Overview of Lecture 5 (part of Lecture 4 in Reader book)

- Bond price listings and Yield to Maturity
  - Treasury Bills
  - Treasury Notes and Bonds

- Inflation, Real and Nominal Interest Rates
A bond is defined by its **maturity date, coupon rate, and face (par) value**.

- The bond makes coupon payments every 6 months until maturity.
- Each coupon payment = $\frac{1}{2}$ (coupon rate)(face value)
- The face value is paid out at the maturity date (together with the last coupon payment).
Treasury Bills are issued by the US Treasury, with 3 months, 6 months and 1 year to maturity.
- 3 and 6 month Bills usually auctioned weekly.
- 1 year Bills usually auctioned monthly.

Treasury Bills are “zero coupon” bonds.
- You pay for them today, and receive the face value, usually $10,000, at maturity.
- Bond prices are quoted relative to a $100 face value
  - E.g. the quoted price is for 1/100 of a T-Bill
The bid price (at which you can sell) and ask price (at which you can buy) are quoted as a discount from the face value, \( d \).

- Are these true interest rates?
- What about the “Ask Yield”?
Treasury Bills: Discount

Given the quoted discount, we can calculate the price:

\[
P = 100 \times \left( 1 - \frac{n \times d}{360} \right).
\]

E.g. the bill with maturity 27 days, quoted (ask) discount 5.88%, has price equal to

\[
P = 100 \times \left( 1 - \frac{n \times d}{360} \right) = 100 \times \left( 1 - \frac{27 \times 0.0588}{360} \right) = \$99.559.
\]

Remember: this is the price per $100 principal.

So for 1 bill you’d pay 99.559 x 10,000/100 = $9,955.90.
The interest rate over the next 27 days is
- \( 99.559 \times (1+r_{27\text{ day}}) = 100. \)
- I.e. \( r_{27\text{ day}} = \frac{100}{99.559} - 1 = 0.443\% . \)

This can be converted to an annual interest rate:
- \( 1+r_{\text{annual}} = (1+r_{27\text{ day}})^{\frac{365}{27}} = 1.0616. \)
  \( r_{\text{annual}} = 6.16\% \)

Thus, the true (effective) annual interest rate is slightly higher than the quoted discount rate.
The “Ask Yield” quoted in the paper is based on the ask price:

\[ y = \frac{365}{n} \times \left( \frac{100 - P_{\text{ask}}}{P_{\text{ask}}} \right) = \frac{d \times 365/360}{1 - nd/360}. \]

For our bill, \( y = 365/27 \times (100-99.559) / 99.559 = 5.99\% \).

- This is larger than the discount, but also smaller than the true interest rate.
- Note that \( (100 - P_{\text{ask}})/P_{\text{ask}} \) is the true n-day interest rate.
- It’s then converted to an annual rate by multiplying by 365/n, rather than raising to the power of 365/n.
Enter T-Bill quotes from the Wall Street Journal:

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<th>Maturity</th>
<th>Days</th>
<th>Bid</th>
<th>Asked</th>
<th>Chg</th>
<th>Ask Yld</th>
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<td>5.88</td>
<td>-0.01</td>
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</table>

- Price, \( P_{ask} = 100 \times (1 - \frac{nd}{360}) = \) 99.559
- True \( r = \frac{(100 - P)}{P} = \) 0.4429534246025
- Annual \( r_A = (1 + r)^{365/n} = \) 6.15695379742831
- Ask yield, \( y = (\frac{365/n}{P}) \times (100-P) / P = \) 5.98807407333009
Treasury Notes and Bonds

- Treasury Notes and Treasury Bonds are coupon paying bonds issued by the US government. The only difference is maturity:
  - Notes have more than 1, and up to 10 yrs. to maturity.
  - Bonds have more than 10, and up to 30, yrs. to maturity.
- Coupon payments are made every 6 months.
### Treasury Bond and Note Listings

(WSJ, Friday 9/8/00)

*Prices are quoted in 32nds of a dollar.*

- E.g. 138:01 means $138 + 1/32 = $138.03125

*Quoted (“clean”) prices are not the actual prices you pay.*

- You have to add “accrued interest”

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<th>Maturity</th>
<th>Rate</th>
<th>Mo/Yr</th>
<th>Bld</th>
<th>Asked</th>
<th>Chg.</th>
<th>Yld.</th>
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<tr>
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Accrued Interest

- If \( t_0 \) is the date of the last coupon payment, \( t_1 \) the date of the next payment, and \( t \) is today's date,

\[
\text{Accrued interest} = \left( \frac{t - t_0}{t_1 - t_0} \right) \times \text{coupon payment}
\]

- For our bond, last coupon was May 15, 2000
  - 116 days ago as of September 8, 2000.
- Next coupon is November 15, 2000
  - 184 days after May 15.
- Accrued interest = \((116 / 184) \times (9.875 / 2) = \$3.11277.
- Price you’d actually pay = $138.03125 + $3.11277.
  = $141.14402.
The “Ask Yield” column show’s the bond’s yield to maturity (YTM).

For annual coupon paying bonds, YTM solves the following equation for y

\[
\text{Bond Price} = \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots
\]

where the C’s are the bond’s cash payments.

For semi-annual coupon paying bonds, it is standard to quote the yield as an APR compounded semiannually:

\[
\text{Bond Price} = \frac{C_{6m}}{1+y/2} + \frac{C_{1y}}{(1+y/2)^2} + \frac{C_{18m}}{(1+y/2)^3} + \ldots
\]

Some of you may recognize that the YTM equals the internal rate of return (IRR) for the bond.
Yield to Maturity

- The YTM represents (roughly) the average return to an investor that purchases the bond and holds it until maturity.
- However, the YTM is not a spot or forward interest rate.
  - Spot and forward interest rates relate to investments with just 2 cash flows: at the start date and at the end date.
  - The bond, however, has a sequence of cash flows.
  - The YTM is a sort of weighted average of the spot rates.
  - The weights depend (roughly) on the size of the coupon payments and the number of periods until maturity.
  - What is the YTM of a zero coupon bond?
- Though commonly quoted, it is not a very useful measure
  - Only relevant for one bond.
  - See BM p. 677 for warnings regarding YTM.
Real interest rates, nominal interest rates and inflation

- A nominal cash flow is simply the number of dollars you pay out or receive.
  - A $1 nominal cash flow will have different purchasing power at different dates due to price level changes.

- A real cash flow is adjusted for inflation.
  - A real dollar always has the same purchasing power.

- $1 real equals $1 nominal today.
Converting between Real and Nominal cash flows

To convert a nominal cash flow to a real cash flow, we need to adjust for the decrease in purchasing power:

- \[
    \text{Real cash flow} = \frac{\text{Nominal cash flow}}{(1 + \text{inflation rate})^t}
    \]

E.g. BM Example p. 48:

- If you invest $1,000 today at 10%, your nominal payout in 20 years will be $6727.50 \((=1000(1.10)^{20})\).
- If the inflation rate over this period is 6% annually, then the real value will be $2097.67 \((=6727.50/1.06^{20})\).
In this example, what was the real return?
- \((2097.67/1000)^{1/20} - 1 = 3.77\%\)
- Note that 1.0377 equals 1.1 / 1.06.

General rule (see reader p. 33 for derivation):
- \((1 + r_{\text{real}})(1 + i) = 1 + r\)
  
  » In our example, \((1.0377)(1.06) = 1.10\).
- Equivalently, \(r_{\text{real}} = (r - i)/(1 + i)\).
- NOT \(r_{\text{real}} = (r - i)\), though it's approximately correct.
Discounting Real and Nominal Cash Flows

The Rule:

– Always discount real cash flows with the real interest rate.
– Always discount nominal cash flows with the nominal interest rate.
Real and Nominal Discounting: Reader Example P. 34

- Real interest rate: \( r_{\text{nominal}} = 12\% \); inflation rate = 8%
  - Compute \( r_{\text{real}} = \frac{r_{\text{nominal}} - i}{1 + i} = \frac{.04}{1.08} = .037037 = 3.7\% \).

- Cash flows: \( C_0 = -100; \ C_1 = 50; \ C_2, C_3 \) growing at inf. rate.
  - Nominal: \( C_2 = 50(1+.08) = 54; \ C_3 = 50(1.08)^2 = 58.32 \)
  - Real: \( C_1 = 50/1.08 = 46.3; \ C_2 = C_3 = 46.3 \)

- What is the NPV?

- 1) Discount nominal cash flows with nominal rate:
  - \( \text{NPV} = -100 + 50/1.12 + 54/1.12^2 + 58.32/1.12^3 = 29.20. \)

- 2) Discount real cash flows with real interest rate:
  - \( \text{NPV} = -100 + 46.3/1.037 + 46.3/1.037^2 + 46.3/1.037^3 = 29.20. \)