Overview of Next Topic

- Choosing among multiple projects
  - Investment opportunity curve
  - Optimal investment decisions
  - Maximizing PV as criterion for investment choice by firm

- Calculating Present Value

- Interest rates
  - Definition
  - Compounding
    » going from short to long periods
    » going from long to short periods
Choosing Among Many Projects

- You set up a firm whose assets are $1,000 in the bank and a corn farm.
  - Each column of corn costs $200 to seed.
  - There are 5 columns.

- The payoff for each column is:
  - Row A: 50, B: 600, C: 400, D: 200, E: 100

- Plot the tradeoff between dividends today and dividends tomorrow.
Planting Columns of Corn

Investment Opportunity Curve

(slope tells us marginal return)

B: $200 cost, $600 crop

C: $200 cost, $400 crop

$1000 in bank

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Firm’s Optimal Investment

Managers should thus
- Take on every project with a return > r (positive NPV)
- Reject every project with a return < r (negative NPV)
- I.e. they should use the NPV rule.

This maximizes the PV of the firm.
- All shareholders agree on this investment policy.
- For alternative views of managers’ objectives, see Micro/OB

Note: the firm’s PV exceeds the cash it starts with.
- Every time it invests in a + NPV project, the firm’s PV increases by the project’s NPV.
What discount rate to use?

- So far, we have assumed everything is riskless.
- Our discount rate, \( r \) has been the riskless interest rate.
- More generally, \( r \) represents the opportunity cost of capital.
  - The expected return on other, “equivalently risky” investments.
  - We’ll see how to calculate this later in the course

- Idea: A project only makes us better off if its return is higher than the return we could get on alternative investments elsewhere.
  - NPV quantifies how much better off.
The present value (PV) of a cash flow $C_1$ one year from today is

$$PV = \frac{C_1}{1 + r} = DF_1 \times C_1$$

where $DF_1 = \frac{1}{1+r}$, the discount factor for period 1 (the present value of $1$).

For cash flow in year $t$, $PV = \frac{C_t}{(1+r)^t} = DF_t \times C_t$.

For cash flows in several periods, $C_0, C_1, C_2, \ldots$,

$$PV = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots$$
Example

- $C_0 = -11$
- $C_1 = 6$
- $C_2 = 7$
- $r = .09$

\[(N)PV = -11 + \frac{6}{1.09} + \frac{7}{(1.09)^2}\]

\[= -11 + 5.50 + 5.89\]

\[= 0.40.\]
Example - Using a Financial Calculator (e.g. HP-12C)

- Step 1: Clear all registers ([f][REG] on HP-12C)
- Step 2: Enter data:

{11}[CHS][g][CF_0]  (The period 0 cash flow)
{6}[g][CF_j]  (The period 1 cash flow)
{7}[g][CF_j]  (The period 2 cash flow)
{9}[i]  (The interest rate)
[f][NPV]  (Asks for the answer)
Example: A Lottery

- You know that the probability of winning the lottery is 1 in 13 million.
- Walking by the news stand you see a big sign, “JACKPOT 20 MILLION!!”
- Should you invest $1 for a ticket?

1Paid in 20 annual equal installments. Payments are tax-free.
Lottery Decision

- First step: calculate the present value of the payoff.
  - A quick examination of the *Wall Street Journal* shows that the appropriate interest rate is 8%.

\[ PV = \frac{1M}{1.08^1} + \frac{1M}{1.08^2} + \frac{1M}{1.08^3} + \ldots + \frac{1M}{1.08^{20}} = 9.8M \]

- Odds are 1 in 13 M, so PV of expected payoff is 
  \[ \frac{9.8M}{13 M} = 0.75. \]
- NPV = 0.75 – 1.00 = -0.25
- Maybe next time…
Basics of Interest Rates

Definition: Let $r_n$ represents an \textit{n period interest rate}. If you invest $C$ dollars for $n$ periods, you will end up with $C(1+r_n)$ dollars.

For the purposes of this course, only numbers that meet the above definition are interest rates.
Compounding: How to go from short periods to long periods

Example: The 1 month interest rate is 1%. What is the 1 year rate?

There are 12 months in 1 year.

\[(1.01)^{12} = (1 + r_{\text{yearly}})\]

Interest received after 12 months in the bank.

Interest received after 1 year in the bank.

\[r_{\text{yearly}} = (1.01)^{12} - 1 = 0.1268 = 12.68\% .\]
Example: The annual interest rate is 14%. What is the daily rate?

In 1 year there are 365 days.

\[
1.14 = (1+r_{\text{daily}})^{365}
\]

Interest received after 1 year in the bank.

\[
r_{\text{daily}} = 1.14^{1/365} - 1 = .000359 = 0.0359\%
\]

Interest received after 365 days in the bank.