Lecture 13: CAPM continued

Questions: How are risk and return related?
- What kinds of risk do we care about?
- For bearing more risk, how much extra return do we get?

Readings:
-- Brealey and Myers, Chapters 7 and 8
-- Reader, Chapters 13 and 14 (yes, both chapter 14’s…)
Portfolios of 2 stocks

![Graph showing portfolios of 2 stocks with E(R) on the y-axis and Sigma on the x-axis.]

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Portfolios of more than 2 stocks

Minimum variance frontier
Top half is the “efficient frontier”
Might an investor pick this portfolio?
Combining risky portfolios with the riskless asset

Note: Every investor chooses the same risky portfolio, S.
What is portfolio S?

- Suppose S contained no IBM. What would happen?
- Suppose S contained 90% Pets.com? What would happen?
- In *equilibrium*, when we add together everyone’s holdings of portfolio S, we must have every share in every company.
- So S must be the *market portfolio*, M.
  - A portfolio that contains every investment in the economy in proportion to their total value.
  - Denote its return $r_m$. 
We already have one key finding: All investors ought to split their money between the **market portfolio** and the risk-free asset.

In practice, buy an **index fund**, or an **exchange traded fund (ETF)**, such as a **SPDR**.

These are funds designed to track a market **index**.
- Most commonly used index: S&P 500 (**^SPX**)
  - ETF: **SPY**
- A broader index is the Wilshire 5000 (**^TMW**)
  - ETF: **WFIVX**
  - 5,000 stocks vs. 500
  - S&P stocks represent c. 77% of the Wilshire 5000 by value

Is the Wilshire 5,000 really the market portfolio?
Wilshire 5,000 vs. S&P 500

Exchange provides no volume data.

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What about our original question?

- How does this help with our original question, though: how to calculate r?
- We’re almost there…
Mathematically, the market portfolio being the tangency portfolio tells us that for any asset $i$,

$$r_i = r_f + \frac{\text{Cov}(\tilde{r}_i, \tilde{r}_m)}{\text{Var}(\tilde{r}_m)} \left[ E(\tilde{r}_m) - r_f \right],$$

$$= r_f + \beta_i \left[ E(\tilde{r}_m) - r_f \right].$$

- The **Capital Asset Pricing Model (CAPM)**.
- This is the relationship between risk and return we have been looking for.
The CAPM

- Expected return is related to the stock’s “beta”
  - Depends on covariance with the market: Market Risk
  - Does not depend on its variance: Firm Specific Risk
  - Remember our diversification example…

\[
\begin{align*}
  r_i &= r_f + \beta_i [E(\tilde{r}_m) - r_f], \\
  \beta_i &= \frac{\text{Cov}(\tilde{r}_i, \tilde{r}_m)}{\text{Var}(\tilde{r}_m)}.
\end{align*}
\]
The Security Market Line

Do all stocks lie on the Security Market Line?
# Capital Market Line versus Security Market Line

<table>
<thead>
<tr>
<th>Capital Market Line</th>
<th>Security Market Line</th>
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</thead>
<tbody>
<tr>
<td><strong>Vertical Axis</strong></td>
<td></td>
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<tr>
<td>Expected return</td>
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</tr>
<tr>
<td><strong>Horizontal Axis</strong></td>
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<tr>
<td>Standard Deviation</td>
<td>Beta</td>
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<td><strong>Line shows</strong></td>
<td></td>
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<tr>
<td>Portfolios mixing optimal portfolio with risk-free asset</td>
<td>Individual stock expected return as function of its Beta</td>
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<td><strong>What lies on the line?</strong></td>
<td></td>
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<td>Only combinations of optimal portfolio and $r_f$</td>
<td>All stocks</td>
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**Individual stock expected return as a function of its Beta**

**Line shows:**
- **Capital Market Line**
  - Portfolios mixing optimal portfolio with risk-free asset
- **Security Market Line**
  - All stocks

**What lies on the line?**
- **Capital Market Line**
  - Only combinations of optimal portfolio and $r_f$
- **Security Market Line**
  - All stocks
What is the expected return on a stock with $\beta = 0$?
- Answer is $r_f$: Same return as risk-free asset! Why?
- Think about insuring lots of houses against fire.
  » Each individually is risky, but
  » When you diversify by insuring lots of houses, overall portfolio becomes riskless.

What is the expected return on a stock with $\beta < 0$?

If $\beta < 0$, then $r_i < r_f$! You’d accept a return lower than the risk-free rate? Why?