeness of valuation rules tends to follow the “general theory” approach by conducting cross-sectional tests (e.g., Gheyara and Boatsman 1980, Ro 1980, and Beaver et al. 1982). Econometric perspective on valuation suggests that empirical tests could benefit from paying more attention to the characteristics of assets of firms and industries to which valuation rules are being applied.

Third, the level of aggregation at which adjustment of historical to current values is carried out has a major impact on the properties of valuation. The FASB’s proposal wisely leaves this issue open.

**Concluding Remarks**

Traditional analyses in accounting theory as well as empirical work tend to examine and compare the properties of individual valuation rules. This note, based on some four decades of theoretical and empirical literature, provides a perspective on valuation rules.
alternative approach. Theories of valuation can be integrated into a unified framework to facilitate direct comparison of their properties in specified environments. When current prices change, and are prone to measurement errors, neither the current nor the general price level valuation is necessarily the minimum mean squared error estimator of the unobserved economic value of resources. Generally, min (MSE) estimator is likely to be a specific price index rule whose actual identity depends on the parameters of the economy. If the measurement errors are sufficiently large relative to movement errors, even historical valuation can be the min (MSE) estimator.

Which valuation rule has minimum mean squared error is a matter of econometrics, not of theory or principle; it all depends on the relative magnitudes of the parameters of the economy. One size shoe does not fit all; neither does valuation.

References


1. Price Movement Error (MSE) of Three Valuation Rules

2. Price Measurement Error (MSE) of Three Valuation Rules

3. Total Error (MSE) of Three Valuation Rules

Figure 1: Schematic Diagram of Valuation Errors (not to scale)