Three Minimal Market Institutions: Theory and Experimental Evidence

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Abstract

We define and examine the performance of three minimal strategic market games in laboratory relative to their theoretical predictions. These closed exchange economies have limited amounts of cash to facilitate transactions, and include feedback unlike open or partial equilibrium settings of most experiments. Without a role for money or credit, and abstracting away from market mechanisms, general equilibrium theory makes no predictions about how paths of convergence to the CE may differ across alternative markets. Our laboratory data reveal different paths of convergence to CE, and different levels of allocative efficiency in the three settings. The results suggest that abstraction from institutional details may have been taken too far, and, for example, the oligopoly effect of feedback from buying an endowed good is missed. Inclusion of mechanism differences into theory may help us understand markets better.

Keywords: strategic market games, laboratory experiments, general equilibrium

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1. MINIMAL MARKET INSTITUTIONS

We define three minimal market institutions, and compare their theoretical properties to the outcomes observed in laboratory experiments with human subjects and in computer simulations. Minimal institutions are mechanisms that are so stripped down of details that it is not possible to simplify them any further without infringing on the basic phenomenon to be considered. Three basic price formation mechanisms are listed here by the nature of the strategy sets in a single market for each trader:

1. The sell-all model (strategy set of dimension 1);
2. The buy-sell model (strategy set of dimension 2);
3. The simultaneous double auction model (strategy set of dimension 2 or 4).

We report here on the first experiment from a planned series designed to examine the roles of markets, money and credit in economic competition. The abovementioned three basic price formation mechanisms utilize a commodity money for trade, and are described in Section 2. Theory indicates that as the number of traders increases, all three markets should converge to competitive equilibrium. This paper documents the cross-mechanism differences when the number of traders is small, and that non-cooperative and competitive general equilibrium (NCE and CGE) solutions provide reasonable but less than perfect benchmarks to organize the laboratory data.

There is a large literature devoted to exchange markets as games in strategic form solved for their non-cooperative equilibria (e.g. Bertrand 1883, Cournot 1897, Dubey 1982, Dubey and Shubik 1978, 1980, Edgeworth 1925, Nash 1951, Quint and Shubik 2005, Shapley 1995, Shapley and Shubik 1977, Shubik 1973, Sorin 1996). There are also two other extensive literatures: one in macro-economics stressing rational expectations (exemplified by Lucas, 1987, 1988, Lucas and Sargent 1981) and the other in mathematical finance mostly on competitive partial equilibrium open models dealing explicitly with money, transactions costs and the constraints on cash flows. All three approaches broadly involve money, markets and financial institutions. There has been considerable gaming activity on bargaining, bidding and on the emergence of competitive
prices in some simple markets with little stress on the explicit role of money (Marimon, Spear and Sunder 1993, and Marimon and Sunder 1993, 1994, 1995). In the present paper, the stress is on gaming involving the roles of money, credit and other financial instruments.

There has also been a considerable amount of experimental gaming on market models to investigate competition (Smith 1982, Plott 1982). That markets with only a few independent individual traders yield outcomes in close neighborhood of the predictions on competitive model is a common thread in this work. Most experimentation has involved trade in a single market. In the spirit of general equilibrium, we consider two markets. We formulate experimentally playable strategic market games where the trade is mediated by money, but the overall system is closed.

After a description of institutions in Section 2, equilibrium predictions—general and non-cooperative—for each institution are given in Section 3. Section 4 describes the experimental setup we used to implement these markets in laboratory with human as well as minimally intelligent (MI) artificial traders (Gode and Sunder 1993, 1997). The results are presented in Section 5, followed by some concluding remarks.

2. THREE MARKET GAMES

2.1 Definitions

Money

In each market game \( n (=1 \text{ or } 2) \) commodities are traded and one or more commodities or instruments are used as a means of payment or money. Money could be treated as an ordinary good, appearing in any standard form in the utility function; or more especially we could consider that it appears as a linear separable good in the trader’s utility function. It could also be represented as fiat or “outside” money in the sense that it is created and distributed by government. There is also the possibility of individual credit where individuals issue and others accept personal currencies or promises to pay in government fiat. These conditions are made precise in the experimental set up described later.
Bids

(1) A money bid: A trader $i$ bids an amount of money $b^i_j$ for the $j$th commodity. He has no reserve price and takes what the market gives him. This provides a simple quantity bid to construct a mechanism similar to Cournot’s (1897). The market clearing mechanism gives the trader quantity $x^i_j = b^i_j / p^j$ where $p^j$ is the market price that is formed collectively by individual bids and offers.

(2) A price-quantity bid: Suppose that a trader $i$ instead of offering an amount of money to buy a good $j$, bids a personal unit price $p^i_j$ he is willing to pay to buy up to an amount $q^i_j$ of the good. It is reasonable to expect that he is willing to buy $q^i_j$ or less for a price less than or equal to $p^i_j$. There is an implicit limit in this bid inasmuch as $q^i_j p^i_j$ must be less than or equal to the individual’s credit line plus cash. Since we do not consider a credit mechanism in the three market institutions considered in this paper, $q^i_j p^i_j$ cannot exceed the available cash. Minor variations of these bids consider any upper or lower bounds on prices or quantities acceptable to the bidder.

Offers

Analogously, there are two simple forms of offers.

(1) A non-contingent offer to sell: Suppose that an individual $i$ owns $a^i_j$ units of good $j$ and wishes to sell some of it. The simplest strategy is for her to offer $q^i_j \leq a^i_j$ units for sale at the market-determined price.

A somewhat more complex action, but still not involving any more information and confined to a single move is:

(2) The price-quantity offer: Suppose that a trader $i$ is willing to sell up to an amount $q^i_j$ of good $j$ at unit price $p^i_j$. It is reasonable to expect that she is willing to sell $q^i_j$ or less for a price more than or equal to $p^i_j$, the outcomes acceptable to her. Since no individual can offer to sell goods she does not have, $q^i_j \leq a^i_j$.

We use observable acts to buy (bids) and sell (offers) as the building blocks to construct three simple market games. Simplifying them any further will prevent any trading. The first two market games involve a single move by every agent, taken simultaneously. The third, double auction, involves sequential multiple moves by various players. Each game can be generalized to multiple plays.
Let $B_i$ be the set of bids available to buyer $i$ and $Q_j$ be the set of offers available to seller $j$. A market mechanism is a mapping $T$ which transforms the bids and offers into trades and prices.

Consider $n$ individuals where $i$ has an endowment $a_j^i$ of good $j$ ($j = 1, \ldots, m$) and an endowment $M_i$ of money. Suppose there are $m$ markets, one for each good $j$ where it can be exchanged for money. A plausible restriction on the market mechanism is that all trades in a given market take place at the same time and the same price. This requires that $p_j^i = p_j$ for $i = 1, \ldots, n$.

In general, we cannot assume that bids in one market are independent of bids in the others. There is at least a credit interlink across markets because, for example, different "margin" requirements may make an individual's credit line a function of his bids.

2.2 Moves and strategies.

A strategy is a plan an individual uses to select his moves as a function of the information available when he is called upon to move. We limit ourselves to markets with simultaneous moves by the buyers and sellers who all have symmetric knowledge about the states of nature. Even in complex market clearing mechanisms, strategy cannot be based on the knowledge of the moves of others. When individuals in identical situations make a single simultaneous move, their strategies and moves are identical.

If one set of individuals moves first, and these moves are announced before the others move, then the strategies of the latter will call for moves to be selected contingent on the behavior of the former. Strategies are contingency plans and proliferate as a function of information.

2.3 The sell-all model

This is the simplest of the three models. Consider $n$ traders trading in $m+1$ goods, where the $m+1$st good has a special operational role, in addition to its possible utility in consumption. Each trader $i$ has an initial bundle of goods $a_i^j = (a_1^i, \ldots, a_m^i, M_i^i)$, where $a_j^i \geq 0$ for all $j = 1, \ldots, m+1$ and $a_{m+1}^i = M_i$, and $u^i = u_i(x_1, \ldots, x_m, x_{m+1})$, where $u^i$ need not actually depend directly on $x_{m+1}$; a fiat money is not excluded.
In order to keep strategies simple, let us suppose that the traders are required to offer for sale all of their holdings of the first \( m \) goods. Instead of owning their initial bundle of endowments outright; the traders own a claim on the proceeds when the bundle is sold at the prevailing market price.

Suppose there is one trading post for each of the first \( m \) commodities, where the total supplies \((a_1, ..., a_m)\) are deposited for sale "on consignment," so to speak. Each trader \( i \) submits bids by allocating amounts \( b^j_i \) of his endowment \( m^i \) of the \( m+1 \)st commodity among the \( m \) trading posts, \( j = 1, ..., m \). There are a number of possible rules governing the permitted range of bids. In the simplest case, with no credit of any kind, the limits on \( b^j_i \) are given by:

\[
\sum_{j=1}^{m} b^j_i \leq M^i, \text{ and } b^j_i \geq 0, \ j = 1, ..., m.
\]

An interpretation of this spending limit is that the traders are required to pay cash in advance for their purchases.

**Price Formation:** The prices are formed from the simultaneously submitted bids of all buyers; we define

\[
p^j_i = b^j_i / a^j, \ j = 1, ..., m.
\]

Thus, bids precede prices. Traders allocate their budgets fiscally, committing specific quantities of their means of payment to the purchase of each good without definite knowledge of what the per-unit price will be (and how many units of each good their bid will get them). At an equilibrium this will not matter, as prices will be what the traders expect them to be. In a multi-period context, moreover, the traders will know the previous prices and may expect that variations in individual behavior in a mass market will not change prices by much. But any deviation from expectations will result in changing the quantities of goods received, and not in the quantities of cash spent. In a mass market, the difference between the outcomes from allocating a portion of one's budget for purchase of a certain good, and from a decision to buy a specific amount at an unspecified price, will not be too different.

The prices in our model are determined so that they will exactly balance the books at each trading post. The amount of the \( j \)th good that the \( i \)th trader receives in return for his bid \( b^j_i \) is
\[ x_j^i = \begin{cases} b_j^i / p_j & \text{if } p_j > 0, \ j = 1, \ldots, m, \\ 0 & \text{if } p_j = 0, \ j = 1, \ldots, m. \end{cases} \]

His final balance of the \( m+1 \)st good, taking account of his sales as well as his purchases, is
\[
\Pi(b^1, \ldots, b^i, \ldots, b^n) = u(x_i, \ldots, x_{m+1}).
\]

His payoff is expressed as a function of all the traders' strategies; thus we can write
\[
x_{m+1}^i = a_{m+1}^i - \sum_{j=1}^{m} b_j^i + \sum_{j=1}^{m} a_j^i p_j.
\]

2.4 The buy-sell model

In the buy-sell model, from the viewpoint of experimental gaming, the individual makes twice as many decisions in each market. Given simultaneous moves, there are no contingencies in this market either. Physical quantities of goods are submitted for sale and quantities of money are submitted for purchases, and the markets are cleared. The mechanism does not permit any underemployment of resources\(^2\).

The amount of the \( j \)th good that the \( i \)th trader receives in return for his bid \( b_j^i \) is:
\[
x_j^i = \begin{cases} b_j^i / p_j & \text{if } p_j > 0, \ j = 1, \ldots, m, \\ 0 & \text{if } p_j = 0, \ j = 1, \ldots, m. \end{cases} \]

However price is somewhat different as it depends on the quantities of each good offered for sale (and not on the endowment of each good):
\[ p_j = b_j^i / q_j, j = 1, \ldots, m. \]

His final amount of the \( m+1 \)st good, taking account of trader \( i \)'s sales as well as his purchases, is
\[
x_{m+1}^i = a_{m+1}^i - \sum_{j=1}^{m} b_j^i + \sum_{j=1}^{m} a_j^i p_j.
\]

2.5 The bid-offer or double auction model

The double auction model doubles the size of the strategy set yet again, changing price into a strategic variable from a mere outcome of the quantity strategies in the sell-all and

\(^2\) Except when there is no bid or offer, in which instance all resources are returned to their owners. If they are ripe tomatoes, the owner is in trouble.
buy-sell models. In each of the \( m \) markets, an individual’s strategy has four components \((p, q; p^*, q^*)\) where the first pair of numbers is interpreted as an offer to sell amount \( q \) or less for a price \( p \) or more, and the next pair is a bid to buy amount \( q^* \) or less at a price \( p^* \) or less.

From the viewpoint of both game theory and experimental gaming the number of decisions in a double auction is more than in the other two markets. Imposing a condition that one can either buy or sell, but not both, is a possible theoretical simplification. In practice, however, an individual can be a buyer or a seller or a trader. Most consumers are buyers and most producers are sellers of specific commodities or services; a trader can be active on both sides of the market.

3. GENERAL AND NONCOOPERATIVE EQUILIBRIA

The non-cooperative equilibrium solution is a fairly natural game theoretic way to approach these games without any direct communication. A non-cooperative equilibrium satisfies the existence of mutually consistent expectations. If each predicts that the other will play his strategy associated with a non-cooperative equilibrium the actions of all will be self-confirming. No one acting individually will have an incentive to deviate from this equilibrium. This could be called an outcome consistent with “rational expectations,” but as the outcome may not be unique and generically is not optimal, the label of “rational” is best avoided.

The general equilibrium solution is defined as the set of prices that clear all markets efficiently. In general, the mathematical structure of a non-cooperative and competitive equilibria differ. However, it can be shown in theory that, as the number of traders in markets increase, under reasonable conditions, the non-cooperative equilibrium approaches the general competitive equilibrium (GCE). In symmetric markets without face-to-face communication experimentation can verify that with as few as 5-10 traders on each side, the outcome approximates the GCE, and any differences between the two can be explained by the non-cooperative equilibrium.

3.1. The Non-cooperative Equilibrium in Sell-All Market

The simplest model to consider is the sell-all model and for experimental purposes we wish to keep the payoff structure simple enough that it can easily be explained to an
undergraduate untutored in economics or mathematics. In order to do so we selected the payoff to be of the form

$$\alpha \sqrt{xy} + M - b + pa$$

where $\alpha$ is an appropriately chosen parameter (explained in the discussion of the game), the square root of $xy$ is a simple Cobb-Douglas utility function whose range of values is furnished in a coarse-grid table in order to ease the computational burden. The linear term measures the money residual ($M$ is the initial amount of money, $b$ is the amount of money bid, and $pa$ represents earnings from selling $a$ units at price $p$).

The full mathematical solutions of this model under differing constraints are given in Appendix B. Table 1 shows the non-cooperative equilibria for markets with 2, 3, 4, 5, 10 and many traders on each side for the parameter values used in the experiment.

3.2. The Non-cooperative Equilibrium in Buy-Sell Market

The basic difference between the sell-all and the buy-sell models is manifested in the freedom for the individuals to control the amount of goods to sell in the latter market (see Table 2). The general formulae for the non-cooperative equilibria are given in Appendix B.

3.3. The Non-cooperative Equilibrium in Double-Auction Model

The bid-offer market is best modeled as a simultaneous sealed bid auction. The clearing method for the one-shot game is simplicity itself. Bids are assembled in a down-sloping histogram and offers in an up-sloping histogram. Market price is formed where the two lines intersect.

The double auction used in stock markets and in our experiment is a continuous process where bids and offers flow in sequentially and a trade takes place whenever they match or cross. We use this continuous double auction rather than the simultaneous sealed bid auction so traders can learn from the order-book and from past prices.

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3 The utilization of a money with a Marshallian or constant marginal utility can be interpreted in terms of a known expectation of the worth of future purchasing power. In this context any change in price level can be attributed to error and learning the equilibrium of the actual game is stationary. This device provides an easy and logically consistent way in an experimental game to provide terminal conditions.

4 It is necessary to take care of several cases; see Dubey and Shubik (1980) or Dubey (1982).
Two individuals on each side of the market are sufficient for the competitive equilibrium to be a non-cooperative equilibrium. A simple example considering optimal response is sufficient to show this. Suppose that there are two individuals of two types. All have the payoff function given above, but individuals of type 1 and 2 have endowments of \((a, 0, M)\) and \((0, a, M)\), respectively, where the first component is the endowment of the first good, the second the endowment of the second good and the third the endowment of money. Suppose \(M > a/2\) and \(\alpha = 2\) (the parameter in the payoff function), a trader of type 1 offers to sell \(a/2\) or less of good 1 at a price of 1 or more and to buy up to \(a/2\) of good 2 at a price of 1 or less, it can be verified that this is an equilibrium outcome and the price of both goods is 1 (\(p_1 = p_2 = 1\)).

There is a considerable amount of experimental evidence that in a single market the double auction mechanism yields efficient allocations. In their single-commodity double auctions, Gode and Sunder (1993 and 1997) found that it requires negligible skills or intelligence from traders for the market outcome to lie in close proximity of the competitive equilibrium.\(^5\)

We consider two markets for two commodities; whether the complementarities between the two make a difference remains open.

In their one-shot versions, the three games are the simplest price formation mechanisms that can be constructed, involving the maximum of one (sell-all), two (buy-sell) and four (double auction) strategic variables. They can all be analyzed for their non-cooperative equilibria. Unlike most other market experiments, these are general equilibrium full feedback models, not partial equilibrium constructs.

The general equilibrium feature, in theory, generates an asymmetry in actions when there are few agents, as can be seen in the sell-all model where a seller obtains an oligopolistic income from buying a commodity to which he has ownership claims (as contrasted with buying a commodity he does not have). This asymmetry is the largest in the buy-sell game, the next largest in the sell-all game and the smallest in the double auction (see tables 1 and 2 for numerical examples for 10 traders, five on each side).

Paradoxically, because MI agents ignore their oligopolistic influence the

\(^5\) From a strictly technical game theoretic point of view there is a continuum of non-cooperative equilibria, all with the same efficiency that are consistent with the competitive equilibrium outcome.
theoretical prediction is that in all markets the price should be as close or closer to the competitive equilibrium than the oligopolistic human traders, but because of the random action there should be a variation in payoffs that is not present in the equilibrium analysis of the three games.

The speed of learning and the variation among players is not predicted by the static non-cooperative or general equilibrium theories. Many learning theories have been proposed by others and we do not examine them. We only conjecture that variations in individual behavior will diminish in the later periods (replications) of the game.

In these games the terminal amount of money held by each individual was added to their dollar payoffs (term $M - b$ in the payoff function). This served to fix the price level that the transactions would be expected to approach towards the end. The observed divergence between these predicted and realized prices in some cases was considerable, and is discussed later.

4. THE EXPERIMENTAL SETUP

We conducted and report on two separate sessions for each of the three market games considered in this paper. In each session, programmed in Z-tree software (see Fischbacher 1999), the participants traded two goods—labeled A and B—for one kind of money. Each session had ten participants, five of them were endowed with some units of A and none of B, while the other five had some units of B and none of A. All had the same starting endowment of money. Each session consisted of ten independent rounds of trading. Subjects’ “consumption” at the end of each round was accumulated in a “bank account” with the experimenter. No goods balances were carried over from one round to the next, and each subject was re-endowed with the ownership claims to goods A or B at the beginning of each round. In the first two treatments money is carried over to the following round, while in the double auction money holdings were reinitialized at the start of each round (see descriptions of specific treatments below and in Table 3).

(Insert Table 3 about here)

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6 In addition, we conducted a few sessions with 20 participants, of whom 10 were of each type.
4.1 Sell-All Call Market

In Treatment 1 (sell-all Market) the initial endowments were 200 or 0 units of A, 0 or 200 units of B, and 6,000 in cash. All units of A and B were sold automatically at a price derived from the set of bids submitted by the traders. In other words, subjects did not have to decide on the number of units they wished to sell; all their holdings of goods were sold at the prevailing market price. Consequently, they had ownership claim to the revenue from selling 200 units of the good they were endowed with. The only decision participants had to make was how much of their money endowment they wished to bid to buy good A and how much to bid to buy good B. (see Appendix A for instructions and the ‘trading screen’). Each sell-all market was repeated for 20 periods.

As outlined above the unit prices of A and B are calculated as the respective sums of money bid for the respective good by all traders divided by the total units of each goods for sale. With 6,000 units of money endowment per trader there is more than enough money to reach general equilibrium at prices of 20 per unit of A and B. At general equilibrium traders would spend 2,000 on each good, A and B, and keep 2,000 of their money endowment unspent. However, in a thin market with only a few traders, deviating from general equilibrium spending level may make sense to traders. When a trader spends more on the good he is endowed with, he raises its price and therefore his revenue from selling a part of his endowment of this good. Apart from the general equilibrium, there also exists a non-cooperative equilibrium in which traders spend 2213.4 on the good they own, 1810.6 on the other good, and keep 1976.0 unspent. Prices are slightly higher at 20.12 for both goods in this equilibrium. We conducted two runs of this treatment.

4.2 Buy-Sell Call Market

Unlike in Market Game 1 (Treatment 1), traders in this treatment directly control the goods they are endowed with, and decide how many, if any, units they wish to sell (in Treatment 1 all units were sold automatically). Again half of the traders are endowed with 200 units of A and none of B, while the other half are endowed with 200 units of B and none of A. Each trader has an initial endowment of 4,000 units of money at the beginning of the first round of the session. Money balances are carried over from one round to the next. Each buy-sell market was repeated for 20 periods.
Traders make two decisions: The amount of their money to buy the good they do not own, and the number of units to sell out of the 200 units of the good they own.

Prices for A and B are calculated by dividing the total investment for the respective good by the number of units put up for sale. Competitive equilibrium prices and conditions are the same as in Treatment 1. Final holdings of goods are (100,100) each (prices are 20/20, each trader spends 2,000 for the good he does not own, and sells 100 units of the good he owns). At the non-cooperative equilibrium with 5 traders on each side of the market traders of type 1 offer 78.05 units of the second good for sale and bid 1560.97 units of money for the first good. Traders of type 2 do the opposite (see Table 2). Final endowments are (78.05, 121.95) for traders of type 1 and (121.95, 78.05) for traders of type 2. Prices are 20/20.

4.3 Double Auction Market

Treatment 3 features a double auction market where participants can trade goods A and B in a continuous market for several periods. We simplify trading by considering only transactions for one unit at a time. To reduce the number of transactions need to reach equilibrium levels, initial endowments of A and B are reduced from (200/0, 0/200) to (20/0, 0/20), so traders own 20 units of a good rather than 200. Each period lasts for 180 seconds to give the participants enough time to allow participants to trade ten units of goods they do not yet own, and by selling ten units of the good they are endowed with, required to reach equilibrium. Traders are endowed with 4,000 units of money, which is more than enough for trading. Competitive equilibrium and non-cooperative equilibrium prices coincide for the closed double auction model as was shown by Dubey (1982). They are 100 for each good. The first run of the double auction market was repeated for 10 periods, the second run for 11 periods.

In the double auction experiments we allow market as well as limit orders. All orders are executed according to price and then time priority. Market orders have priority over limit orders in the order book. This means market orders are always executed.

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Footnote:

7 The results for the non-cooperative equilibrium are delicately dependent on the formulation of details of the game; see Shubik (1959), Wilson (1978), and Schmeidler (1980). In some models it is possible that there is no pure strategy non-cooperative equilibrium, in others there may be a multiplicity of equilibria with the same value.
instantaneously. Again holdings of money and goods are carried over from one round to
the next.

Participants see current information about their cash and stock holdings, their
wealth, and their transactions within the current period on the screen. In the centre of the
screen they see the open order books and they have the opportunity to post limit or
market orders. On the left side of the screen transaction prices of the round are charted
against time.8

5. HYPOTHESES, CONJECTURES AND EXPERIMENTAL RESULTS

We use six aspects of market outcomes (allocative efficiency, money balances,
symmetry of allocation across the two goods, prices, cross-trader dispersion of earnings,
and trading volume) to assess their behavior relative to four different benchmarks
(autarky, competitive general equilibrium, non-cooperative equilibrium, and minimally
intelligent or MI agents). In addition, we examine the velocity of money and kurtosis and
autocorrelation of returns in double auctions.

Allocative efficiency is measured by the total earnings of the traders as a fraction
of the total earnings in competitive equilibrium. Money balances refer to the percentage
of initial money left unspent after buying decisions are made. As participants also receive
income from selling some or all of the goods they are endowed with, end-of-period
money holdings are usually higher than the money balance we refer to. Symmetry of
allocation is the ratio of consumption of good A and B (= min (c_A/c_B, c_B/c_A)). Given the
parameters chosen for these experiments, goods A and B should be allocated
symmetrically at the competitive equilibrium, and the symmetry measure should be 1 in
competitive equilibrium. The behavior of transaction prices is measured by market
clearing prices for sell-all and buy-sell markets, and by average transaction prices
(averaged across transactions within one period) in the double auction markets.

We report these four performance measures relative to four different benchmarks
summarized in Table 4. Autarky (no trade) and competitive general equilibrium (CGE)
are the two obvious benchmarks. Under autarky, efficiency and symmetry are 0, money
balance is 100 percent, and the price is undefined. The competitive general equilibrium
allocations are 100 units each of good A and B in sell-all and buy-sell markets, and 10

8 The chart was shown in one of the two double auction sessions, not in the other.
units of each good in the double auctions, yielding a symmetry measure of 1 in all cases. Prices are 20 in sell-all and buy-sell markets, and 100 in the double auction markets.

(Insert Table 4 about here)

The third benchmark for market performance is non-cooperative equilibrium for 10 traders (five endowed with good A and five endowed with good B). Application of theory to the parameters of these markets yields bids of 2214 and 1811 for the owned and the other good respectively, for final holdings of 110 and 90 units in the sell-all model. In buy-sell model non-cooperative equilibrium requires selling 78 of the 200 units of the owned good and buying 78 with a bid of 1561. In the double auction traders should keep 11 of their 20 units of the good they are endowed with and buy 9 of the other. Unspent money balance is 32.92 percent in sell-all, 60.98 percent unspent in buy-sell, and not defined in the double auction. The resulting measures for symmetry are 0.82 in sell-all, 0.64 in buy-sell, and 0.82 in double auction. Prices are 20.12 in sell-all, 20 in buy-sell and 100 in double auction.

Finally, we compare the results obtained from human traders in these three markets against the minimal intelligence (MI) benchmark (see Gode and Sunder 1993). To obtain the MI results, we simulate each of the three market structures with MI traders defined as follows. In sell-all market, given the money endowment of 6,000, each trader picks two uniformly distributed random numbers between 0 and 3,000 as his bids for goods A and B respectively yielding an average price of 15. The efficiency and symmetry is measured empirically from the individual variations in the simulation. In the buy-sell market, each trader offers to sell a randomly chosen quantity of the endowed good (from uniform distribution between 0-200) and bids a randomly chosen quantity of money for the other good (from uniform distribution between 0-4,000). This yields an average price of 20 and same allocations as the sell-all market. Actual variations in efficiency, symmetry and prices are determined by the randomness. In double auctions, with equal probability and independently, one trader is picked, one of the two markets is picked, and either bid or ask is picked. Given the trader’s current holdings of the two goods and cash, computer calculates the opportunity set (the maximum amount of bid the trader can make without diminishing its net payoff), and draws a random number between the current bid and this calculated upper limit (if the latter is more than the former) and submits it as a
bid from this trader. In case of asks, the computer calculates the minimum amount of ask
the trader can submit without diminishing its net payoff and submits a random number
between this calculated lower limit and the current ask (if the latter is above the former),
as the ask. Higher bids replace lower ones as market bids, and lower asks replaced
higher ones as market asks. Whenever market bids and market asks cross, a transaction is
recorded at the price equal to the bid or ask, depending on which of the two is submitted
earlier (see Appendix C).

5.1 Efficiency

Allocative efficiency of the markets is measured each period by the average
amount earned by traders as a percentage of the competitive general equilibrium amount
(1,000 points). Six panels of Figure 1 show the time series of efficiency in two
replications of each of the three market games, charted against the four benchmarks
mentioned above. The autarky (efficiency = 0) and the competitive general equilibrium
(efficiency = 100) frame the charts at the bottom and the top. The ‘--’ and ‘o’ markers
denote the non-cooperative equilibrium (for 5+5 = 10 players) and the efficiencies
observed in a simulation with MI traders. In the following paragraphs we compare the
efficiencies observed for specific market games against these benchmarks across the
three market games.

The first obvious observation is that the allocative efficiency of all six sessions of
three market games is much closer to the predictions of competitive equilibrium (100
percent) and far from the autarky prediction of zero. A second observation is that the
efficiency of markets with profit-seeking human traders (as compared to markets with MI
traders) is higher in sell-all markets, about equal in buy-sell markets, and lower in double
auctions. Third, the departure from symmetry of the competitive equilibrium (CE)
holdings at the non-cooperative equilibrium (NE) is clearly seen when there are five
competitors on each side. We expect this to be around 20 percent as the NE approaches
the CE as O(1/n).

---

9 This means that bids are randomly distributed \( -U(\text{Current Bid}, ((100/0.5) ((c_A+1)c_B)^{0.5} - (c_Ac_B)^{0.5}); \text{asks}
are randomly distributed \( -U((100/0.5) (-(c_B-1)c_A)^{0.5} + (c_Ac_B)^{0.5}), \text{Current Ask}). After each transaction,
current bid is set to 0 and current ask is set to the initial cash balance of 4,000.
Comparisons across market games indicate that the allocative efficiency is the highest in sell-all markets (average 97.7 percent), medium in buy-sell markets (average 93.0 percent), and lower in double auction (average = 90.5 percent). Most experimental gaming results from double auction markets tend to report higher efficiencies (close to 100 percent). However, virtually all such experiments have been conducted in single market partial equilibrium settings. With human subjects, the efficiency dominance of double auction is not preserved in general equilibrium settings in the presence of complementarities across two or more markets. If the values across the markets were not complementary, the efficiencies would be higher. In MI simulations when traders are allowed to trade indefinitely and randomly, subject only the constraint that they do not submit bids or asks that might reduce their net payoff, the double auction markets always reach 100 percent efficiency.

The first panel of Figure 2 shows the efficiencies observed in the buy-sell market with 20 (10+10) human traders. The average efficiency across the 20 periods is 98.1 percent, as compared to 91.4 and 94.6 percent respectively in the two sessions with 10 (5+5) traders. The data are consistent with the conjecture that the market outcomes approach GCE as the number of trader increases.

5.2 Prices

Figure 3 shows the price charts for goods A and B for the six sessions of three market games, along with the respective competitive general equilibrium, non-cooperative equilibrium, and MI prices. In both sessions of the sell-all market, prices of goods A as well as B appear to be close to the competitive general equilibrium prediction of 20 (and NCE prediction of 20.12). Since the price support selected for MI simulations predicted a price of 15, the MI price series for both goods are located near 15.

Prices in the two sessions of buy-sell markets appear to be qualitatively different from the results of the sell-all markets and across the two sessions. In the first session, the prices of goods A and B lie around 10 which is about one half of the CGE and NCE price of 20. In the second session, prices of both goods lie much closer to the CGE price of 20.

---

10 See Gode, Spear and Sunder (2004) for an exception.
The prices of both goods from the two buy-sell sessions populated by MI traders, in contrast are distributed around 20, albeit with much higher variability as one would expect from randomness in agent behavior.

In the first session of the double auction market, range of prices (235-275) lay far above the CGE and NCE prediction of 100. In the second double auction session, these prices are lower (in the 155-265 range), but still significantly above the CGE-NCE prediction of 100. It is remarkable that these large deviations from equilibrium prices result in only a relatively small drop in the allocative efficiency of these auctions. As pointed out by Gode and Sunder (1993), the allocations (and therefore the efficiency) properties of the markets tend to be more robust than the prices.

A possible explanation for the divergence between the predicted price level and the actual price level in some of the games is that in spite of the theoretical power of backward induction in games of finite duration, the terminal conditions are not taken into account until close to the end of the session.

The mean transaction prices in the double auction simulations with MI traders are about 145, considerably above the CGE-NCE price of 100. However, towards the end of every trading period, the transaction prices converge consistently to the close neighbourhood of the CGE-NCE price of 100 (see Figure 4). In contrast, the DA markets with human traders exhibit no such tendency and most transactions are distributed around the period mean.

(Insert Figure 4 about here)

Table 5 shows that the double auction markets saw active trading. Each period was allowed to run for three minutes and we ran 10 and 11 periods in run 5 and 6 respectively for a total duration of 30 and 33 minutes, and 994 and 1,114 transactions respectively. This translates into about one transaction every two seconds and 20 transactions per trader per period on average in both runs. Recall that 20 (18) transactions per trader are necessary to reach CGE (NCE), with each trader buying 10 (9) units of the good he does not have and selling 10 (9) units of the good he is endowed with. However, as we saw in discussion of efficiency and symmetry, while the total number of transactions was close to CGE prediction, their distribution across traders showed greater variation. Beside, some traders traded bought as well as sold within the same market.
Finally, the second panel of Figure 2 shows the prices observed in the buy-sell market with 20 (10+10) human traders. The average price across the 20 periods is 16.44 for A for B, as compared to 11.3/9.2 and 19.9/16.3 respectively in the two sessions with 10 (5+5) traders. The price data do not show any marked tendency to be closer to GCE price as the number of trader increases from 10 to 20.

5.3 Symmetry

Figure 5 shows the asymmetry introduced by the oligopoly effect: when prices are the same trader gains an advantage from buying more of the good he is endowed with because of a feedback effect of income. This is independent of the dispersion of results. We see that symmetry is highest in the sell-all markets, lower in the buy-sell, and lowest in the double auction setting. The lower the symmetry the lower the average earnings, because skewed investment leads to lower earnings in the earning functions used in these experiments. Since in MI simulations of double auction traders are allowed to trade indefinitely, all traders do so until their holdings are perfectly symmetrical (and their payoffs reach the individual maximum).

The third panel of Figure 2 shows the symmetry of holdings observed in the buy-sell market with 20 (10+10) human traders. The average symmetry across the 20 periods is 0.81, as compared to 0.60 and 0.71 respectively in the two sessions with 10 (5+5) traders. The data are consistent with the conjecture that the market outcomes approach GCE as the number of trader increases.

5.4 Money Balances

The payoff functions were parameterized so that beyond a certain level we would expect that individuals would prefer to hold back rather than spend cash. Figure 6 compares actual money balances (money left unspent) against the four benchmarks of competitive equilibrium, non-cooperative equilibrium, autarky, and MI traders in sell-all and buy-sell games. Since money balances remain unchanged in double auction, they are not shown. We find that money balances in the sell-all markets came close to the CGE level of 33.33 percent, while in the buy-sell markets traders kept more of their money than CGE would predict, but close to the non-cooperative equilibrium of 60.98 percent.
As a consequence prices in sell-all markets are close to CGE-levels of 20 in the sell-all markets, but much lower in buy-sell markets. Our understanding for this finding is that traders in buy-sell markets were much more aware of their influence on prices of other people’s goods, than they were in the sell-all market.

(Insert Figure 6 about here)

The fourth panel of Figure 2 shows the unspent money holdings observed in the buy-sell market with 20 (10+10) human traders. The average unspent money across the 20 periods is 62 percent of the initial endowment, as compared to 74 and 60 percent respectively in the two sessions with 10 (5+5) traders. The money holdings data do not show a marked tendency to be closer to GCE prediction of 50 percent as the number of trader increases from 10 to 20.

5.5 Cross-sectional Standard Deviation of Individual Traders’ Earnings

The cross-sectional standard deviation of individual traders’ period earnings for the 10 (5+5) trader sessions is shown in Figure 7. It is 16 percent of the CGE earnings in the sell-all markets. In buy-sell markets (24 to 46 percent) and double auctions (26 to 36 percent) the standard deviation is higher. There is no evidence that the standard deviation declines through the replications over the periods of a session. In contrast, in the only 20 (10+10) trader session we ran for buy-sell market, the cross sectional standard deviation is much lower (an average of 11 percent) and declines steadily from approximately 24 percent in the first period to about 5 percent in the 20th period. It seems reasonable to concluded that there are no significant differences among the standard deviation of earnings across the three mechanisms, and no consistent tendency of the standard deviation to decrease over replications.

(Insert Figure 7 about here)

5.6 Trading Volume as a Percent of CGE Volume

Observed trading volume as a percent of CGE is shown in Figure 8. This volume is slightly higher in the buy-sell sessions (105.16 and 88.75 percent), although it is highly variable. Volume is lower in the sell-all and the double auctions (84.8 and 74.7 percent on average respectively). Both the (5+5) trader double auctions as well as the (10+10) trader buy-sell market exhibit a tendency for the trading volume to increase over periods of a session. No such tendency is present in the (5+5) trader sell-all and buy-sell markets.
5.7 Velocity and Quantity theory

Sell-all and buy-sell games do not allow much leeway for variations in velocity of money. Except for being able to hoard there is no strategic component to timing of trades. As the market meets only once per period and the quantity of money is well defined, in essence, the quantity theory of money holds by definition. In contrast, double auction allows the opportunity for money to turn over many times through trading within the same period.

Obtaining an operationally tight definitions of money, its velocity and the endogenous variations in velocity is a theoretical challenge. Without detailed microstructure, the concept of the velocity of money is not operational. To define velocity, one needs a clear understanding of what is meant by money; a measure of its quantity; and an operational descriptions of the individuals’ trading opportunity sets and strategies.

Our gaming set up assigns operational meaning to all of them albeit in a limited way. There is only one means of payment in the game. In the double auction market, in each period there is an implicitly defined minimal trading grid size, the minimal time for a trade to be offered and completed. The individuals have the strategic choice as to when to bid and thus influence velocity.

Table 5 shows the velocity (turnover) of both, money and goods. During the ten 180-second trading periods with 10 traders, 200 units of goods generated a volume of 994 (turnover rate of 5.0) in Session 5 and 1114 (turnover rate of 5.6) in Session 6. Total money stock of 40,000 was used to make gross payments/receipts of 252,363 (turnover of 6.3) in Session 5 and 214,716 (turnover of 5.4) in Session 6. In other words, each unit of money changed hands about six times during each session, and each unit of goods was traded more than five times. Because of the continuous trading in single units of goods, the total amount money needed to facilitate this trading was much less than what we provided. At the prices we observed (the maximum was 500) one can argue that 5,000 units of money should have been enough to move from initial endowment to CGE position in single unit transactions by traders if they alternate between selling an
endowed unit and buying a unit of the other good.\footnote{We have not yet conducted an experiment to verify whether providing a smaller amount of money will affect its velocity.} There is no straight forward way of translating this velocity observed in the laboratory to natural economies; these data would be useful in comparative studies of alternative mechanisms in laboratory environments.

5.8 Kurtosis and Autocorrelation

Financial market returns are known to exhibit excess kurtosis (fat tails relative to Gaussian distribution), show no significant autocorrelation of returns, but have significant autocorrelations of simple derivatives of returns, e.g. absolute or squared returns. The last finding hints at volatility clustering, as a significant autocorrelation of absolute returns shows that large price changes are more likely to occur after other price changes (e.g. Mandelbrot 1963a,b, Plott and Sunder 1982, Bouchaud and Potters 2001, Plerou et al. 1999, Cont 1997, 2001, and Voit 2003).

In the data generated from the double auction markets we find excess kurtosis (8.9 for good A and 9.0 for good B in Session 5 and 28.8 and 11.7 for goods A and B respectively in Session 6). These numbers are comparable to the excess kurtosis in the range of 5 to 20 found in stock market returns (variations depending on time horizon and whether you use tick data or daily closing prices).

As shown in Figure 9, there is no significant autocorrelation after lag 1 in the four series of laboratory returns we have, which is consistent with the price series being random walks. The lag 1 autocorrelation is a well-known result of bounce between bids and asks in the double-auction mechanism. The autocorrelation function of absolute returns, however, is consistently outside the significance bounds for both goods in Session 5 (but not in Session 6), suggesting the possibility—but no certainty—of volatility clustering in these laboratory markets.

(Insert Figure 9 about here)

6. Discussion

The study of three markets in this experiment is a first step toward a planned investigation of trade using individual IOU notes and trade utilizing bank loans with the possibility of default (see Huber, Shubik and Sunder, 2007). All three markets are closed, full feedback models with explicit price-formation mechanisms and trade involving some
form of money. Our second experiment examines the conditions under which personal credit can serve as a substitute for commodity, bank or government money. However, prior to formulating and running such an experiment it is desirable to study the price formation and market mechanisms first, without heavy emphasis on money and credit mechanisms.

The experiment reveals that (1) the non-cooperative and general competitive equilibrium models provide a reasonable anchor to locate most but not all the observed outcomes of the three market mechanisms; (2) there is some evidence that outcomes tend to get closer to GCE predictions as the number of players increases; (3) unlike well known results from many partial equilibrium double auctions, prices and allocations in our double auctions with full feedback reveal significant and apparently persistent deviations from CGE predictions; and (4) the outcome paths from the three market mechanisms exhibit significant and persistent differences among them.

As social institutions, mass market mechanisms may have evolved to minimize the importance of individual social psychological factors and that these experiments support this observation. They also suggest that the non-cooperative equilibrium approach is more fundamental than the competitive equilibrium, with the former encompassing the latter as a special limiting case. Furthermore the former requires the full specification of price formation mechanisms and the simplest of such mechanisms are studied here.

Given the structure presented here several natural extensions are to investigate “everyone a banker,” i.e. the use of personal credit; borrowing from and depositing in a government bank and the role of private banks in financing risky investment. These can all be modeled as straightforward extensions of the models presented here.
References


Shapley L.S. 1995 Class Notes  Mimeographed UCLA


Table 1: Non-cooperative Equilibria for Sell-all Model

(Parameter values used in the laboratory experiments: $a = 200; M = 6,000; \alpha = 10$)

<table>
<thead>
<tr>
<th>Number of Agents</th>
<th>Price(1)</th>
<th>Price(2)</th>
<th>Quantity(1)</th>
<th>Quantity(2)</th>
<th>Unspent money</th>
<th>Payoff</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>21.14</td>
<td>21.14</td>
<td>0.6277a</td>
<td>0.3723a</td>
<td>0.2953M</td>
<td>0.4834a</td>
</tr>
<tr>
<td>3</td>
<td>20.40</td>
<td>20.40</td>
<td>0.5838a</td>
<td>0.4162a</td>
<td>0.3200M</td>
<td>0.4929a</td>
</tr>
<tr>
<td>4</td>
<td>20.20</td>
<td>20.20</td>
<td>0.5626a</td>
<td>0.4374a</td>
<td>0.3267M</td>
<td>0.4961a</td>
</tr>
<tr>
<td>5</td>
<td>20.12</td>
<td>20.12</td>
<td>0.5501a</td>
<td>0.4499a</td>
<td>0.3293M</td>
<td>0.4975a</td>
</tr>
<tr>
<td>10</td>
<td>20.03</td>
<td>20.03</td>
<td>0.5250a</td>
<td>0.4750a</td>
<td>0.3323M</td>
<td>0.4994a</td>
</tr>
<tr>
<td>many</td>
<td>20.00</td>
<td>20.00</td>
<td>0.5000a</td>
<td>0.5000a</td>
<td>0.3333M</td>
<td>0.5000a</td>
</tr>
</tbody>
</table>

Table 2: Non-cooperative Equilibria in Buy-sell Market

(Parameter values used in the laboratory experiments: $a = 200; M = 4,000; \alpha = 10$)

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<thead>
<tr>
<th>Number of Agents</th>
<th>Price(1)</th>
<th>Price(2)</th>
<th>Quantity(1)</th>
<th>Quantity(2)</th>
<th>Unspent money</th>
<th>Payoff</th>
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<td>2</td>
<td>20.00</td>
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<td>0.8000a</td>
<td>0.2000a</td>
<td>0.8000M</td>
<td>0.4000a</td>
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<tr>
<td>3</td>
<td>20.00</td>
<td>20.00</td>
<td>0.6923a</td>
<td>0.3077a</td>
<td>0.6923M</td>
<td>0.4615a</td>
</tr>
<tr>
<td>4</td>
<td>20.00</td>
<td>20.00</td>
<td>0.6400a</td>
<td>0.3600a</td>
<td>0.6400M</td>
<td>0.4800a</td>
</tr>
<tr>
<td>5</td>
<td>20.00</td>
<td>20.00</td>
<td>0.6098a</td>
<td>0.3902a</td>
<td>0.6098M</td>
<td>0.4878a</td>
</tr>
<tr>
<td>10</td>
<td>20.00</td>
<td>20.00</td>
<td>0.5525a</td>
<td>0.4475a</td>
<td>0.5525M</td>
<td>0.4972a</td>
</tr>
<tr>
<td>many</td>
<td>20.00</td>
<td>20.00</td>
<td>0.5000a</td>
<td>0.5000a</td>
<td>0.5000M</td>
<td>0.5000a</td>
</tr>
</tbody>
</table>
Table 3: Design Parameters for Six Sessions of Three Market Games

<table>
<thead>
<tr>
<th>Runs</th>
<th>Market Game</th>
<th>Endowments of Individuals</th>
<th>Money carried over?</th>
<th>Payoff function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1+2</td>
<td>Sell-All</td>
<td>200 for 5 traders; 0 for 5 others</td>
<td>Yes</td>
<td>$10(c_{ACB})^{0.5}$ each period +0.25 final money bal.</td>
</tr>
<tr>
<td>3+4</td>
<td>Buy-Sell</td>
<td>200 for 5 traders; 0 for 5 others</td>
<td>Yes</td>
<td>$10(c_{ACB})^{0.5}$ each period +0.25 final money bal.</td>
</tr>
<tr>
<td>5+6</td>
<td>Double Auction</td>
<td>20 for 5 traders; 0 for 5 others</td>
<td>No</td>
<td>$100(c_{ACB})^{0.5}$ +0.5 final money bal.</td>
</tr>
</tbody>
</table>

Table 4 Equilibrium Predictions for the Three Market Games

<table>
<thead>
<tr>
<th>Runs</th>
<th>Market Game</th>
<th>Autarky</th>
<th>General Equilibrium</th>
<th>Non-cooperative Equilibrium</th>
<th>Minimally Intelligent (MI) Traders</th>
</tr>
</thead>
<tbody>
<tr>
<td>1+2</td>
<td>Sell-All</td>
<td>$P_A = P_B = NA$</td>
<td>$X_A = 200 or 0$ $X_B = 200 or 0$ Net money = 0 Points = 0</td>
<td>$P_A = P_B = 20$ $X_A = X_B = 100$ Net money = 0 Points = 1,000</td>
<td>$P_A = P_B = 14.50$ $X_{own} = 100$ (avg.) $X_{other} = 100$ (avg.) Net money = 0 Points = 897</td>
</tr>
<tr>
<td>3+4</td>
<td>Buy-Sell</td>
<td>$P_A = P_B = NA$</td>
<td>$X_A = 200 or 0$ $X_B = 200 or 0$ Net money = 0 Points = 0</td>
<td>$P_A = P_B = 20$ $X_A = X_B = 100$ Net money = 0 Points = 1,000</td>
<td>$P_A = P_B = 20$ $X_{own} = 100$ (avg.) $X_{other} = 100$ (avg.) Net money = 0 Points = 897</td>
</tr>
<tr>
<td>5+6</td>
<td>Double Auction</td>
<td>$P_A = P_B = NA$</td>
<td>$X_A = 20 or 0$ $X_B = 20 or 0$ Net money = 0 Points = 0</td>
<td>$P_A = P_B = 100$ $X_A = 20 or 0$ $X_B = 20 or 0$ Net money = 0 Points = 1,000</td>
<td>$P_A = P_B = 145$ $X_A = 10$ (avg.) $X_B = 10$ (avg.) Net money = 0 Points = 1,000</td>
</tr>
<tr>
<td></td>
<td>Goods in market</td>
<td>Money in market</td>
<td>Goods traded</td>
<td>Money paid</td>
<td>Turnover stocks</td>
</tr>
<tr>
<td>-------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-------------</td>
<td>------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Run 5</td>
<td>200</td>
<td>40,000</td>
<td>994</td>
<td>252,362</td>
<td>5.0</td>
</tr>
<tr>
<td>Run 6</td>
<td>200</td>
<td>40,000</td>
<td>1,114</td>
<td>214,716</td>
<td>5.6</td>
</tr>
</tbody>
</table>
Figure 1: Efficiency of Allocations (Average Earnings) for n = 5+5

Sell-All (Run 1, Mean=97.7)

Sell-All (Run 2, Mean=97.6)

Buy-Sell (Run 3, Mean=91.4)

Buy-Sell (Run 4, Mean=94.6)

Double Auction (Run 5, Mean=87.8)

Double Auction (Run 6, Mean=93.2)
Figure 2: Performance of Buy-Sell Market with $n = 10 + 10$

Legend: please refer to the corresponding figures 1, 3, and 5 to 8.

- Efficiency (Mean=98.1)
- Prices (Mean=16.4 for A, 16.5 for B)
- Symmetry (Mean=0.81)
- Unspent money (Mean=62.45 percent)
- Standard Dev. Of Earnings (Mean=118)
- Trade as % of trade needed to achieve GE
Figure 3: Transaction Price Levels for $n = 5+5$

Sell-All (Run 1, Avg. $A=18.92$; $B=20.90$)

Sell-All (Run 2, Avg. $A=21.52$; $B=20.49$)

Buy-Sell (Run 3, Avg. $A=11.32$; $B=9.24$)

Buy-Sell (Run 4, Avg. $A=19.89$; $B=16.34$)

Double Auction (Run 5, Avg. $A=261$; $B=246$)

Double Auction (Run 6, Avg. $A=225$; $B=170$)

Huber, Shubik, and Sunder, Three Minimal Market Institutions, 7/24/2008
Figure 4: Double Auction Transaction Price Paths within individual Trading Periods with MI traders

Run 5: Transaction Sequence No.

Prices

Run 6: Transaction Sequence No.

Prices

Huber, Shubik, and Sunder, Three Minimal Market Institutions, 7/24/2008
Figure 5: Symmetry of Allocations for n = 5+5

Sell-All (Session 1, Mean=0.76)

Sell-All (Session 2, Mean=0.71)

Buy-Sell (Session 3, Mean=0.60)

Buy-Sell (Session 4, Mean=0.71)

Double Auction (Session 5, Mean=0.53)

Double Auction (Session 6, Mean=0.60)
Figure 6: Unspent money as a percentage of initial endowment for \( n = 5+5 \) traders

Sell-All (Session 1, Mean=33.63)  
Sell-All (Session 2, Mean=29.99)

Buy-Sell (Session 3, Mean=73.72)  
Buy-Sell (Session 4, Mean=60.47)
Figure 7: Standard Deviation of Earnings per Period

- **Sell-All (Session 1, Mean=150)**
- **Sell-All (Session 2, Mean=165)**
- **Buy-Sell (Session 3, Mean=463)**
- **Buy-Sell (Session 4, Mean=243)**
- **Double Auction (Session 5, Mean=261)**
- **Double Auction (Session 6, Mean=362)**

Huber, Shubik, and Sunder, Three Minimal Market Institutions, 7/24/2008 36
Figure 8: Goods traded as Percentage of Trade needed to achieve GE

Sell-All (Session 1, Mean=86.63)

Sell-All (Session 2, Mean=82.97)

Buy-Sell (Session 3, Mean=105.16)

Buy-Sell (Session 4, Mean=88.75)

Double Auction (Session 5, Mean=68.40)

Double Auction (Session 6, Mean=80.91)

Huber, Shubik, and Sunder, Three Minimal Market Institutions, 7/24/2008 37
Figure 9: Autocorrelation functions of returns and absolute returns

Run 5

Autocorrelation functions of returns and absolute returns for good A in run 1

Autocorrelation functions of returns and absolute returns for good B in run 1

Run 6

Autocorrelation functions of returns and absolute returns for good A in run 2

Autocorrelation functions of returns and absolute returns for good B in run 2
Appendix A: Experimental instructions

Market Game 1: Sell-All (with money carried over), Sessions 1 and 2

This is an experiment in market decision making. The instructions are simple, and if you follow them carefully and make good decisions, you will earn more money, which will be paid to you at the end of the session.

This session consists of several periods and has 10 participants. At the beginning of each period, five of the participants will receive as income the proceeds from selling 200 units of good A, for which they have ownership claim. The other five are entitled to the proceeds from selling 200 units of good B. In addition you will get 6,000 units of money at the start of the experiment. Depending on how many goods A and B you buy and on the proceeds from selling your goods this amount will change from period to period.

During each period we shall conduct a market in which the price per unit of A and B will be determined. All units of A and B will be sold at this price, and you can buy units of A and B at this price. The following paragraph describes how the price per unit of A and B will be determined.

In each period, you are asked to enter the amount of cash you are willing to pay to buy good A, and the amount you are willing to pay to buy good B (see the center of Screen 1). The sum of these two amounts cannot exceed your current holdings of money at the beginning of the period.

The computer will calculate the sum of the amounts offered by all participants for good A. (= Sum_A). It will also calculate the total number of units of A available for sale (n_A, which will be 1,000 if we have five participants each with ownership claim to 200 units of good A). The computer then calculates the price of A, P_A = Sum_A/n_A.

If you offered to pay b_A to buy good A, you will get b_A/P_A units of good A. The same procedure is carried out for good B.

Your final money balance will be your money at the beginning of the period plus the money from the sales of your initial entitlement to proceeds from A or B less the amount you pay to buy A and B:

New money holdings = Money at start of period + P_A*#A + P_B*#B – b_A – b_B

With #A and #B being either 200 or zero.

The number of units of A and B you buy (and consume), will determine the number of points you earn for the period:

Points earned = 10 * (b_A/P_A * b_B/P_B)\(^0.5\)

Example: If you buy 100 units of A and 100 units of B in the market you earn

\[ 10 * (100 * 100)^{0.5} = 1,000 \text{ points}. \]

Your money holdings will only be relevant in the last period. At this time the starting endowment of 6,000 units of money will be deducted from your final money holdings. The net holdings, positive or negative, will be divided by 4 and this number will be added to your total points earned.

Screen 2 shows the example of calculations for Period 3. There are 10 participants in the market, and half of them have 200 units of A, the other half 200 units of B. Here we see a subject entitled to proceeds from 200 units of good A.
Screen 1:

Period 1

You have:

Ownership claims for units of good A: 0
Ownership claims for units of good B: 200
Units of money: 6000

Amount you offer to pay to buy A: 1
Amount you offer to pay to buy B: 1

Screen 2:

Period 2

Total amount offered for A: 18870
Price of A: 18.9

Units of A sold for you: 200
Proceeds from sales of A: 3775

Units of A you bought and consumed: 117
Payment for buying A: 2200

Total amount offered for B: 21470
Price of B: 21.5

Units of B sold for you: 9
Proceeds from sales of B: 39

Points earned:
Money at start of period: 6210
Proceeds from selling A and B: 3775
Payment for buying A and B: 4100
New money holdings: 5884

Total earnings this period: 1015

<table>
<thead>
<tr>
<th>period</th>
<th>price A</th>
<th>consumption of A</th>
<th>price B</th>
<th>consumption of B</th>
<th>Money</th>
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<th>cumulative earnings</th>
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</table>
The earnings of each period (shown in the last column in the lower part of Screen 2) will be added up at the end of session. At the end they will be converted into real Dollars at the rate of 1,000 points = 1 US$, and this amount will be paid out to you.

**How to calculate the points you earn:**

Points earned = 10 * \((b_A/P_A * b_B/P_B)^{0.5}\)

To give you an understanding for the formula the following table might be useful. It shows the resulting points from different combinations of goods A and B. It is obvious, that more goods mean more points, however, to get more goods you usually have to pay more, thereby reducing your money balance, which will limit your ability to buy in later periods.

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<td>1936</td>
<td>2092</td>
<td>2236</td>
<td>2372</td>
<td>2500</td>
</tr>
</tbody>
</table>

Examples:

1) If you buy 50 units of good A and 75 units of good B and both prices are 20, then your points from consuming the goods are 612. Your net change in money is 200 (A or B) * 20 = 4,000 minus 50 * 20 – 75 * 20 = 1,500, so you have 1,500 more to spend or save in the next period.

2) If you buy 150 units of good A and 125 units of good B and both prices are 20, then your points from consuming the goods are 1369. Your net cash balance is 200 (A or B) * 20 = 4,000 minus 150 * 20 – 125 * 20 = -1,500, so you have 1,500 less to spend or save in the next period.
Market Game 2: Buy-Sell (with money carried over), Sessions 3 and 4

This is an experiment in market decision making. The instructions are simple, and if you follow them carefully and make good decisions, you will earn more money, which will be paid to you at the end of the session.

This session consists of several periods and has 10 participants. At the beginning of each period, five of the participants will receive ownership claim to 200 units of good A, and the other five will receive ownership claim to 200 units of good B. In addition each participant will get 4,000 units of money at the start of period 1 of the experiment.

Each participant is free to sell any or all the goods he/she owns for units of money. The amount of your money balance will change depending on the proceeds from selling your goods, and how many units of goods A and B you buy, and this balance will be carried over from period to period.

During each period we shall conduct a market in which the price per unit of A and B will be determined. All units of A and B will be sold at this price, and you can buy units of A and B at this price. The following paragraphs describe how the price per unit of A and B will be determined.

In each period, you are asked to enter the cash you are willing to pay to buy the good you do not own (say A), and the number of units of the good you own that you are willing to sell (say B) (see the center of Screen 1). The cash you bid to buy cannot exceed your money balance at the beginning of the current period, and the units you offer to sell cannot exceed your ownership claim of that good (200).

The computer will calculate the sum of the amounts of money offered by all participants for good A. (= Sum_A). It will also calculate the total number of units of A offered for sale (q_A), and determine the price of A, P_A = Sum_A/q_A.

If you offered to pay b_A to buy good A, you will get to buy b_A/P_A units of good A. The same procedure is carried out for good B to arrive at the price P_B = Sum_B/q_B and the number of units you buy = b_B/P_B.

The amount of money you pay to buy one good is subtracted, and the proceeds from the sale of the other good are added, to your initial money balance of 4,000, in order to arrive at your final money balance.

Both goods are perishable and must be either sold or consumed in the current period. The number of units of A and B you own at the end of the period, c_A and c_B (unsold units of owned good and purchased units of the other good) will be consumed and determine the number of points you earn for the period:

Points earned = 10 * (c_A * c_B)^0.5

Example: If you sell 75 units of A and buy 90 units of B in the market you earn

10 * ((200-75) * 90)^0.5 = 1,061 points.

Your cash balance holdings will help determine the points you earn only in the last period. At this time the starting endowment of 4,000 units of money will be deducted from your final money holdings. The net holdings (which may be negative) will be divided by 2 and this number will be added to (or subtracted from) your total points earned.
Screen 1:

You have:
- Units of good A you own: 0
- Units of good B you own: 200
- Units of money: 5600

Amount you offer to pay to buy A

Units of B you sell

Screen 2 shows an example of calculations for Period 2. There are 10 participants in the market, and half of them have 200 units of A, the other half 200 units of B. Here we see a subject starting with 200 units of good A.

Information on bids and transactions in good A
Information on bids and transactions in good B
Earnings calculation
Cumulative earnings so far. This number/1000 will be the US-$ you get

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<th>Period</th>
<th>Remaining time [sec]</th>
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<table>
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<th>period</th>
<th>price A</th>
<th>consumption of A</th>
<th>price B</th>
<th>consumption of B</th>
<th>Money</th>
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<th>cumulative earnings</th>
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The earnings of each period (shown in the last column in the lower part of Screen 2) will be added up at the end of session. At the end they will be converted into real Dollars at the rate of 1,000 points = 1 US$ and this amount will be paid out to you.

**How to calculate the points you earn:**

The points earn each period are calculated with the following formula:

Points earned = 10 * \((c_A * c_B)^{0.5}\)

The following table may be useful to understand this relationship. It shows the resulting points from different combinations of goods A and B. Consuming more goods means more points. However, to **consume** more goods now you usually have to buy more and sell less, reducing your cash balance carried into the future.

<table>
<thead>
<tr>
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<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
<th>225</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units of A you <strong>buy</strong> and consume</td>
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<td></td>
<td></td>
<td></td>
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<td>2092</td>
<td>2236</td>
<td>2372</td>
<td>2500</td>
<td></td>
</tr>
</tbody>
</table>

Examples:

1) If you sell 150 units of good A at a price of 25 (keeping 50) and buy 125 units of good B at a price of 22, you earn 612 (= 50*125) points from consuming the goods in the current period, and your net cash balance carried over to the following period changes by +1,000 (= 150 * 25 – 125 *22). You have 1,000 in cash to spend in the future.

2) If you buy 150 units of good A and sell 75 units of good B (keeping 125) and both prices are 20, then your points from consuming the goods are 1369. Your net cash balance changes by -1,500 (= -150 * 20 + 75* 20), so you have 1,500 less to spend in the future.
Market Game 3: Double Auction (money not carried over), Sessions 5 and 6

This is an experiment in market decision making. The instructions are simple, and if you follow them carefully and make good decisions, you will earn more money, which will be paid to you at the end of the session.

This session consists of several periods and has 10 participants. At the beginning of each period, five of the participants will receive 20 units of good A, and the other five will receive 20 units of good B. In addition each participant will get 4,000 units of money at the start of period 1 of the experiment (see top of Screen 1).

Each participant is free to sell any or all the goods he/she owns, or buy more units for money. The amount of your money balance will change depending on the proceeds from selling or buying goods A and B, and this balance will be carried over from period to period.

During each period we shall conduct a market in which t A and B will be traded in a double auction. The following paragraphs describe how A and B can be traded.

Trading

See Screen 1. There is a chart of transaction prices on the left, followed by two columns to trade Good A and two columns to trade Good B.

You can buy or sell one unit of either good in each transaction. You can buy goods in one of two ways:

1. Enter a bid price in the light blue box above the red BID button on your screen, click on this red button, and wait for some trader to accept your bid (i.e., sell to you at your bid price); or
2. Click on the red BUY button to buy one unit of the good at the price listed at the top of the ASK column above this red button.

Similarly, you can sell one unit of either good in one of two ways:

1. Enter an ask price in the light blue box above the red ASK button on your screen, click on this red button, and wait for someone else to accept your ask (i.e., buy from you at your ask price); or
2. Click on the SELL red button to sell one unit of a good at the price listed at the top of the BID column above this red button.

You may enter as many bids and asks as you wish. A new bid (to buy) is allowed only if you have sufficient amount of cash on hand in case all your outstanding bids are accepted (to prevent your cash holdings from dropping below zero). A new ask (to sell) is allowed if you have sufficient units of goods to sell in case all your asks are accepted (to prevent your units of goods from falling below zero).

Both goods are perishable and must be either sold or consumed in the current period. The number of units of A and B you own at the end of the period, cA and cB will be consumed and determine the number of points you earn for the period:

Points earned = 100 * (cA * cB)^{0.5}

Example: If you sell own 7 units of A and 12 units of B at the end of period, you earn

100 * (7 * 12)^{0.5} = 916.5 points.

Your cash balance holdings will help determine the points you earn only in the last period. At this time the starting endowment of 4,000 units of money will be deducted from your final money holdings. The net holdings (which may be negative) will be divided by 2 and this number will be added to (or subtracted from) your total points earned.
Screen 1 shows an example of calculations for Period 2.

Screen 2 shows an example of calculations for Period 2.

The earnings of each period (shown in the last column in the lower part of Screen 2) will be added up at the end of session. At the end they will be converted into real Dollars at the rate of 500 points = 1 US$ and this amount will be paid out to you.
How to calculate the points you earn:
The points earned each period are calculated with the following formula:
Points earned = 100 * (c_A * c_B)^0.5
The following table may be useful to understand this relationship. It shows the resulting points from different combinations of goods A and B. Consuming more goods means more points. However, to consume more goods now you usually have to buy more and sell less, reducing your cash balance carried into the future.

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<thead>
<tr>
<th>Units of A you consume</th>
<th>Units of good B you consume</th>
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<td>35</td>
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<td>40</td>
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</table>

Example: If you sell 15 units of good A (keeping 5) and buy 12 units of good B you earn 775 (= 100*(5 * 12)^0.5) points from consuming the goods in the current period.
Appendix B
Calculations for Sell-All
Notation
\( b_{ij} = \) the bid of individual \( i \) \((i=1,\ldots,n)\) of type \( j \) \((j=1,2)\) in market \( k \) \((k=1,2)\)
\( A = \) utility function scaling parameter
\( p_k = \) price of commodity \( k \)
\( m = \) initial money holding of each trader
\( (a,0) = \) initial holding of goods of type 1
\( (0,a) = \) initial holdings of goods of type 2.

The individual of type 2 wishes to maximize his payoff function which is of the form:

\[
\Pi = A \left[ \frac{b_1^1 b_2^1}{p_1 p_2} + (m - b_1^1 - b_2^2 + p_2 a) \right]
\]

The calculation for the sell-all model requires to solution of the two equations derived for each trader from the first order conditions on the bidding in the two goods markets. By symmetry we need only be concerned with one type of trader.

We obtain the equation

\[
b_2 \left( (n-1)b_1 + nb_2 \right) = \frac{n}{n-1} \left( nb_1 + (n-1)b_2 \right)
\]

As \( n \) becomes large this yields \( b_1 = b_2 \). Substituting in for \( b_1 \) in terms of \( b_2 \) we can calculate Table 1.

Calculations for buy-sell

The payoff function for the buy-sell market is given by

\[
\Pi = A \left[ \frac{b_1^1 (a - q_2^1)}{p_1 p_2} + (m - b_1^1 - b_2^2 + p_2 q_2^1) \right]
\]

where \( q_k^i \) is the amount of good \( k \) offered for sale by individual \( i \) in market \( j \)

We obtain from individual maximization of these equations the following values

\[
b = \frac{Aa(n-1)^2}{2(n^2 + n - 1)}
\]

\[
q = \frac{a(n-1)^2}{2(n^2 + n - 1)}
\]

These are utilized to calculate Table 2.
APPENDIX C: Algorithm Used for Double Auction with Minimally Intelligent (MI) Traders

Total number of traders = n
Endowment: \( E_A/E_B/M \)
Current balances at any point of time during trading: \( c_A/c_B/m \)

Randomly pick one of the n traders in the market with equal probability (with replacement)
For the chosen trader, randomly pick one of the two markets with equal probability (with replacement).

For the chosen market, randomly pick bid or ask with equal probability (with replacement)
1. If bid is picked for the chosen trader for the chosen market A:
   Calculate \( d = \frac{2}{3} 100 \left( (c_A+1)c_B^{0.5} - (c_Ac_B)^{0.5} \right) \). Pick a uniform random number \( U \sim (0,d) \), and submit it as a bid for A.

2. 1. If bid is picked for the chosen trader for the chosen market B:
   Calculate \( d = \frac{2}{3} 100 \left( (c_B+1)c_A^{0.5} - (c_Ac_B)^{0.5} \right) \). Pick a uniform random number \( U \sim (0,d) \), and submit it as a bid for B.

3. If ask is picked for the chosen trader for the chosen market A:
   Calculate \( e = (2) 100 \left( -(c_A-1)c_B^{0.5} + (c_Ac_B)^{0.5} \right) \). Pick a uniform random number \( U \sim (e, M) \), and submit it as an ask for A.

4. 1. If ask is picked for the chosen trader for the chosen market B:
   Calculate \( e = (2) 100 \left( -(c_B-1)c_A^{0.5} + (c_Ac_B)^{0.5} \right) \). Pick a uniform random number \( U \sim (e, M) \), and submit it as an ask for B.

Let it run for sufficient number of periods until twice the time after the last transaction.
Use the final \( c_A, c_B, \) and \( m \) for calculating earnings of each trader.

\[
q = \frac{a(n-1)^2}{2(n^2 + n - 1)}
\]