Lecture 16: Capital Budgeting, Beta, and Cash Flows

Reading:
- Brealey and Myers, Chapter 9
- Lecture Reader, Chapter 15

Topics:
- Final topics on basic CAPM
- Debt, Equity, and Asset Betas
- Leveraged Betas
- Operating Leverage
Consider a portfolio of $n$ securities, with weights $w_1, \ldots, w_n$.

The beta of the portfolio, $\beta_p$, is given by

$$\beta_p = w_1\beta_1 + w_2\beta_2 + \ldots + w_n\beta_n.$$ 

The expected return on the portfolio is

$$r_p = w_1r_1 + w_2r_2 + \ldots + w_nr_n = r_f + \beta_p(r_m - r_f).$$
### Example ($100 Total Value Portfolio)

<table>
<thead>
<tr>
<th>Asset</th>
<th>$\beta$</th>
<th>$w$</th>
<th>$$$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-.25</td>
<td>-$25</td>
</tr>
<tr>
<td>2</td>
<td>.5</td>
<td>1.5</td>
<td>$150</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-.75</td>
<td>-$75</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>.5</td>
<td>$50</td>
</tr>
</tbody>
</table>

The portfolio $\beta$ is:

$$\beta_p = -.25(0) + 1.5(.5) - .75(1) + .5(1.5) = .75$$
Beta and the Market Portfolio

The market portfolio’s beta equals exactly 1 since:

$$\beta_m = \frac{\text{Cov}(\bar{r}_m, \bar{r}_m)}{\text{Var}(\bar{r}_m)} = \frac{\text{Var}(\bar{r}_m)}{\text{Var}(\bar{r}_m)} = 1$$

Therefore the weighted sum of all the betas in the economy equals one.
Graham and Harvey (1999) sent a survey to CFOs of all Fortune 500 companies, plus the 4,440 members of the Financial Executives Institute.

Main findings:

- 74.9% of respondents (almost) always use NPV!!
  » Compares with 9.8% found by Gitman/Forrester (1977).
- 73.5% set discount rate using CAPM!!
- 58.8% would use a single company-wide discount rate for all projects, regardless of type..

See BM pp. 199-211 for more discussion
Any cash flow can be an asset with its own $\beta$.

For example, a company with many divisions may have a different $\beta$ for each division’s revenues.

- a division with $\beta = 1.5$ will require a higher discount rate than a division with $\beta = .75$.

Even individual machine cash flows may have $\beta$:

- Further, the machine’s expense cash flows and its revenue cash flows may have different $\beta$.

Now, what effect does Debt have on $\beta$?
Debt, Equity and Asset Betas

- The profits generated by a firm's assets are distributed to its debt and equity holders.

- Therefore, one can think of a firm's assets as equivalent to a portfolio of debt and equity.
  - \( A = \) dollar value of the firm's assets.
  - \( D = \) dollar value of the firm's debt.
  - \( E = \) dollar value of the firm's equity.

- By accounting definition: \( A = D + E \).
The Asset Beta

Since $A = D+E$, we can write the asset beta as a function of the debt and equity betas:

$$\beta_A = \beta_D \frac{D}{D+E} + \beta_E \frac{E}{D+E} = (d)(\beta_D) + (e)(\beta_E),$$

where $d = \frac{D}{D+E}$; $e = \frac{E}{D+E}$.

- We can also write $\beta_E = (\beta_A - d\beta_d)/(e)$
- If firm’s debt is risk free, equity beta has form:
  $$\beta_E = \frac{\beta_A}{e} = \frac{\beta_A A}{E} = \frac{\beta_A A}{(A-D)}.$$
The Effect of Risk-Free Debt on Beta

- A firm’s asset beta depends on the assets alone; it does not change as the amount of debt changes.

- But a firm’s equity beta does depend on its risk-free debt: $\beta_E = \beta_A/e = \beta_A A/E = \beta_A A/(A-D)$.
  - The linkage between the firm's equity beta and its debt-equity mix is often overlooked.
  - In fact, we see that $\beta_E$ rises in tandem with debt D.
  - What does this tell you about the effect of more debt on the firm’s riskiness?
Is Debt Less Costly than Equity?
(No!)

People often think debt is less costly than equity, since bond interest rates < expected stock returns.

- This misses point that more debt makes equity riskier.

In fact, a firm’s cost of capital is independent of how it finances a project. Proof follows.

Assume firm can issue all the risk free debt it wishes.

We know the following:

- \( r_D = r_f \) (since the debt is risk free.)
- \( \beta_E = \beta_A/e \) (again, the debt is risk free)
- \( r_E = r_f + \beta_E(r_m - r_f) \)
Define the total financing cost $r_T$:

$$r_T = (e)(r_E) + (d)(r_D)$$

Now substitute out $r_E$, $r_D$ and use $\beta_E = \beta_A/e$ to get

$$r_T = (e) \left[ r_f + \frac{\beta_A}{e} (r_m - r_f) \right] + (d)(r_f).$$

Finally, with just a bit of algebra:

$$r_T = r_f + \beta_A (r_m - r_f)$$

Total cost of capital is independent of financing!
Example 1:

But Leverage does Raise $r_E$

Suppose: $\beta_A = 2$, $A = 100$, $r_f = .05$, $r_m - r_f = .1$

Let’s see how $r_E$ rises as amount of debt, $D$, rises:

<table>
<thead>
<tr>
<th>D</th>
<th>$\beta_E$</th>
<th>$r_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>.25</td>
</tr>
<tr>
<td>10</td>
<td>2.22</td>
<td>.272</td>
</tr>
<tr>
<td>50</td>
<td>4</td>
<td>.45</td>
</tr>
<tr>
<td>90</td>
<td>20</td>
<td>2.05 (Yes, it really is 2.05!)</td>
</tr>
</tbody>
</table>
You can buy the well diversified mutual fund Get Rich Yesterday (GRY) containing only stocks.

- The mutual fund beta ($\beta_G$) = .8. Market portfolio SD = 20%.
- Your portfolio goal is a standard deviation of 12%.
- In what proportions should you mix GRY and the risk free asset?

(Step 1) SD(GRY) = .8(20) = 16. (All diversified portfolios with $\beta = .8$, must be 80% as risky as the market portfolio.)

(Step 2) If you combine GRY with the risk free asset:

$$\text{SD} \left( w_G \tilde{r}_G + (1 - w_G) r_f \right) = w_G \sigma_G = w_G (16) = 12 .$$

So, $w_G = 3 / 4 = .75$. 
Now let every firm held by GRY change from 60% equity financing to 40% equity financing.

- All the debt is risk free
- What change is required for $w_G$?

Recall that $\beta_E = \beta_A/e$, so $\beta_A = e\beta_E$.

- With initial 60% equity financed, firms’ $\beta_A = (.6)(.8) = .48$.
- With final 40% equity financed, $.48 = .4\beta_E$, so $\beta_E = 1.2$.
- If the new $\beta_E = 1.2$, then the S.D. of GRY must be $1.2(20) = 24$.
- This implies $w_G(24) = 12$, or $W_G = .5$.
- So you reduce your holdings in GRY to 50% of your portfolio.
Operating Leverage

- An asset creates a portfolio of 3 cash flows:
  - Revenues from the asset (R)
  - Fixed cost to purchase and use the asset (F)
  - Variable cost of using the asset (V)

- Asset cash flow (A) is written as: \( A = R - F - V \)
  - Rearrange to get: \( R = A + F + V \).
  - Treat revenues as a portfolio of asset, fixed cost and variable cost.
  - Then use portfolio formula to get revenues beta:
    \[ \beta_R = \beta_A \frac{A}{R} + \beta_F \frac{F}{R} + \beta_V \frac{V}{R}. \]
Now start with: $\beta_R = \beta_A \frac{A}{R} + \beta_F \frac{F}{R} + \beta_V \frac{V}{R}$.

- The beta of the fixed costs equals 0.
- Also assume revenue beta equals the variable cost beta.
- So set $\beta_F = 0$ and $\beta_V = \beta_R$, and we get

$$\beta_R = \beta_A \frac{A}{R} + \beta_R \frac{V}{R}.$$ 

- Now solve for $\beta_A$, using $R-V = A+F$ from above:

$$\beta_A = \beta_R \frac{R-V}{A}.$$
Operating Leverage, Conclusion

Holding the asset value constant, an increase in the fixed costs increases the asset beta.

Thus, projects with large fixed costs have their cash flows discounted at a higher rate than projects with low fixed costs.

\[ \beta_A = \beta_R \left[ 1 + \frac{F}{A} \right]. \]