Lecture 12: Portfolio Theory & CAPM

How are risk and return related?
- What kinds of risk do we care about?
- For bearing more risk, how much extra return should we get?

Brealey and Myers, Chapters 7 and 8
Reader, Chapters 13 and 14
Variance as a measure of Risk

- A person's utility depends only on the mean and variance of his/her portfolio if either of these assumptions holds:
  - Stock returns are normally distributed; or
  - The utility function is quadratic in return.

- In this case, the variance or standard deviation of return completely measures the risk of an investment.

- Now consider how an investor should invest
  - The investor faces many securities (stocks) to invest in.
  - Means and variances of each stock are known.
Choices of Possible Securities

Could this be the “optimal” stock?

Dominating Security Region

Risk-free Interest rate

Expected return < r_f

What if we form a portfolio of two stocks?
Forming portfolios of 2 stocks

Why does line arc more, the lower the correlation?
Combining more than 2 stocks

How far to the left can we go?

E(R1) 15.000%
E(R2) 25.000%
E(R3) 35.000%
SD(R1) 24.000%
SD(R2) 20.000%
SD(R3) 27.000%

Portfolios of 3 stocks
Minimum variance frontier

Might an investor pick this portfolio?
The optimal portfolio for any investor must lie on the top half of the *minimum variance frontier*
  – This is called the *efficient frontier*.
  – No other portfolios offer a higher return for a given SD.
  – Different investors may choose different portfolios.

How would you find the portfolios on the minimum variance frontier?
  – Minimize variance subject to expected return = \( r \).
Combining the risk-free asset with a risky portfolio

Form a portfolio, \( P \), by investing \( w_s \) in risky portfolio \( S \), and \((1 - w_s)\) in risk-free asset:

- **Expected return:** \( r_P = (1-w_s)r_f + w_s r_S = r_f + w_s(r_S-r_f) \)
- **Variance:**
  \[
  \text{Var}[(1 - w_S)r_f + w_S\tilde{r}_S] = (1 - w_S)^2 \times 0 + w_S^2 \text{Var}(\tilde{r}_S) + 2w_S(1 - w_S) \times 0 \\
  = w_S^2 \text{Var}(\tilde{r}_S).
  \]

- Or, more compactly, \( \sigma_p = w_s \sigma_s \)
- Both standard deviation and expected return are **linear** functions of \( w_S \).
Combining risky portfolios with the riskless asset

S.D.

\[
\text{Slope} = \frac{(r_P - r_f)}{\sigma_P}
\]

Portfolio P

Portfolio Q

\[
\sigma_P
\]

\[
(r_P - r_f)
\]
Example combining risk-free asset with risky portfolios

- Mighty Big Returns (MBR) has an expected return of 15%, and a standard deviation of 40%.
- Sleep Well Fund (SWF) has an expected return of 5%, and a standard deviation of 10%.
- The risk-free rate is 3%.
- You can only invest in one of the two funds (e.g. because of minimum purchase requirements)
  - You may also buy the risk-free asset.
- Which fund would you choose?
Goal: Find the fund with the higher slope, \((r_s - r_f) / \sigma_s\).

- This slope (Sharpe ratio) measures extra return for each unit of risk.

- MBR: \((15 - 3) / 40 = .3\)
- SWF: \((5 - 3) / 10 = .2\)
Generally, what’s the optimal combination? \( (r_f = 10\%) \)

Portfolios of 3 stocks

- Capital market line
- Portfolio S plus lending
- Portfolio S plus borrowing

\( E(R) \) vs. \( \Sigma \)
The Tangency Portfolio, S

- The optimal portfolio combines S with the risk free asset in some proportions.
- All other portfolios produce a lower line, leading to a lower return for the same standard deviation.
- The line connecting the risk free asset to S is known as the capital market line.
- Key point: Every investor chooses the same risky portfolio S.
  - Differ only in the percent allocated to the risk-free asset
- Q: Do all stocks lie on the capital market line?