Capital Immobility and the Reach for Yield

Alan Moreira*

November 2015

Abstract

In this paper, I build a model where financial intermediation slows the flow of capital. Investors optimally learn from intermediary performance to allocate capital toward profitable intermediaries. Intermediaries reach for yield, i.e. invest in high tail risk assets, in an attempt to drive flows and reduce liquidation risk. Reaching for yield is stronger among intermediaries with weak opportunities, resulting in a reduction in the informativeness of performance; investors take longer to learn and capital flows become less responsive to performance. Capital becomes slow moving because the reach for yield dampens learning. The model predicts capital immobility to be stronger when tail risk is high; when tail risk is underpriced; and in asset classes with large cross-sectional variation in tail risk exposures.

JEL Codes: D82, G14, G23

Keywords: financial intermediation, reaching for yield, slow-moving capital.

*Yale University School of Management, alan.moreira@yale.edu.
1 Introduction

Despite the large and increasing share of wealth that is managed by financial intermediaries, a growing body of work documents that financial capital moves slowly (Duffie, 2010; Pedersen, Mitchell and Pulvino, 2007). At the same time, there is an emerging consensus that reaching for yield is widespread in the financial sector (Rajan, 2005, 2008, 2012; Stein, 2013). Financial intermediaries seem to over-invest in high-yield high tail risk assets.\(^1\) These facts have important aggregate implications on their own, but they are also deeply intertwined. How can investors know where to allocate their capital, if intermediaries can manufacture “alpha” by loading on low probability tail risks?

In this paper, I build a dynamic model of financial intermediation to study this connection. The model is centered on the interaction between intermediaries and investors’ capital allocation decisions. The key result of my model is that the reach for yield leads to capital immobility. Capital is endogenously slow moving as a result of excessive risk-taking by some intermediaries. The basic mechanism works as follows: financial intermediaries have strong incentives to improve short-term performance by loading on tail risks. Better performance improves their track record, attracts capital, and reduces the risk of liquidation. Incentives are particularly strong for intermediaries without good investment opportunities. These “bad” intermediaries reach for yield more, dampening the performance advantage of “good” intermediaries. Intermediary performance becomes less informative about the underlying quality of the intermediary. Investors take longer to learn and capital misallocation persists for longer.

Thus, capital immobility is an endogenous result of investors’ optimal response to the investment incentives financial intermediaries face. The key assumption is that investors cannot directly measure tail risks in the intermediary portfolio. As a result, loading on tail risks is particularly attractive to an intermediary. It boosts short-term performance without appearing in observable measures of risk. The reach for yield delays the speed of learning to the extent it is stronger among bad intermediaries, which turns out always to hold in equilibrium. Intuitively, if all intermediaries were equally aggressive in their reach for yield, performance in a tail event would be completely uninformative. A more aggressive loading on tail risks would enable the bad intermediary to attract flows in

\(^{1}\)For example, Rajan (2005) argues: “A number of insurance companies and pension funds have entered in the credit derivative market to sell guarantees against a company defaulting. Essentially, these investment managers collect premia in ordinary times from people buying the guarantees. With very small probability, however, the company will default, forcing the guarantor to pay out a large amount. The investment managers are, thus taking on tail risks, which produce positive returns most of the time as a compensation for a rare very negative return. These strategies have the appearance of producing very high alphas, so managers have incentives to load on them.”
the short term without impacting flows during a tail event. Thus, in equilibrium, the bad intermediary must reach for yield more aggressively, resulting in less learning in the short term and more learning during a tail event. Because the bad intermediary optimally balances out these competing learning channels, the result is that the reach for yield by the bad intermediary reduces the speed of learning and distorts the intertemporal allocation of capital.

Reaching for yield by a good intermediary, on the other hand, has the opposite effect. It increases the speed at which investors learn as it makes it harder for the bad type to keep up and it makes short-term performance more informative. However, it generates inefficient cross-sectional variation in investment flows across assets, reducing the expected returns earned by fund investors. Intuitively, as the good intermediary concentrates investments in high tail risk assets, the portfolio becomes poorly diversified, resulting in a fall in the portfolio Sharpe ratio and expected returns. Assets that are heavily exposed to tail risks are quickly inundated with capital as the intermediary attracts new capital flows. Capital allocation in these assets overshoots their long-run equilibrium. Assets that pay well during a tail event experience the opposite phenomenon. As investors pour capital into the intermediary, capital flows into these assets only at very low speeds. Thus, the intertemporal allocation of capital improves as the good type reaches for yield more aggressively, but the cross-sectional allocation of capital gets worse.

An example is useful here to illustrate how the decisions of the good intermediary shape the allocation of capital. Suppose the maximum alpha of the good type is 5% and achieved with a zero tail exposure. Suppose further that within a given investment opportunity set, any intermediary can manufacture 5% alpha by loading on tail risks. If only the bad intermediary reaches for yield, short-term performance differences across types disappear. As a result, learning stalls and capital ceases to move in the short term. Given the allocation of capital to each intermediary, the allocation of capital is the most efficient as it achieves the highest expected return. Now consider the case where the good type also reaches for yield. The fund alpha declines to 4% as his or her portfolio deviates from the maximum expected return portfolio, but the short-term performance increases to 7% as the portfolio becomes more concentrated on high tail risk assets that over-perform in the short term. In this case, short-term performance is again informative and capital less immobile. Capital flows faster even though expected returns are lower and the cross-sectional allocation of capital is worse. The trade-off between the intertemporal and cross-sectional allocation of capital is a key insight that emerges from the model.

A novel insight that emerges from the model is the feedback loop between the reach-for-yield behavior of both investors and intermediaries. The closer an intermediary is to
The increase in incentives to reach for yield is particularly strong among good intermediaries who have the most to lose from liquidation. The relative change in incentives drives investors to expect larger short-term performance differences across intermediaries. Thus, investors naturally respond more aggressively to short-term performance as short-term performance becomes more informative. This response further amplifies the incentives to reach for yield. This reach-for-yield spiral implies that flow-performance sensitivity is a nonlinear function of past performance.

The model makes several predictions that apply most directly to mutual funds, hedge funds, and private equity funds. The first set of predictions relates capital immobility to fund characteristics. Funds with more flexible investment mandates, or with mandates to invest in assets with relatively greater cross-sectional variation in tail risk, exhibit a weaker and more concave relationship between flows and performance. Intuitively, an opportunity set with greater variation in tail risk provides the intermediary with more freedom to invest in high tail risk assets without changing observable measures of portfolio risk. There is evidence that these predictions hold in the data for asset classes in which the manager has more flexibility, such as private equity (Kaplan and Schoar, 2005) and hedge funds (Goetzmann, Ingersoll Jr and Ross, 2003), and asset classes with a relatively large cross-sectional variation in tail risk, such as corporate bonds (Goldstein, Jiang and Ng, 2015).

The model provides a novel explanation for the empirical evidence in Kacperczyk and Schnabl (2012), who documented a large increase in reach for yield across money market fund managers and investors, and attributed this increase to a lack of market discipline.\(^2\) In the model, investors’ understanding of fund manager incentives amplifies the reach-for-yield behavior of both investors and managers. The market discipline imposed by investors drives the rampant reach-for-yield behavior.

Perhaps more strikingly, the model is consistent not only with the very persistent overpricing of senior tranches of collateralized debt obligations (CDOs) (and underpricing of junior tranches) documented by Coval, Jurek and Stafford (2009), but can also be consistent with the fact that this overpricing increased in summer 2007, as a large tail event in the U.S. housing market became more likely and more widely discussed in the news media. Intuitively, the model dynamics implies capital flows faster toward the most senior tranches that have relatively higher tail exposure. This results in under-investment in the junior tranches and over-investment in the senior tranches. When the probability

\(^2\)For example, investors either believed in an implicit government guarantee or neglected the magnitude of the tail risks.
of a tail event is initially very low, an increase in the probability of a tail event further amplifies these relative investment incentives as the short-term performance benefit of investing in high tail risk assets initially increases with the probability of a tail event.

The model has several implications for financial stability. First, tail risks tend to concentrate in the portfolios of financial intermediaries. Second, it suggests that periods when tail events are more likely are periods when capital reallocation is particularly slow. Third, tail risks are more likely to build up in relatively low-risk asset classes, where normal-times volatility risk is a particularly poor proxy for asset tail exposure. And fourth, the model predicts that reaching for yield is stronger in low-interest-rate environments, which is consistent with recent empirical evidence (Choi and Kronlund, 2014) and the view of leading policy makers (Rajan, 2005, 2012; Stein, 2013).

For the remainder of the paper, I proceed as follows. After a brief discussion of the literature, I present the model setup and characterize the model solution in Section 3. I study two economies: a benchmark economy where tail risk is readily observable by investors and an economy where tail risk cannot be directly measured. In Section 4, I use a numerical calibration to illustrate the implications of the model.

2 Literature review

This paper relates to a growing literature that focuses on implicit incentives that are induced by investor behavior. Chevalier and Ellison (1997), Basak, Pavlova and Shapiro (2007), Chapman, Evans and Xu (2009), and Basak and Makarov (2014) studied implications for manager portfolio choice, and Brennan and Li (1993), Shleifer and Vishny (1997), Vayanos (2004), Cuoco and Kaniel (2011), Basak and Pavlova (2013), and Kaniel and Kondor (2013) studied the implications of these implicit incentives for asset pricing. These authors took the behavior of investors as given and studied the implications for portfolio choice and equilibrium pricing.

A second strand of literature this paper relates to is that of learning in money management. Berk and Green (2004) showed that the behavior of fund flows and lack of persistence in fund performance could be explained by the idea that investors use past fund performance to learn about their managers. Pastor and Stambaugh (2010) relied on a similar idea to explain the dynamics of the size of the money management industry. Berk and Stanton (2007) built on these ideas to explain the closed-end fund discount, and

---

3 Extreme examples of this disconnect between volatility risk and tail risk are the money market funds studied in Kacperczyk and Schnabl (2012) and the different CDO tranches analyzed by Coval, Jurek and Stafford (2009).
Dangl, Wu and Zechner (2008) studied the effect of learning on the optimal replacement of a manager.

This paper connects these two sides of the literature, and studies an environment where learning is endogenous to the manager’s trading behavior. The previous work that emphasized the dynamics of investor learning has mostly abstracted from the interaction between portfolio choice and learning by either simplifying the investment opportunity set or arguing that the investment opportunity set was “sufficient non stationary,” which made this type of endogenous response by fund investors unfeasible (Shleifer and Vishny, 1997). This paper contributes to the agency literature by showing that learning and manager incentives interact in powerful ways. A novel feedback loop between the reach-for-yield behavior of investors and managers emerges, producing amplification and time variation of reach-for-yield incentives.

There is also some recent work that connects the agency and learning views. Dasgupta and Prat (2008) and Dasgupta, Prat and Verardo (2011) studied the effects on asset pricing of fund managers’ reputation concerns in a model where prices were determined by a market maker. Guerrieri and Kondor (2012), Malliaris and Yan (2009), and Di Maggio (2014) studied how reputation concerns impacted the willingness of a fund manager to invest in strategies that paid well with low probability. Makarov and Platin (2015) studied a setting with symmetric information between investors and managers and showed that contracts that deferred compensation as a function of past performance could neutralize incentives to reach for yield. Vayanos and Woolley (2013) showed that learning about manager efficiency had the potential to explain the momentum effect. Acharya, Pagano and Volpin (2013) also developed a model in which reputation concerns slowed down the identification of good managers. The authors assumed that learning had to start again every time the manager switched to a new project. So, managers switch projects inefficiently to mitigate their reputation risk. This paper focuses instead, on the choice of the payoff distribution and applies more directly to financial intermediaries. To a varying extent, these papers and mine build on the signal-jamming framework of Holmstrom (1999). He formally showed the role of reputation concerns in generating implicit incentives and agency distortions. My contribution to this literature is to show how reputation concerns not only create incentives to reach for yield but also reduce the speed at which capital is reallocated to good intermediaries.
3 Model

Time is continuous $t \in [0, \infty)$ and the economy is populated by two types of agents: investors (denoted by I) and financial experts (denoted by E). Both agents are risk-neutral and discount the future at risk-free rate of interest $\rho$.

The financial expert has no wealth, earns zero if not working as an intermediary, and can raise capital from investors to invest in a risk-free technology, whose net return is $\rho dt$, and in $n$ risky technologies, whose net returns are given by

$$dR_t = (\rho + \alpha_{\theta}\mu + \lambda \kappa)dt + \sigma dB_t - Y_t dJ_t,$$  \hspace{1cm} (1)

where $\alpha_{\theta}$ is a scalar that captures expert specific differences in expected returns, $\mu$ and $\kappa$ are $n \times 1$ vectors that capture differences in excess returns and tail exposures across technologies, and $\sigma$ is a $n \times n$ matrix, where $\Sigma = \sigma\sigma'$ is the return covariance matrix during normal times. $B_t$ denotes an $n$-dimensional Brownian motion capturing normal-times risk, $J_t$ is a Poisson process with constant intensity $\lambda$ and captures tail risk. In a crash, return realization are distributed as $Y_t \sim N(\kappa, \phi \Sigma)$, where $\phi > 0$ controls the increase in variance during a tail event.

These technologies should be thought of as asset classes, investment strategies, or different portfolios sorts. Note that asset expected returns are $E_t[dR_t] = \mu_{\alpha_{\theta}}$. I normalize $\sqrt{\mu^\top \Sigma^{-1} \mu} = 1$, so $\alpha_{\theta}$ is the maximum Sharpe ratio across these assets$^4$.

Reputation concerns arise because of incomplete information about the expert investment opportunity set. Specifically, the expert can be of two possible types: The skilled ($\theta = S$) type has a positive Sharpe ratio $\alpha_S > 0$, while the opportunistic ($\theta = O$) Sharpe ratio is zero $\alpha_O = 0$. Investors do not know the manager type $\theta$ or the fund portfolio $X$, but observe fund returns. Their information can be represented by $\mathcal{F}_t^I = \{dR_s : s \leq t\}$. The opaqueness of the fund portfolio is motivated by the idea that the intermediary has a unique understanding of the assets they trades, and even though investors are sophisticated and able to fully understand the environment, they are unable to evaluate specific portfolio positions.$^5$ Experts know their type and their information can be represented by $\mathcal{F}_t^E = \{(dR_s, \theta) : s \leq t\}$. At date-0, investors believe the expert is skilled ($\theta = S$) with probability $P_0$. I refer to $P_t$ as the expert reputation.

The expert faces decreasing returns in it’s ability to invest in profitable opportunities,

$^4$Precisely speaking $\alpha_{\theta}$ is the maximum ratio of expected returns to the square root of the instantaneous quadratic variation (normal-times return volatility) across these technologies.

$^5$Another motivation for this assumption is optimal secrecy arising from the need to preserve the profitability of a trading strategy.
which I model as transaction costs following Berk and Green (2004). Let $Q_t$ be a $1 \times n$ row vector of position weights that describe how the expert allocates capital across the $n$ technologies, $a_t$ is the fund dollar size, and $\sigma_{r,t} = \sqrt{Q_t \Sigma Q_t^\top}$ is the fund volatility. The expert faces transaction costs that scale with the total amount of “capital-at-risk” $a_t \sigma_{r,t}$, specifically $a_t \sigma_{r,t} c(a_t \sigma_{r,t}) dt$, with $c'(0) \geq 0$.

The expert raises capital from investors through a fund management contract. I assume one-period contracts, where the expert quotes an intermediation fee $f_t$ per dollar managed period by period. As standard in the principal agent literature, I assume that when the expert is indifferent across portfolio, the expert does the best thing for the fund’s investors. Specifically, I assume the expert places weight $\varepsilon > 0$ on maximizing expected returns. The weight $\varepsilon$ has the interpretation of a symmetric performance fee or a dollar stake the intermediary has in the fund. Buffa, Vayanos and Woolley (2014) for example, worked with an identical contract.

I discuss in detail the implications of the different assumptions in Appendix B.

### 3.1 Equilibrium concept

There are two key frictions in the information environment: the expert has no direct channel to communicate the quality of their opportunities and the tail risk of their portfolio. The first assumption implies that investors use performance to learn about the investment opportunities the expert has. The second assumption implies the expert does not internalize how investors respond to unobserved changes in the portfolio composition. This leads to the following notion of equilibria.

I use perfect Bayesian equilibria as the equilibrium concept. In this model, this concept has three implications: (i) investors take as given their own beliefs and the behavior of experts when choosing how much to invest in the fund; (ii) experts take as given the behavior of investors and investors’ beliefs when choosing how to allocate capital; (iii) investors’ beliefs about expert choices are consistent with expert choices and investors update their beliefs about the type of expert according to Bayes’ rule. I look for a stationary Markov equilibrium in which the only state variable is manager’s reputation $P_t$.

### 3.2 Preliminaries

The solution of the equilibrium involves two steps. I first take as given the investor’s beliefs about how the expert’s portfolio policy depends on reputation, and solve for the evolution of the reputation process. Given the investor’s learning behavior, I express the
expert problem recursively and solve for the optimal portfolio policy. I then characterize equilibrium by imposing mutual consistency between investor and expert behavior. Proofs are in Appendix C.

Investors do not observe the fund portfolio, but can measure the fund’s instantaneous volatility, so it is useful to decompose the portfolio policy in the observable volatility component $\sigma_{r,t}$ and the hidden component $X_t \equiv Q_t \sigma_{r,t}$. Define $\Omega = \{X \in \mathbb{R}^M | X \Sigma H X^\top = 1\}$ and then given the volatility choice, the intermediary can choose any vector $X_t \in \Omega$. This decomposition is without loss of generality because I could always choose $Q_I$ freely and construct the $\{X_I, \sigma_{F,t}\}$ representation. This decomposition implies that fund excess returns per unit of volatility can be written as

$$dr_t^e = \frac{dr_t - \rho dt}{\sigma_{r,t}} = X_t \left[ (\alpha \theta \mu + \kappa \lambda) dt - \kappa dJ_t \right] - c(a_t \sigma_{r,t}) dt + dB_t + \sqrt{\phi} \epsilon_t dJ_t. \quad (2)$$

Holding expected return constant, a higher tail exposure increases fund performance in any period without a crash. For this reason, I refer to the uncompensated drift $X_t (\alpha \theta \mu + \lambda \kappa)$ as the fund normal-times or short-term performance. The univariate shocks $dB_t$ and $\epsilon_t$ are linear combinations of the original return shocks in Equation (1) (i.e., $dB_t = X_t^\top \sigma dB_t$ and $\epsilon_t = X_t^\top \sigma \epsilon_t$) and are themselves distributed as Brownian motion and a standard normal.

It is convenient to have in mind that in this unit-variance space, the maximum expected return portfolio is $\frac{\Sigma^{-1} \mu}{\sqrt{\mu^\top \Sigma^{-1} \mu}}$ and it earns $\alpha_\theta$. I refer to this portfolio as the mean-variance efficient (MVE) portfolio. The maximum tail risk portfolio is given by $\frac{\Sigma^{-1} \kappa}{\sqrt{\kappa^\top \Sigma^{-1} \kappa}}$ and has tail exposure $\kappa^+ = \sqrt{\kappa^\top \Sigma^{-1} \kappa}$. I refer to this as the tail risk portfolio.

An observation about policy functions and equilibrium prices is useful here. In the current environment, any equilibrium with asymmetric information features pooling of opportunists with skilled intermediaries on observable quantities. Opportunists never choose to separate because that would imply immediate liquidation because the opportunistic type has zero alpha. This implies that fund volatility and fees can depend only on reputation, but not fund type. Importantly, portfolio opaqueness implies that expert portfolio allocation policy can depend on the intermediary type as well: $X_t = X(P_t, \theta)$. I use the superscript $I$ to denote variables from investor’s vantage point: $X_I(P_t, \theta) = \mathbb{E}^I [X(P_t, \theta) | \theta]$.

The financial expert maximizes the net present value of their expected compensation
flow plus the weight they places on maximizing expected returns:

$$
\sup_{f, \sigma, X} \mathbb{E}_t^E \left[ \int_u^\infty e^{-\rho(t-u)} (a_t f_t dt + \epsilon d\rho_t) \right] \quad \text{s.t.} \quad X_t \in \Omega, f_t \geq 0, \sigma_{r,t} > 0.
$$

(3)

Where the nonnegative fee constraint implies the expert cannot pay investors to avoid liquidation. It is the investors’ behavior that makes the expert problem dynamic. A sufficiently bad sequence of returns drive the expert reputation so low that investors liquidated the fund. Proposition 1 shows that the threshold of fund liquidation $P$ can be characterized using the investor’s break-even condition:

**Proposition 1.** There is a reputation cutoff $P$ such that the fund is liquidated permanently the first time $P_t < P$. Given policies $f(P), \sigma_t(P)$ and $X_t(P, \theta)$, $P$ satisfies $P = \frac{f(P) + c(0)}{\alpha S X_t(P, \theta) \mu}$.

The expert is liquidated with a positive reputation if the intermediation fee or transaction costs are positive as the fund size approaches zero. I assume $c(0) > 0$ throughout the paper to guarantee that the expert faces liquidation risk.

Investors’ optimal investment policy is to invest up to the point that they break-even given their beliefs about the expert type, portfolio, and fees. It follows that the fund expected excess returns net of transaction costs and fees must be exactly zero in equilibrium:

$$
\sigma_{r,t} P_t \alpha S X_t(P_t, \theta) \mu - \sigma_{r,t} c(a_t \sigma_{r,t}) - f_t = 0,
$$

(4)

where $P_t \alpha S X_t(P_t, \theta) \mu$ is the intermediary Sharpe ratio from the investor’s vantage point. Investors pour in capital so that after fees, they break even.

### 3.3 Learning

I now solve for the evolution of investors’ beliefs about the intermediary type. Optimal learning in this two-type environment consists of comparing the likelihood that a given return realization was generated by each alternative statistical model, i.e. the two types of financial experts: $\theta \in \{S, O\}$. I divide the learning process into normal times and periods with a tail event. In both situations, the statistical learning problem investors face is equivalent to that of an econometrician who tries to differentiate between two normal random variables of identical variance but different means.

For convenience, I transform belief from the probability space to the log-likelihood ratio space. This mapping is injective, and therefore without loss. Given a liquidation threshold $P$, the function $P(p) = \frac{Pe^p}{1-Pe^p}$ maps $p$, the log-likelihood distance to liqui-
dation into the probability $P$ that the intermediary is skilled. Note that by construction, $P[0] = P$. I refer to both $P$ and $p$ as reputation, and I abuse notation and refer to functions that were defined in the probability space as if they were defined in the log-likelihood space. A simple application of Bayes’ rule in continuous time implies the following dynamics for investors’ beliefs:

**Proposition 2.** Given beliefs $X_I(p, \theta)$, the reputation $p_t = \ln \left( \frac{\text{Prob}(\theta = S)}{\text{Prob}(\theta = O)} \frac{1 - P}{P} \right)$ satisfies,

$$dp_t = \vartheta(p_t) \times \left( dr^*_t - E_t^I [dr^*_t] \right) + \nu(p_t) \times \left( dr^*_t - E_t^I [dr^*_t | dJ_t = 1] \right) dJ_t$$

(5)

for $p_t > 0$ and $dp_t = 0$ if there exists $s \leq t$ such that $p_s \leq 0$, with learning coefficients given by

$$\vartheta(p_t) = \mu \left( X_I(p_t, S) \right) + \lambda \left( X_I(p_t, S) - X_I(p_t, O) \right) \kappa$$

$$\nu(p_t) = \frac{E_t^I [dr^*_t | S, dJ_t = 1] - E_t^I [dr^*_t | O, dJ_t = 1]}{\phi} = - \frac{(X_I(p_t, S) - X_I(p_t, O)) \kappa}{\phi}.$$

The filter described in Proposition 2 works as follows: if investors expect the skilled type to perform better than the opportunistic type in normal times, $\vartheta(p_t) > 0$, then positive return surprises, $dr^*_t - E_t^I [dr^*_t] > 0$, signal good news about the expert’s quality and fund’s expected returns, where performance news is measured relative what is expected from a $p_t$ reputation manager. The endogenous informational content during normal times is measured by $\vartheta(p)$, which is the difference in Sharpe ratios across types. During tail events, learning works in a similar fashion. In contrast to the literature on herding behavior (e.g., Scharfstein and Stein (1990) and Zwiebel (1995)), reputation dynamic depends exclusively on investors’ beliefs and the manager’s own actions and not on the actions of other managers.

It is important to note that when the fund is liquidated, reputation ceases to evolve and the manager stays liquidated forever. This implies that a sufficiently low reputation today reduces the expert earnings for the entire future. This nonlinearity at low reputation values makes the skilled type averse with respect to reputational shocks.

### 3.4 Financial expert

The expert optimization problem depends on investors’ beliefs $p_t$ through their decision to liquidate and the fee the expert can charge investors. Thus, the Bellman equation for
the expert can be written as,

$$\rho V(p_t, \theta) \ dt = \sup_{\{X, \sigma_r, f\} \in \Omega \times \mathbb{R}_+^2} \mathbb{E}^F_t \left[ a_t f_t dt + e \mathbb{E}^r_t \right] + V_p(p_t, \theta) \phi(p_t) \mathbb{E}^E \left[ d r^e_t - \mathbb{E}^L_t \left[ d r^e_t \right] \right]$$

$$+ \frac{1}{2} V_{pp}(p_t, \theta) \phi^2(p_t) dt + \lambda \mathbb{E}^F_t \left[ V(p_{t+}, \theta) - V(p_t, \theta) \right] dt = 1 \ dt, \ p_t \geq 0.$$

with boundary condition $V(p_t, \theta) = 0$ for any $p \leq 0$, and where the reputation after a tail event realization jumps to $p_{t+} = p_t + \nu(p_t) \times (dr^e_t - \mathbb{E}^L_t [dr^e_t | dJ_t = 1])$.

On the left hand side (LHS) of Equation (6) we have discounting. The first term on the right hand side (RHS) is the instantaneous compensation flow, which I will discuss shortly. The following three terms capture the valuation consequences of the reputation dynamics. The first two capture growth and risk during normal times. The last term captures what happens after a tail event realization: the expert reputation jumps to a new (and uncertain) value because of the lumpy amount of information revealed during a crash.

Note that neither fees nor volatility appear in the reputation dynamics. Intuitively, because both are observable, they do not impact learning, as they can be completely filtered out by fund investors. Substituting the investor investment policy (Equation (4)) into the Bellman equation (Equation (6)) and taking first-order conditions with respect to the total "capital-at-risk" $a_t \sigma_{r,t}$, I determine that the optimal amount of capital-at-risk must satisfy

$$a_t \sigma_{r,t} c'(a_t \sigma_{r,t}) = p_t \alpha_s X_t(p_t, S) \mu - c(a_t \sigma_{r,t}).$$

At the optimum, the increase in dollar returns (RHS) and the increase in transaction costs (LHS) are exactly equal. I consider a quadratic cost function and a cost function that implies a constant scale.

**Proposition 3.** Let $A(p_t) = a_t \sigma_{r,t}$ be the equilibrium capital at risk.

(i) (Decreasing returns to scale): If $c(x) = \psi_0 + \psi_1 x$, then

$$A(p) = \frac{\mathbb{P}(p) \alpha_s X_t(p, \theta) \mu - \psi_0}{2 \psi_1}, a_t f_t = \frac{(\mathbb{P}(p) \alpha_s X_t(p, \theta) \mu - \psi_0)^2}{4 \psi_1}, \text{ and } \sigma_{r,t} \in (0, \infty);$$

(ii) (Constant scale): If $c(x) = \psi_0 \times 1_{x \leq 1} + \infty 1_{x > 1}$, then

$$A(p_t) = 1, a_t f_t = \mathbb{P}(p_t) \alpha_s X_t(p_t, \theta) \mu - \psi_0, \text{ and } \sigma_{r,t} \in (0, \infty).$$

Only the total capital-at-risk is determined here because of the assumption that transaction costs grow with the total risk of the fund portfolio. If there were, for example, higher transaction costs for taking leverage, this would be different.
3.4.1 Transparent portfolio benchmark

As a benchmark, I first study the equilibrium in which the portfolio tail exposure is directly observable by investors. In this case, the opportunistic expert must pool with the skilled expert on the portfolio choice in order to remain undetected. This implies that the portfolio tail exposure does not impact the reputation dynamics. Intuitively, both types have the same tail exposure, thus tail event performance is uninformative, and differences in normal-times performance solely reflect differences in expected returns.

Let $\zeta_t$ be the Lagrange multiplier associated with the unit-variance constraint, then the first-order condition for the expert portfolio allocation is

$$\varepsilon \mu \alpha_\theta + V_p(p_t, \theta) \vartheta(p_t) \mu \alpha_\theta - \zeta_t \Sigma X_t = 0, \quad (8)$$

where the first term is the weight the expert places on maximizing expected returns, and the second term, the normal-times reputation incentives. Importantly, none of the terms depend on the vector of tail exposures. If $V_p(p, \theta) \vartheta(p_t) + \varepsilon \geq 0$, Equation (8) implies the skilled type chooses the maximum expected return portfolio and the opportunistic type pools, choosing any portfolio that generates the same tail exposure. Indeed, in this case $\vartheta(p_t) = \alpha_S > 0$ and $V_p(p, \theta) \geq 0$ for any $p \in [0, \infty)$ because fees increase and the probability of liquidation decreases with reputation. Proposition 5 summarizes the equilibrium in this case:

**Proposition 4.** If $X^\top_t \kappa \in \mathcal{F}_t^I$, then (i) $X(p, \theta) = \frac{\mu \Sigma^{-1} \mu}{\sqrt{\mu \Sigma^{-1} \mu}}$, (ii) $\mathbb{E}[d p_t | \theta] = (1 - 2 \mathbb{I}_{\theta = O}) \frac{1}{2} \alpha_S^2$.

Because investors can adjust their learning behavior to the expert’s tail risk choice, the skilled expert invests in the MVE portfolio and the opportunistic expert is forced to replicate the portfolio tail exposure. Reputation growth is determined by the skilled expert’s alpha because there are no hidden differences in exposure across the portfolios of the two types. The speed of learning depends only on the profitability of the opportunities to which the skilled intermediary has access. In the transparent case, the equilibrium is identical to that studied by Berk and Green (2004). I obtain closed-form solutions for the expert’s valuation in the particular case where the fees and scale are constant:

**Proposition 5.** If $X^\top_t \kappa \in \mathcal{F}_t^I$, $\frac{f_t}{\sigma^t_{i-j}} = \overline{f}, \varepsilon = 0$, and $c(\cdot)$ is constant scale, then

$$V(p, \theta) = \frac{\overline{f}}{\rho} \left(1 - \exp \left(-\iota_\theta \times p\right)\right), \quad (9)$$

where $\iota_S = \frac{\sqrt{1 + \frac{8}{\alpha_S^2} + 1}}{2}$ and $\iota_O = \frac{\sqrt{1 + \frac{8}{\alpha_S^2} - 1}}{2}$.
The expert valuation increases at a decreasing rate, with reputation reaching in the limit \( p \to \infty \), the value of their compensation flow discounted at discount rate \( \rho \). Note that the compensation flow is constant and independent of reputation. Valuation only changes because lower reputations increase the probability of fund liquidation. The root \( \iota \) informs us of how fast this liquidation risk decays with reputation, decaying faster for the skilled type \( \iota_S - \iota_O = 1 \), implying the skilled type has a more concave value function than the opportunistic type. Thus, the heterogeneity in reputation concerns is an result of the different reputation growth each expert expects. Intuitively, because the skilled reputation has a positive drift, a small increase in reputation has a strong effect on the probability of liquidation. The opportunist’ reputation on the other hand drifts toward zero, and increases in reputation only produce a temporary delay in liquidation.

3.4.2 Opaque portfolio

Now I turn to the case where the portfolio is opaque and investors cannot measure the fund tail risk. The opportunistic type no longer has to pool and is free to choose the portfolio tail exposure freely. Because tail risk is properly priced, the opportunistic exposure choice has no direct bearing on the investor’s decision to invest with the expert, but it impacts equilibrium through two endogenous channels that interact with each other: (i) the learning channel changes the dynamics of reputation; and (ii) the incentive channel introduces incentives for the skilled expert to deviate from the maximum expected return portfolio. These incentives can be seen in the portfolio allocation first-order condition,

\[
\alpha \mu + V_p \vartheta(p_t) (\alpha \mu + \lambda \kappa) - v(p_t) \mathbb{E}^E [V_p (p_{t+}, \theta)] \lambda \kappa - \zeta_t \Sigma X_t = 0
\]  

Together with the unit-variance constraint, Equation (10) is a set of \( n + 1 \) nonlinear equations that determine the vector of portfolio allocations \( X \) and the Lagrangian multiplier as a function of performance and reputation incentives. As before, in the first term we have the expected return incentives and in the second term, normal-times reputation incentives, but now normal-times reputation incentives depend on the vector of tail exposures. Intuitively, because \( X^\top_i \kappa \notin \mathcal{F}_t^I \), investors filter performance using only the equilibrium expectation of each intermediary portfolio tail exposure. In the third term, we have the reputation incentive that arises from investor learning during tail events. I will show that in equilibrium \( \vartheta(p_t) > 0 \) and \( v(p_t) \geq 0 \), thus an increase in the portfolio tail exposure increases reputation growth during normal times at the cost of a reduction in reputation growth during tail events.

It is useful to represent the solution of the skilled expert problem in terms of a de-
violation of the MVE portfolio, where the strength of the deviation is summarized by a scalar that measures how much weight the expert places on the portfolio tail exposure. I refer to this weight as the (reach for yield) incentive function. The incentive function $F(p, \kappa_r, \theta)$ that implements the solution to the system of equations in (10) is constructed by collecting the terms that multiply the vector of expected returns $\mu_{\theta}$ and the vector of tail exposures $\kappa$, and normalizing the latter by the former. After recognizing that

$$p_+ = p + v(p) \times (-\kappa_r + \mathbb{E}[-X_1 \kappa]) + v(p)\epsilon,$$

where $\kappa_r = X^\top \kappa$ is the tail risk associated with the expect allocation choice, I obtain,

$$F(p, \kappa_r, \theta) = \frac{\theta(p)V_p(p, \theta) - \lambda v(p)\mathbb{E}_E[V_p(p + v(p) \times (-\kappa_r - \mathbb{E}[-X_1 \kappa]) + v(p)\epsilon, \theta)]}{\epsilon + \theta(p)V_p(p, \theta)}.$$

(11)

Because the marginal benefit of reputation growth $V_p$ changes with reputation, tail event reputation incentives vary with the expert tail risk choice. In the numerator of Equation (11), normal-times and tail event incentives push the expert’s choice in opposite directions. In the denominator, we have the expected return and normal-times reputation incentives to generate alpha. Performance incentives shrink the effect of reputation incentives toward zero, pushing the expert closer to the MVE portfolio. I now can represent the optimal portfolio as follows:

**Proposition 6.** Let $X : \mathbb{R} \to \mathbb{R}^N$ be given by $X_\theta(p) \equiv \frac{(\mu_{\theta} + \pi \lambda \kappa)^\top \Sigma^{-1}}{\sqrt{(\mu_{\theta} + \pi \lambda \kappa)^\top \Sigma^{-1} (\mu_{\theta} + \pi \lambda \kappa)}}$, then the expert optimal portfolio is given by

(i) $X(p, S) = X_S(p)$, where $p$ solves $p = F(p, X_S(p) \kappa, S)$;

(ii) $X(p, O) = \{ X \in \Omega \mid X \kappa = \max \{ \min \{ \kappa_r, \kappa^+ \}, -\kappa^+ \} \},$ where $\kappa_r$ solves $0 = F(p, \kappa_r, O)}$.

Starting with the skilled expert, relative to the transparent benchmark, the optimal portfolio has a tilt toward the high tail risk technologies. The strength of the tilt is determined by the competing incentives discussed in the previous paragraph. A fixed-point characterization of these incentives is needed because Equation (11) is nonlinear in the choice of tail exposure.

While the skilled expert has to balance tail risk and true alpha in the portfolio allocation, the opportunist expert has no alpha, so they relies exclusively on tail risk as the source of reputation growth. The optimal portfolio policy tries to balance normal-times and tail event reputation incentives. When the interior optimum is not feasible, the portfolio choice is constrained by the investment opportunity set, which naturally places a bound on the amount of tail risk one can take without increasing portfolio variance.
3.5 Equilibria

To characterize equilibrium, I must now recognize that the incentive function in Equation (11) is an equilibrium object. Specifically, the learning functions $\vartheta(p)$ and $\nu(p)$ depend on investor’s beliefs about the expert’s optimal policies, which must be correct in equilibrium. Proposition 7 start with the opportunistic type. It takes as given the optimal behavior of the skilled type and imposes equilibrium between investor’s beliefs and the opportunistic expert’s choice.

**Proposition 7.** *(Opportunistic best response)* Let $\kappa_O : \mathbb{R}_+ \times \mathbb{R}^N \to \mathbb{R}$, be defined as $\kappa_O(p, X) = \min \{\kappa_r, \kappa^+\}$, where $\kappa_r$ solves

$$(\alpha z \mu - (\kappa_r - X\kappa) \lambda) V_p(p, O) = \frac{(\kappa_r - X\kappa) \lambda}{\phi} \mathbb{E}_c \left[ V_p \left( p - \frac{(\kappa_r - X\kappa)^2}{2\phi} + \frac{\kappa_r - X\kappa}{\phi} \epsilon, O \right) \right].$$

(12)

Given beliefs $X_I(p, S)$, the opportunistic policy is $X(p, O) = \{X \in \Omega | X\kappa = \kappa_O(p, X_I(p, S))\}$.

The function $\kappa_O$ describes the best response of the opportunistic expert to investor’s beliefs about the skilled portfolio. Specifically, Equation (12) shows that equilibrium tail risk choice of the opportunistic expert aims at balancing normal-times (LHS) and tail-event reputation incentives (RHS). Intuitively, a relative increase in tail risk taking by the opportunistic type reduces the informativeness of normal-times performance and increases the informativeness of tail event performance, so there is always a difference between tail exposures for which both incentives exactly balance. However, when normal-times incentives are much stronger than tail event incentives, this balance requires a tail exposure higher than the feasible exposure in the investment opportunity set. In this case, the opportunist expert’s tail exposure choice is constrained at $\kappa^+$.

It is useful to represent the opportunistic incentives and best response directly in terms of the skilled expert incentives $\pi(p, S)$. Note that the portfolio choice is deterministic conditional on the intermediary type because intermediary incentives are common knowledge.

**Definition 1.** Let $\delta(p, \pi) = \kappa_O(p, \mathcal{X}_S(\pi)) - \mathcal{X}_S(\pi)\kappa$ and $\pi(p, O) = F(p, \kappa_O(p, \mathcal{X}_S(\pi)), O)$.

The function $\delta(p, \pi)$ expresses the opportunistic expert’s behavior relative to the skilled expert’s behavior. Equation (12) implies the difference $\delta$ between the tail risk of their...

---

6Formally, at $z - y = 0$ the left side of equation (12) is lower than the right side, and as $z - y \to \infty$, the opposite is true. Since the left side is a continuous function of $z - y$, it must be the case that they cross at least once.
portfolios must always be positive. That is, the opportunist always reach for yield more aggressively. However, this difference cannot be too large. Intuitively, if \( \delta(p, \pi) \) ever negative, it would imply that reputation is increasing in performance during normal times and decreasing in performance during tail events. This would allow the opportunist to easily increase reputation growth state-by-state by simply reducing the portfolio tail risk, as a result this cannot be an equilibrium. If \( \delta \) is so large that reputation decreases in performance, then again, the opportunist can increase reputation growth state-by-state by increasing the portfolio tail risk. Corollary 7.1 follows:

**Corollary 7.1.** (Reputation is increasing in performance) \( v(p), \vartheta(p) \geq 0 \) for any \( p \geq 0 \).

Proposition 8 substitutes the best response of the opportunistic expert into the incentive function of the skilled type to determine equilibrium. The equilibrium restriction imposes that the incentive function has to be self-generating: the expected return and tail exposure difference induced by the incentive function \( \pi(p, S) \) must in turn induce incentives \( \pi(p, S) \). Intuitively, incentives are an equilibrium object because reputation incentives depends on investor learning behavior, which in turn depend on investor expectations about the expert’s choices, which ultimately depend on the incentives the expert faces.

**Proposition 8.** (Equilibrium) The portfolio \( X_I(p, S) = X_S(\pi(p, S)) \) is consistent with equilibrium for any incentive function that satisfies \( \pi(p, S) = G(p, \pi(p, S)) \), where

\[
G(p, \pi) = \frac{V_p(p, S) (\alpha_s X_S(\pi) \mu - \lambda \delta(p, \pi)) - \frac{\delta(p, \pi)}{\phi} \mathbb{E} \left[ V_p \left( p + \frac{1}{2 \phi} (\delta(p, \pi))^2 + \frac{\delta(p, \pi)}{\phi} \epsilon, S \right) \right]}{\epsilon + (\alpha_s X_S(\pi) \mu - \lambda \delta(p, \pi)) V_p(p, S)}.
\]

(13)

Note that the RHS of Equation (13) is increasing in \( \pi \). Intuitively, if incentives for investing in high tail risk technologies are strong, investors expect more tail risk taking and normal-times performance to be more informative, inducing strong incentives to invest in these assets. Strategic complementarity between investors’ beliefs and the skilled expert’s choice can lead to multiplicity. Because the focus of this paper is to highlight the link between reach for yield and slow moving capital, I focus on the equilibria that feature more learning and faster capital flows:

**Definition 2.** For any \( p \geq 0 \), let \( \mathcal{M}(p) = \{ \pi \in \mathbb{R} | \pi = G(p, \pi) \} \) be the set of incentives consistent with equilibria for a given reputation, then the fastest equilibria are given by

\[
\pi^*(p, S) = \arg \max_{\pi \in \mathcal{M}(p)} (\alpha_s X(\pi) \mu - \lambda \delta(p, \pi))^2 + \lambda \left( \frac{\delta(p, \pi)}{\phi} \right)^2.
\]

(14)
It is instructive to consider the case where tail event returns are sufficiently noisy ($\phi \rightarrow \infty$) to make learning during tail events unimportant. In this case, the opportunistic type wants to choose $\kappa_O = \lambda^{-1}X_I(p, S)(\mu_\alpha S + \lambda \kappa)$ because it does not have to tradeoff normal-times and tail event reputational incentives. If $\lambda^{-1}(\mu_\alpha S + \lambda \kappa) \leq \kappa^+$, and the skilled expert does not reach for yield, then the choice $\kappa_O$ is feasible, normal-times performance ceases to have information, and reputation becomes stuck at reputation $p$; learning stops completely. In this case, the skilled type has no reputation concerns ($\pi(p, S) = 0$), invests in the MVE portfolio, and our initial conjecture is valid that they does not reach for yield is correct in equilibrium.

However, because the RHS of Equation (13) is increasing in $\pi$, another equilibrium can exist. Suppose now that the skilled expert reaches for yield. If there is a $\pi < 1$ such that $\lambda^{-1}X_S(\pi)(\mu_\alpha S + \lambda \kappa) > \kappa^+$, then there can be a solution of Equation (13) with a positive incentive $\pi$. On the other hand when tail risk taking opportunities are not too rich, $\lambda^{-1}(\mu_\alpha S + \lambda \kappa) > \kappa^+$, the unconstrained optimal choice $\kappa_O$ is unfeasible, normal-times performance is informative and equilibrium incentives are always positive. Corollary C in the Appendix formalizes this discussion.

4 Analysis

I now use a numerical example to expand on the implications of the model. I solve the model by characterizing the solution $V(p, \theta)$ of the integro-differential equation in (6), which I solve numerically using finite-difference methods. Appendix D provides details. Given equilibrium value functions, I characterize the equilibrium incentive function following Propositions 7 and 8. The baseline parameter values are given below. I choose a parameter combination to illustrate the model predictions. I discuss the parameter choice in Appendix A.

Numerical example:

$s = 1, \mu \propto 1_n, \kappa^+ = 2, \mu^\top \Sigma \kappa = 0, N = 9, \lambda = 0.25, \sqrt{\phi} = 3, \rho = 0.05, c(x) = 0.5 + 0.25x$.

4.1 Value functions and endogenous incentives to reach for yield

Incentives to reach for yield arise as a result of an investor’s inability to measure portfolio tail risk directly. The strength of the incentive to reach for yield is an endogenous result of the dynamics of reputation and it’s impact on experts’ valuations. Figures 1(a) and (b) show that valuations increase with reputation. The valuation of a skilled expert is always
higher than for the opportunistic type. Valuations decrease as reputation falls because fees decrease, and more importantly, because the probability of liquidation increases.

In log-likelihood space (Panel (a)), valuations are concave. Concavity implies that reaching-for-yield incentives increase as reputation shrinks (see Equation (13)). Intuitively, as the expert’s reputation get close to the liquidation threshold, the probability of hitting the threshold becomes more sensitive to changes in reputation. As a result, increases in reputation have a much larger effect on the skilled expert’s valuation. Overall, Figure 1(a) shows that the skilled expert is risk-averse with respect to reputation shocks, and particularly so as they approaches liquidation. For comparison, I show valuations for the transparent benchmark as well.

Panel 1(b) shows valuations in the probability space. The vertical line denotes the threshold where the opaque fund is liquidated. The opaque fund attracts capital only for higher reputations because its expected returns are endogenously lower as a result of reaching for yield. Investors pull out earlier because they rationally expect the intermediary to be investing poorly. This is yet another source of amplification. The increase in the liquidation threshold increases incentives to reach for yield for any given level of reputation. This leads to a spiral. The intermediary reaches for yield and investors pull out earlier, further increasing incentives to reach for yield.

Figure 1 about here.

Figure 1(c) shows the reaching-for-yield incentives that result from these valuation dynamics. Incentives trend down with reputation, especially for the skilled intermediary, for whom the probability of liquidation is much more sensitive to changes in reputation at low reputation levels. This difference is a result of the higher expected reputation growth of the skilled intermediary. This mechanism can be seen explicitly in Proposition (5).

For the opportunist, incentives are mostly flat, only falling steeply in the unlikely event that his or her reputation reaches one. As we will see, this implies that the opportunist expert always finds it optimal to invest only the tail risk portfolio. In general, the incentive to reach for yield can reach zero for reputations below one, but it never cross zero in equilibrium (see Propositions 6 and 7). We will see next that this property has direct implications for the opportunist expert’s portfolio.

For the transparent benchmark, incentives are flat at zero because experts anticipate that investors would fully adjust their learning if intermediaries were to reach for yield. Reaching-for-yield incentives are a result of investors’ inability to condition on the actual risk profile of the portfolio when learning about the expert’s quality.
The expert’s portfolio can be represented as a portfolio of time-varying weights on the two base portfolios: the MVE portfolio $X_{\mu}$ and the tail risk portfolio $X_{\kappa}$,

$$X(p, s) = \frac{\alpha_s}{\psi(p, \theta)} X_{\mu} + w_{\kappa}(p, \theta) X_{\kappa},$$  

(15)

where $\psi(\cdot)$ is a scalar (function) that guarantees that the portfolio has unit variance and $w_{\kappa}(p, \theta) = \pi(p, \theta) \lambda \kappa^+ / \psi(p, \theta)$ is the weight on the tail risk portfolio. Figure 1(d) shows the composition of each base portfolio, which itself is a function of the opportunity set experts face. Figure 1(e) depicts the investment opportunity set. Specifically, it shows the relationship between expected returns and tail risk across technologies. In the baseline calibration, I assume tail risk is fairly priced, so expected returns do not change with the technology tail exposure. I later study the case where the tail risk premium is different from zero.

The MVE portfolio loads equally on all assets, while the tail risk portfolio is a long-short portfolio that goes long for the technologies that pay poorly during tail events (high $\kappa$ technologies) and shorts the ones that pay well. For reaching-for-yield incentives to generate a distortion, it is important that these two portfolios are different, so that there is a wedge between a portfolio that maximizes expected returns and one that maximizes normal-times performance.

Figure 1(f) shows a plot of the portfolio weight $w_{\kappa}(p, \theta)$ on the tail risk portfolio. As reputation grows, the skilled expert reduces the position in the tail risk portfolio and increases the position on the MVE portfolio. This contrasts with the transparent benchmark, where the expert always holds the MVE portfolio. The opportunist on the other hand is always fully invested in the tail risk portfolio, as a result of the positive reaching-for-yield incentives shown in Figure 1(c).

Overall, the model predicts that the opportunist always reaches for yield more aggressively than the skilled expert, and the skilled expert’s reaching-for-yield incentives peak as the fund approaches liquidation.

4.2 Performance

The model has implications for fund performance per unit of fund return volatility, as the investor’s behavior only determines the total capital-at-risk $a_t \sigma_{r,t}$ managed by the expert, so I report performance in the unit-variance space, that is in terms of Sharpe ratios.

---

7 The unit variance comes from Proposition 6 and is given by $\psi(p, \theta) = \sqrt{(\mu \alpha + \pi(p, \theta) \lambda \kappa)^\top \Sigma^{-1} (\mu \alpha + \pi(p, \theta) \lambda \kappa)}$
Figure 2(a) shows how the skilled expert’s normal-times performance (gross of transaction costs) increases as reputation decreases and they reach for yield more aggressively. Intuitively, reach for yield pushes the intermediary toward assets with higher tail exposure that are expected to over-perform in the short term. Even though expected returns are constant across technologies, reaching for yield leaves the intermediary with a less diversified portfolio. The portfolio’s expected returns and (dollar profits) fall as a result of this under-diversification as the total capital-at-risk of the fund is determined by fund investors. Reaching for yield is costly because it pushes the intermediary portfolio away from the MVE portfolio (Figure 2(c)). The opportunistic expert in this example is always constrained and reaches for yield by as much as feasible given the investment opportunity set. Recall that the opportunistic expert’s expected returns are zero by assumption ($\alpha_O = 0$).

The transparent benchmark features constant expected returns exactly equal to the normal-times performance, as the expert does not reach for yield in this case and the MVE portfolio has zero tail risk in this example. In general, the transparent portfolio features a constant performance profile, but could have nonzero tail exposure. It inherits the properties of the MVE portfolio, which are exogenous in the model.

Figure 2(c) shows that the tail event performance of the skilled expert only gradually improves with reputation, while the opportunist is always expected to perform equally poorly. Because reputation is a weighted average of past performance, this dynamic has implications for how a fund’s tail exposure evolves with performance. In particular, bad performance should predict higher tail risk.

Note that the skilled expert’s expected returns and normal-times performance net of fees increase as reputation falls, but this is entirely a result of investors demanding compensation for the higher probability that the expert is of the opportunistic type, who has no alpha, wastes transaction costs, and charge the same fees.

The dynamics of expected returns and liquidation has similarities with the mechanism proposed in Shleifer and Vishny (1997), where the risk of investors pulling out induces the manager to choose low-expected-return strategies. In their setting, trading strategies that are more volatile led to higher liquidation risk because investors were assumed not to adjust their behavior appropriately. An important difference between their mechanism and mine is that here investor behavior is derived endogenously.
4.3 Learning speed

Investors learn from performance by forming beliefs about how each expert type invests. These beliefs give investors an expectation of how different the performance across types ought to be in different states of the world. When investors expect that only the opportunistic type is reaching for yield aggressively, they weight recent normal-times performance less, and learn more from performance during tail events. Figures 2(a) and (b) depict the endogenous learning coefficients $\nu(p)$ and $\vartheta(p)$, which describe how investors’ beliefs about the expert type change with fund returns.

Consistent with the dynamics of reach for yield we saw in the last section, the normal-times performance coefficient decreases as reputation rises, while the tail event performance coefficient increases. Intuitively, as the skilled expert reaches for yield less aggressively, the difference in normal-times performance across types shrinks. The reach for yield by the skilled type distorts allocations and decreases expected returns, but it increases the speed of learning. Thus, given the opportunistic type’s behavior, there is a real trade-off even from the vantage point of society as a whole. Transitory distortions in the allocation of capital lead to a faster convergence in the efficient allocation of capital. Portfolio opaqueness introduces a trade-off between the static and intertemporal efficiencies in the allocation of capital.

The speed of reputation growth is a function of the leaning coefficients $\nu(p)$ and $\vartheta(p)$, but it is useful to express reputation growth directly in terms of the skilled expert’s equilibrium incentives $\pi$:

$$
\mathbb{E}[dp_t | \theta = S] = \frac{(\alpha_s \chi_S(\pi) \mu)^2}{2} \times \left[ \left( 1 - \frac{\delta(p, \pi) \lambda}{\alpha_s \chi_S(\pi) \mu} \right)^2 + \frac{1}{\lambda \phi^2} \left( \frac{\delta(p, \pi) \lambda}{\alpha_s \chi_S(\pi) \mu} \right)^2 \right]. \quad (16)
$$

The ratio $\delta(p, \pi) \lambda / \alpha_s \chi_S(\pi) \mu$ measures the difference in reach-for-yield behavior across experts relative to the amount of true alpha generated by the skilled expert. The expression above implies that provided tail event returns are sufficiently volatile, reputation grows slower than in the transparent benchmark. Intuitively, reach for yield by the opportunistic type transfers performance differences across types from normal times to tail events. This transfer leads to an overall decrease in performance informativeness whenever tail event returns are more volatile. The following result provides a sufficient condition for reputation to grow slower in the opaque economy.

**Proposition 9.** If $\phi > \sqrt{1/\lambda}$, then reputation grows slower in the opaque equilibrium.

Figure 2(c) shows the rate of learning; specifically, it shows the expected growth in the reputation of the skilled expert. Because reputation is a martingale, the process is sym-
metric and $\mathbb{E}[dp_t|\theta = 0] = -\mathbb{E}[dp_t|\theta = S]$. Learning is initially faster as the skilled type reaches for yield more aggressively, and gradually falls as their portfolio converges to the MVE portfolio. However, even when the learning rate is maximum, it is substantially lower than in the transparent benchmark. Investors’ beliefs are endogenously less sensitive to returns, but this sensitivity increases as reputation falls. From this perspective, it is natural that investors reach for yield more aggressively as reach-for-yield behavior becomes more widespread across experts. Aggressive reaching for yield by both skilled and opportunistic types make normal-times performance more informative.

### 4.4 Aggregate capital flows

The total amount of capital-at-risk allocated to a fund is a function of investors’ beliefs about the fund’s expected return; from Proposition 3, $A(p_t) = \frac{(P(p_t)\alpha_s \chi_S(\tau(p)))\mu - \psi_1}{\lambda p_t}$. Figure 3(a) shows the evolution of capital-at-risk as a function of reputation. For a given reputation, the opaque fund is always smaller, reaching liquidation earlier. As reputation rises and reaching-for-yield incentives decrease, the opaque portfolio converges to the MVE portfolio.

Figure 3 about here.

The inflow of capital into a fund is determined by the rate at which investors learn and how the skilled expert is expected to change how they allocate capital. Intuitively, if the skilled expert’s incentives are expected to improve with reputation, the investor will choose to invest more for a given change in reputation. Applying Itô’s lemma to the equilibrium capital-at-risk, I can write the expected capital inflow as,

$$\mathbb{E}[dA(p_t)|S, p_t] = \left( A_p(p_t) + A_{pp}(p_t) \right) \frac{\psi_2(p_t)}{\lambda} + \mathbb{E}[A(p_t) - A(p_t)|S, p_t].$$

Figure 3(b) shows this rate as a function of the capital gap, $A(p_\infty) - A(p_t)$, the difference between the long-run and the current level of capital, a measure of the scarcity of capital. The slope of this plot is the rate of convergence in years. For example, the slope of 0.25 for the transparent case implies that the capital gap shrinks by 25% every year. Consistent with the previous section, capital flows at a much slower pace. Capital converges instead at 13% when the fund is opaque. Convergence rates are almost 50% slower than for the transparent portfolio. Convergence is lower for the opaque intermediary throughout, and only when the capital gap is close to zero do the capital flows gain speed relative to the transparent case. This burst of capital flows happens when the skilled expert’s portfolio allocation improves quickly with reputation, inducing investors to invest more.
Capital flows slower into the opaque fund mainly because of the learning dynamics seen in Figure 2(f), which is driven by the fact that the opportunist type reaches for yield more aggressively than the skilled type. This property is a robust feature of any equilibrium in which investors think that positive-return surprises are good news about the intermediary type. Thus, even though the model has several specific assumptions, the connection between reach-for-yield behavior and capital immobility is much more general. I expect it to hold in any model where the expert knows more about their own type and fund’s investors are not able to measure tail risk directly.

4.5 Cross section of capital flows

As investors pour capital into the fund, the skilled expert chooses how to distribute the capital across the different technologies. Thus, the amount of capital in a specific technology depends on the aggregate flows and how the expert invests for a given level of reputation. Here I will focus on the skilled expert only because all the technologies in the opportunistic portfolio earn zero expected returns. Thus, excluding the impact on the learning dynamics, there are no costs associated with the opportunistic capital allocation.

Figure 3(c) shows the technology-specific investment \( \chi_S(\pi_S(p)) A(p) \) as a function of reputation. I show three technologies: high, average, and low tail risk. The high tail risk converges quickly. The level of investment over-shoots its long-run level. Over-investment in these technologies arises because they pay particularly well during normal times, and increase performance and reputation with very high probability.

For the low tail risk strategy, we have exactly the opposite. Initially, these technologies experience capital outflows as investors pour more capital into the intermediary. Capital flows out even though they have the same expected returns as other technologies and provide diversification gains. Investment in these technologies is perceived as costly by the expert because the profits take longer to appear in the fund’s performance, as they perform well during tail events but lag high tail risk technologies the rest of the time. Convergence happens once the expert’s reputation becomes sufficiently high and barely no new capital is flowing into the intermediary. An improvement in incentives leads to the rapid change in capital allocation.

In Figure 3(c), we can see the technology-specific investment growth \( \mathbb{E}[d (\chi_S(\pi(p,S)) A(p)) | S] \) as a function of the capital gap

\[
\chi_S(\pi(\infty, S)) A(\infty) - \chi_S(\pi(p, S)) A(p),
\]  

(17)
which measures how scarce capital is in the specific technology. Sharp differences between high tail risk and low tail risk technologies emerge. While capital flows into a high tail risk technology extremely quickly, capital initially flows out of the low tail risk technologies. As investors pour capital into the fund, less capital is invested in these technologies.

We see in both Figures 3(c) and (d) that capital flows at the same rate to all technologies when the fund is transparent. Capital is equally scarce in each of them (the equal-weighted portfolio is MVE), so the expert allocates investments equally as new capital flows in.

From an empiricist vantage point, the pattern in Figure 3(d) is suggestive of market segmentation or investors’ neglect of tail risk, but here it emerges in an environment where these markets are perfectly integrated and investors are fully aware of the risks involved. Capital flows into perfectly integrated markets at very different rates. This result is driven by the endogenous investment incentives that arise from investors’ capital allocation decisions. Importantly, these distortions arise even though investors fully understand the environment and the tail risks they are exposed to. The lack of measurement leads to incentive distortions.

4.6 Empirical implications

4.6.1 The reach for yield and fund flows

The model provides a different interpretation of the facts documented by Kacperczyk and Schnabl (2012). They documented an expansion of tail risk-taking opportunities for money market funds, showed that investors’ flows were sensitive to the fund yield, and showed that high-flow-sensitivity funds allocated a larger fraction of capital to assets that proved to be highly exposed to the financial crisis. The academic literature and policy makers have interpreted this evidence as a result of a lack of market discipline. This was driven either by investor’s neglect of the link between higher fund yields and tail risk or because investors had confidence in a government bailout of these funds.

In the model, it is market discipline, by which poorly performing managers are liquidated, together with investors not being able to measure directly the tail exposure of the fund, that drives rampant reaching-for-yield. Investors’ full understanding of fund manager incentives amplifies the incentives to reach-for-yield of investors and managers.

An additional prediction that arises from the model is that the reach for yield should become stronger after poor performance and asset outflows. Kacperczyk and Schnabl (2012) did not test this hypothesis directly, but they showed that funds with higher ex-
pense ratios experienced dramatically large outflows and also reached for yield more strongly, which is indirect evidence for the prediction that outflows and poor performance should predict reach-for-yield behavior.

4.6.2 Capital is less mobile in funds with more flexible mandates

I interpret greater investment mandate flexibility as expanding the set of assets that the expert is allowed to invest. An increase in flexibility can have two effects: higher alpha for the skilled expert and greater opportunities to reach for yield for both types. Thus, the optimal mandate would be determined by balancing out these forces. Here I focus on showing that an increase in the reach-for-yield opportunities ($\kappa^+$) leads to a slower flow of capital.

Figure 4 about here.

Figure 4(a) shows the speed of capital flows as a function of the capital gap for three values of $\kappa^+$, the maximum tail risk (per unit of volatility) in the investment opportunity set. The higher $\kappa^+$, the slower capital flows. This result follows directly from the fact that the opportunistic type reaches for yield more aggressively than the skilled type. An increase in the performance that can be manufactured through tail risk reduces the ability of the skilled types to differentiate themselves, resulting in slower capital flows. These same comparative statics also produce more performance persistence and a more concave relation between flow and performance. Performance persistence is a direct result of slower capital flows. The increasing sensitivity of flows to bad performance is the result of stronger time variation in reach for yield by the skilled type.

These predictions are consistent with evidence documented in Kaplan and Schoar (2005), who showed that in private equity funds, performance was more persistent and the flow–performance relationship more concave than in equity mutual funds. Both findings are consistent with a model in which private equity fund managers have more discretion and a greater ability to take on tail risks without increasing the more conventional measures of risk.

A different driver of variation in reach-for-yield opportunities is the asset class that the manager operates; in particular, the degree that the manager can change the portfolio’s tail risk without simultaneously increasing the portfolio (normal-times) risk, which can be easily measured by investors. The corporate bond market is an example of an asset class that exhibits low volatility but substantial negative skewness (Bessembinder et al., 2009; Stein, 2013). This is intuitive and expected given that corporate bonds are
naturally more exposed to downside risk. The model logic implies that capital should move particularly slowly into bond funds. This suggests that when one contrasts with equity mutual funds, bond funds should have performance that is more persistent, and the flow–performance relationship should be weaker for positive returns. There is recent evidence along these lines. Goldstein, Jiang and Ng (2015) found that corporate bond fund flows were more sensitive to negative performance, but I am not aware of any empirical evidence on the persistence of bond fund returns.

Further evidence for the model’s main mechanism comes from Coval, Jurek and Stafford (2009). They showed that senior CDO tranches were persistently overpriced and junior CDO tranches persistently underpriced relative to option markets. Under the reasonable assumption that it is easier for investors to measure tail risks in an option portfolio than in a structured product portfolio, the model is consistent with both the underpricing of the junior tranche and overpricing of the senior tranche. Intermediaries dislike the junior tranche because it is very risky during normal times and has a poor tail-to-volatility risk ratio, while the senior tranche is close to safe during normal times and is exposed to much tail risk per unit of volatility. To be clear, the model has no explicit implications for pricing, but an extension where the intermediary trades against a mean-variance investor would produce exactly these results. Intuitively, a mean-variance investor transforms the quantity distortions across high and low tail risk assets into asset-pricing distortions: the very low allocation into the low tail assets translates into high expected returns, while the very high allocation to high tail risk assets translates into low expected returns.

4.6.3 Capital is less mobile when a tail event is more likely

When tail risk is correctly priced, an increase in the probability of a tail event increases the normal-times performance that can be created by taking on tail risk. Intuitively, if the probability of a tail event were close to zero, the performance benefit of loading on tail risk would also be close to zero. Thus, an increase in the tail event probability increases the short performance that can be manufactured through tail risk-taking.

Figure 4(b) shows the speed of capital flows as a function of the capital gap for three values of $\lambda$, the tail event intensity. The higher the $\lambda$, the slower capital flows. An increase in $\lambda$ increases the amount of performance that can be manufactured by reaching for yield. This reduces the short-term performance advantage of the skilled type. Performance becomes less informative and capital flows slower.

To be clear, this is not a statement about aggregate flows into intermediaries, but about how related these flows are to the intermediary’s performance. The claim here is that in
times of heightened risk of a tail event, flows become less sensitive to performance. Thus, capital is less mobile as there is less reallocation across managers. I am not aware of any paper that directly tests this prediction. However, there is indirect evidence for the link between reach for yield and tail event probability from the analysis in Coval, Jurek and Stafford (2009). As discussed in the previous section, they showed that senior (junior) tranches of structured products were overpriced (underpriced) relative to option markets. A less appreciated empirical finding is that the mispricing increased in the summer of 2007 as the possibility of a large shock hitting the U.S. economy increased.\(^8\) This pattern of divergence is inconsistent with the neglected risks view (Gennaioli, Shleifer and Vishny, 2012), which predicts a convergence once tail risks become more salient. In my framework, it arises naturally as the incentive to load on high tail risk assets increases as the probability of a tail event rises from a low level. Intuitively, a higher tail event likelihood increases the short-term performance differential across high and low tail risk assets and results in a higher short-term performance benefit of tail risk-taking.

### 4.6.4 Capital is less mobile when tail risk is underpriced

In the model, the trade-off between normal-times performance and expected returns emerges from the fact that the tail risk portfolio and the MVE portfolio are different. This difference implies that the maximization of normal-times performance—that is, the reach for yield—must reduce expected returns. The magnitude of the wedge between these portfolios depends critically on how expected returns are related to tail risk.

Thus far, I have assumed a flat relationship, where tail risk is fairly priced in the cross section, so there is a zero tail risk premium. Figure 4(c) shows the speed of capital flows as a function of the capital gap for three alternative assumptions for the relation between expected returns and tail risk. It shows the baseline case where there is no relation between expected returns and tail risk in the cross section of technologies \(\text{cov}(\mu_j, \kappa_j) = 0\), and the cases where expected returns either increase or decrease with tail risk.

Capital moves more slowly when the relation between expected returns and tail risk is negative. That is, when tail risk is underpriced. When tail risk is underpriced, there is a larger wedge between the portfolio that maximizes expected returns and the one that maximizes normal-times performance, further attenuating the short-term performance advantage of the skilled expert when they invest in the MVE portfolio.

The intuition for this result is as follows: the negative relation between tail risk and

---

\(^8\)This can be seen in their Figure 7, which contrasts the market prices of the different CDO tranches with the option-implied spreads. Toward the end of the sample (summer 2007), both market and option-implied spreads spiked but the senior CDO tranches increased by less.
expected returns implies that the MVE portfolio has a negative tail exposure. In equilibrium, this implies a smaller difference between the normal-times performance of skilled and opportunistic types. Thus, normal-times performance is less informative and capital moves more slowly.

4.6.5 Reach for yield is stronger when interest rates are low

As noted by Rajan (2005, 2012) and many others, sustained low interest rates seem to be associated with strong reach-for-yield behavior in asset markets. In the model, this relation between reach for yield and the level of interest rates emerges naturally as incentives to reach for yield are tightly linked to the present value of future fees. Specifically, a lower interest rate leads the expert to value future fees more. The expert puts more weight on future rents relative to current performance incentives. As a result, the expert assigns a higher value to an increase in reputation and reaches for yield more strongly when interest rates are low. Figure 4(d) shows the skilled expert portfolio weight on the tail risk portfolio across different levels of reputation. It shows the equilibrium amount of reach for yield for three different levels to the risk-free rate. A lower interest rate leads to a higher portfolio weight in the tail risk portfolio.

5 Conclusion

In this paper, I have developed a fully consistent narrative of how intermediaries and investors interact. I have shown how this interaction leads to a time variation in reaching-for-yield incentives and has repercussions for the speed at which capital flows. This implies capital flows only slowly to profitable opportunities and that the reduction in flows is particularly severe for strategies that are good hedges against tail risks. The endogenous learning dynamics produces a feedback loop between liquidation risk and reaching-for-yield. In contrast with previous literature, the model results are driven by investors’ sophisticated understanding of the environment. Information and capital flows shape and are shaped by the incentives intermediaries face.

The importance of reach-for-yield opportunities in slowing the flow of capital is the fundamental new insight of this framework. It complements the slow-moving capital literature by developing a mechanism that can account for a substantially more persistent misallocation of capital.
References


**Goldstein, Itay, Hao Jiang, and David T Ng.** 2015. “Investor Flows and Fragility in Corporate Bond Funds.” *Available at SSRN 2596948.*


**Kacperczyk, Marcin, and Philipp Schnabl.** 2012. “How Safe are Money Market Funds?”


Appendix

This appendix contains proofs and derivations used in the paper.

A Parameter choice

The model has five key parameters - $\alpha_S$, $\kappa_+$, $\phi$, and $\lambda$. In what follows I discuss the empirical plausibility of my parameter choice.

The scarcity of capital is measured by the Sharpe ratio of the skilled intermediary before transaction costs and fees, $\alpha_S$. I calibrate $\alpha_S = 1$, which is reasonable since this is the Sharpe ratio of the skilled type. The average intermediary generates much less than this value. The model quantitative implications for the speed of capital flows are stronger for lower values of $\alpha_S$, holding $\lambda \kappa_+$ constant, and are unchanged if $\kappa_+$ goes down proportionally to $\alpha_S$. The intuition for this result is that what matters for the reduction in capital flows is how much of the short-term performance advantage of the skilled type ($\alpha_S$) can be closed by taking on tail risk, which is $\lambda \kappa_+$ when the skilled invests in the MVE portfolio and $\mu^\top \kappa = 0$. This can be seen clearly in Equation (16).

The second key parameter is $\kappa_+$, which is the maximum tail exposure a portfolio of unit-variance can achieve. I set $\eta_+$ to 2, what implies that in a portfolio with 10% standard deviation, the intermediary can find (hidden) opportunities to have a tail loss of 20%. Given the recent losses experienced in some fixed income markets, this tail exposure seems plausible. For example, many of the economic catastrophic bonds of Coval, Jurek and Stafford (2009) dropped to almost zero in the aftermath of the crisis. This parameter should strongly vary by asset class and investment mandate. What is important is that $\lambda \kappa_+$ is of the same order of magnitude as the $\alpha_S$. For example, in this calibration $\lambda \kappa_+ = \frac{\alpha_S}{2}$. If it is much lower than this, than the effects on the speed of capital flows are small.

Tail risk volatility determines how informative tail-event performance is. My baseline calibration uses $\sqrt{\phi} = 3$, which is in line with recent experience. For example, some measures of realized stock market volatility in the Fall of 2008 were 80%. This compares to an average market volatility of 15%. In fixed income markets, this difference is likely to be larger. As $\phi$ grows, the economy converges to the case of Proposition C. As $\phi$ shrinks to zero, tail event performance becomes completely revealing about the intermediary portfolio. Such a parametrization greatly increase the cope for multiple equilibria.

Tail event frequency $\lambda$ determines by how much tail risk taking can boost performance during normal-times, but also determine how quickly the tail event arrives. The first effect increases the temptation of the intermediary, the second effect acts in the op-
posite direction as a disciplining force. I calibrate $\lambda$ to 0.25, consistent with a tail event every four years. This number is in line with recent experience. For example, in the last 20 years the world economy experienced the “Tequilla crisis” (1994), the “Asian crisis” (1997-8), the “Tech bubble” bust (2000), the more recent financial crisis (2007-2009), and the European sovereign debt crisis (2011-2012).

Other less important parameters include $\rho$, $\psi_0$, $\psi_1$, and $\mu^\top \Sigma \kappa$. I set $\rho = 0.05$. Here the patience parameter does not play much of a role, but in a setting with large performance incentives, time discounting becomes important because it controls how the intermediary trades off contemporaneous increases in performance compensations against the future value of a higher reputation.\footnote{Counter-intuitively, in such a setting a more patient intermediary distorts more towards technologies that pay well in the short-term} I set $\psi_0$ to target a liquidation threshold of 0.5, but this choice is completely arbitrary. The model works identically for any positive choice of $\psi_0$ only changing the liquidation threshold and the long runs size of the fund. It is important that $\psi_0 > 0$ so that managers face liquidation risk. The decreasing returns to scale parameter $\psi_1$ is set so that the amount of capital allocated to the intermediary peaks at 1.

I assume $\mu^\top \Sigma \kappa = 0$ and $\mu \propto 1_n$ throughout most of the paper. This implies that the MVE and tail risk portfolio are exactly orthogonal and expected returns are constant across technologies. Economically this means that the MVE portfolio has zero tail exposure and the price of tail risk is zero. These assumptions control how much more tail risk can be invested relative to the MVE portfolio. For the choice $\mu^\top \Sigma \kappa = 0$, this difference is given by $\kappa^+ = \frac{\kappa^\top \Sigma^{-1} \kappa}{\sqrt{\kappa^\top \Sigma^{-1} \kappa}}$. For the more general case this would be given by

$$\frac{\kappa^\top \Sigma^{-1} \kappa}{\sqrt{\kappa^\top \Sigma^{-1} \kappa}} - \frac{\mu^\top \Sigma^{-1} \kappa}{\sqrt{\mu^\top \Sigma^{-1} \mu}}$$

(A.18)

Both an increase in the average tail risk across assets and an increase in the price of risk have the effect of reducing the spread in tail risk across portfolios. As discussed before, the results of the speed of capital flows rest on the assumption that these differences are large.

B Assumptions

A key modeling assumption is that intermediaries face a investment opportunity set that is rich enough to allow some reach for yield, but not too rich to allow unbounded gambling. This fits well most of the investment management industry where managers are constrained by investment mandates to a varying degree. Managers cannot trade arbi-
trary payoffs without raising red flags, but can easily distort their portfolio towards asset with high tail risk exposure that are otherwise identical. Coval, Jurek and Stafford (2009) provide a useful example of such differences within the fixed income market. The assumption that tail risk cannot be perfectly measured even \textit{ex-post} is realistic and also theoretically important. It avoids that small performance differences during tail events to be perfectly informative about the intermediary portfolio. This would be unrealistic and an artificial by-product of the continuous time environment. It would also introduce a lot of new equilibria. Intuitively, as the signal to noise ratio grows to infinity all managers have strong incentives to conform exactly with investors expectations. In this sense tail risk volatility does something similar as private information in the global games literature.

Another critical assumption is that intermediaries can only use performance to signal their type. The assumption that there are no other signaling mechanism is likely to be counterfactual. However all it is required for the model fundamental mechanism to work is that there some asymmetric information about intermediary skill left. As long this is the case, investor will use performance to learn about intermediary type and the link between capital immobility and reach for yield will exist, at least qualitatively.

The assumption that there are only two intermediary types is obviously stylized, but necessary for tractability in an environment where asymmetric information is persistent. Previous work that studies the interaction of learning and investment decisions in money management (For example, Makarov and Platin, 2015) work in an environment where information about manager quality is symmetric across investors and managers. The assumption that the manager knows more about his quality is key to generate the link between reaching for yield and slow moving capital. It is essential that managers that are observationally identical (same reputation), behave differently as function of their privately known skill. This produces variation in reaching for yield incentives and render performance less informative.

The contractual environment fits well most of the mutual fund industry contracts, and is in line with how previous work have modeled asset management contracts (For example, Berk and Green, 2004). However, it is admittedly stylized and abstracts from many realistic features present in the contracts of more sophisticated financial intermediaries, such as hedge funds.

Before discussing how the introduction of performance contracts would change the model results, it is useful to contrast the incentive problem that arises here with the one that shows up in the moral-hazard based models (For example, He and Krishnamurthy (2013)). There performance incentives are needed to induce work or avoid tunneling by the intermediary. Here, it is driven by reputation incentives that arise endoge-
nously from the sensitivity of the intermediary human capital to his performance track record. Provision of full insurance to the intermediary would implement the first best, but it is not feasible if there is competition among investors. Intuitively, investors outside of the relationship would make outside offers to intermediaries that are performing well, unraveling the insurance scheme.

The introduction of symmetric performance fees has no impact on the investment behavior of the opportunistic type, but pushes the skilled experts towards the MVE portfolio. Intuitively, no matter his portfolio choice the opportunistic generates zero expected excess return, so a linear performance schedule does not impact his choice. Thus, linear incentive fees have the effect of further slowing down the flow of capital into the skilled intermediary, but it reduces the dispersion in flows across technologies.

Nonlinear contracts could potentially increase the speed of capital flows if it induces incentives on the opportunist type to perform well during a tail event. If the contract is sufficiently nonlinear it could induce gambling on the opportunistic type, counter-balancing the reputation incentives. In practice, the typical nonlinear contract used among financial intermediaries, the high-water-mark contract, is likely to have a mixed effect on incentives. When the fund is sufficiently far from the high-water-mark, the contract places more weight on large return realizations. This would nudge the intermediary in the right direction. But when the manager is close to the high water mark, the contract is linear on small return realizations, but places less weight on really bad return realizations. This would nudge the manager in the wrong direction.

C Proofs

Proposition 1.

Proof. Investors liquidate when they can no longer break even for any positive fund size, that is when \( \mathbb{E}_t[\sigma_r(t)dr_t^i - f dt] < 0 \) for any positive investment \( a_t \) in the fund. Plugging Equation (4), I obtain

\[
\sigma_r(P)PX_t(P,S)\mu_\alpha_s - \sigma_r(P)c(0) - f = 0,
\]

which proves the proposition. $\square$

Proposition 2.

Proof. Consider the following cumulative realized excess return history \( r_\Delta \) in an interval \( \Delta \), and let \( P_t \) the perceived probability the intermediary is of type \( \theta = S \) in the beginning
of the interval. Bayes law implies,

\[ P_{t+\Delta} = \frac{f(r_{\Delta}|\theta = g, P_t)P_t}{f(r_{\Delta}|\theta = g, P_t)P_t + f(r_{\Delta}|\theta = b, P_t)(1 - P_t)}, \]  

(C.1)

where \( f(r|\theta, P_0) \) is the probability distribution of a return history \( r \) if the intermediary is of type \( \theta \) and initial reputation \( P_t \). In our setting this density is a complex object since the distribution of realized returns is time-varying due to the time-variation in the intermediary portfolio. However, in the limit \( dt \) the problem simplifies as portfolios are approximately constant as the interval becomes arbitrary short. The problem is further simplified because in any interval \( dt \), \( dJ_t \) equals to zero or one. The learning problem boils down to distinguish between two statistical models is two different observable states. For this proof I will consider I abstract form transaction costs just to economize on notation, but transaction costs have no impact on the learning dynamics.

Let me start with normal times, \( dJ = 0 \). In this case, from the investor vantage point returns are distributed as,

\[ r_{\theta}^dt \sim N(r_{\theta}X_I(p, \theta), \mu_\alpha + \lambda \kappa) dt, \sigma_r^2 dt). \]  

(C.2)

Let \( N(dR|\mu, \sigma^2) \) be the normal pdf with mean \( \mu \) and variance \( \sigma^2 \). Applying equation (C.1) I obtain

\[
P_{t+dt} = \frac{N(r_{dt}|\sigma_rX_I(p, S), (\mu_\alpha + \lambda \kappa) dt, \sigma_r^2 dt) \times P_t}{N(r_{dt}|\sigma_rX_I(p, S), (\mu_\alpha + \lambda \kappa) dt, \sigma_r^2 dt) \times P_t + N(r_{dt}|\sigma_rX_I(p, O), (\mu_\alpha + \lambda \kappa) dt, \sigma_r^2 dt) \times (1 - P_t)} \]

\[
= \frac{N(\frac{\mu}{\sigma_r}X_I(p, S), (\mu_\alpha + \lambda \kappa) dt, dt) \times P_t}{N(\frac{\mu}{\sigma_r}X_I(p, S), (\mu_\alpha + \lambda \kappa) dt, dt) \times P_t + N(\frac{\mu}{\sigma_r}X_I(p, O), (\mu_\alpha + \lambda \kappa) dt, dt) \times (1 - P_t)}, \]  

(C.3)

Where \( \frac{\mu}{\sigma_r} = dr^d_t \) from Equation (2). Now lets go to log-likelihood space, define

\[ p_{t+dt} = ln \left( \frac{P_{t+dt}}{1 - P_{t+dt}} \right) \]  

(C.4)
and substitute equation (16) to get,

\[
p_{t+dt} = \ln \left( N\left( \frac{R_t}{\sigma_r} | X_t(p, S) (\mu_S + \lambda \kappa) \, dt, \, dt \right) \right) - \ln \left( N\left( \frac{R_t}{\sigma_r} | X_t(p, O) (\mu_O + \lambda \kappa) \, dt, \, dt \right) \right) (1 - P_t) - P_t + \ln \left( \exp \left( - (dr_t - X_t(p, S) (\mu_S + \lambda \kappa) \, dt)^2 \left( 2dt \right)^{-1} \right) \right)
\]

\[
= - \ln \left( \exp \left( - (dr_t - X_t(p, O) (\mu_O + \lambda \kappa) \, dt)^2 \left( 2dt \right)^{-1} \right) \right)
\]

\[
= p_t - \frac{(dr_t - X_t(p, S) (\mu_S + \lambda \kappa) \, dt)}{2dt} + \frac{(dr_t - X_t(p, O) (\mu_O + \lambda \kappa) \, dt)^2}{2dt^2}
\]

\[
= p_t + \frac{(X_t(p, S) (\mu_S + \lambda \kappa) - X_t(p, O) (\mu_O + \lambda \kappa)) (dr_t - X_t(p, S) (\mu_S + \lambda \kappa) - X_t(p, O) (\mu_O + \lambda \kappa))}{2}
\]

(C.5)

Now let's focus on instants with a tail event realization, \( dJ = 1 \), and \( R^\theta_{dt} \sim N(-X_t(p, \theta) \kappa \sigma_r, \phi \sigma_r^2) \).

Repeating exactly the same algebra as in the \( dJ = 0 \) case I obtain,

\[
p_{t+dt} = p_t - \frac{(-X_t(p, S) \kappa - X_t(p, O) \kappa)}{\phi} \left( \frac{R^\theta_{dt}}{\sigma_R} + \frac{X_t(p, S) \kappa + X_t(p, O) \kappa}{2} \right).
\]

(C.6)

Equations (C.5) and (C.6) imply the result in Proposition (2).

\( \square \)

**Proposition 3.**

**Proof.** Plugging equation (4) into \( \Gamma_t = \epsilon \mathbb{E} \left[ dr_t^e \right] + a_t f_t \) I obtain

\[
\epsilon \mathbb{E} \left[ dr_t^e \right] + a_t \sigma_{r,t} P X_t(P, S) \mu_S - a_t \sigma_{r,t} c(a_t \sigma_{r,t}).
\]

(C.7)

Differentiating with respect to \( a_t \sigma_{r,t} \), Equation (7) follows. Substituting the transaction cost functions \( c(\cdot) \) I can solve for the optimal capital-at-risk \( A = a_t \sigma_{r,t} \) and total dollar fees \( a_t f_t \). For example, in the case of quadratic cost function I obtain

\[
A \psi_1 = P \alpha_S X(P, S) \mu - (\psi_0 + \psi_1 A),
\]

which yields \( A = (P \alpha_S X(P, S) \mu - \psi_0) / (2 \psi_1) \) as enunciated in this Proposition. To obtain the optimal fee that implements the optimal size choice I substitute in the investors break-even condition (Equation (4)).

\( \square \)

**Proposition 4.**
Proof. Solving Equation (8) I obtain \( X = \mu^\top \Sigma^{-1} \times y \), where \( y = \zeta^{-1} \alpha (\varepsilon + V_p(p, \theta) \theta(p)) \) is a scalar. Plugging in the unit-variance constraint, \( X \Sigma X^\top = 1 \), we get,

\[
\mu^\top \Sigma^{-1} \times y \Sigma (\mu^\top \Sigma^{-1} \times y)^\top = y^2 \mu^\top \Sigma^{-1} \mu = 1,
\]

so it must be that \( \zeta^{-1} = \frac{\sqrt{\mu^\top \Sigma^{-1} \mu}}{\alpha (\varepsilon + V_p(p, \theta) \theta(p))} \) and \( X(p, \theta) = \frac{\mu^\top \Sigma^{-1}}{\sqrt{\mu^\top \Sigma^{-1} \mu}} \). The expected growth in reputation obtains directly from substituting \( X(p, \theta) \) on Equation (5).

Proposition 5.

Proof. Substituting in the Bellman Equation (6) the optimal choice \( X(p, \theta) = \frac{\mu^\top \Sigma^{-1}}{\sqrt{\mu^\top \Sigma^{-1} \mu}} \) I obtain

\[
\rho V(p, S) = a_t f_t + V_p(p, S) \frac{\alpha^2}{2} + V_{pp}(p, S) \frac{\alpha^2}{2},
\]

\[
\rho V(p, O) = a_t f_t - V_p(p, O) \frac{\alpha^2}{2} + V_{pp}(p, O) \frac{\alpha^2}{2}.
\]

Using that \( a_t \sigma_{r,t} = 1 \) when \( \varepsilon \) is constant scale, and the assumption that \( f_t / \sigma_{r,t} = \bar{f} \), I get that \( a_t f_t = \bar{f} \). Substituting in the ODE above I obtain the solution

\[
V(p, S) = \frac{\bar{f}}{\rho} + K_1 e^{-\frac{1-\sqrt{1+\frac{\rho}{\alpha^2}}}{2} p} + K_2 e^{-\frac{1+\sqrt{1+\frac{\rho}{\alpha^2}}}{2} p},
\]

\[
V(p, O) = \frac{\bar{f}}{\rho} + K_3 e^{-\frac{-1-\sqrt{1+\frac{\rho}{\alpha^2}}}{2} p} + K_4 e^{-\frac{-1+\sqrt{1+\frac{\rho}{\alpha^2}}}{2} p}.
\]

Imposing the boundary conditions \( V(0, \theta) = 0 \) and that \( V(\infty, \theta) = \frac{\bar{f}}{\rho} \) I obtain the solution.

Proposition 6.

Proof. Let me start isolating the terms in the Bellman equation (Equation (6)) that depend directly on the expert choice,
\[
\sup_{X \in \Omega} \varepsilon X \mu \alpha_\theta + V_p \theta(p_I) X (\mu \alpha_\theta + \lambda \kappa) + \lambda \mathbb{E}_t^E \left[ V \left( p + v(p) \times \left( -X \kappa - \mathbb{E}^I [-X_I \kappa | p] \right) + v(p) \varepsilon, \theta \right) \right] \left| dJ_t = 1 \right]. \tag{C.14}
\]

I conjecture for now that value function are positive, increasing and concave in reputation \((V(p, \theta), V_p(p, \theta) \geq 0, V_{pp} < 0)\) and positive return surprises are always good news about expert type \((\theta(p), v(p) \geq 0)\). We will later show that these properties hold in equilibrium. So it follows that the third term is decreasing in the portfolio tail-exposure \(X \kappa\), while the other terms are linear in the portfolio expected return and normal times performance. Note that

\[
\kappa \kappa^\top v(p)^2 \lambda \mathbb{E}_t^E \left[ V_{pp} \left( p + v(p) \times \left( -X \kappa - \mathbb{E}^I [-X_I \kappa | p] \right) + v(p) \varepsilon, \theta \right) \right] \left| dJ_t = 1 \right] < 0, \tag{C.15}
\]

so the first order condition is is necessary and sufficient to characterize the optimal policy.

Let me start with the skilled type and assume the uni-variance constraint binds. Differentiating with respect to \(X\) I obtain Equation (10). Solving for \(X\) I obtain

\[
X = \zeta^{-1} (\alpha_S \times y_1 + y_2 \times \lambda \times \kappa)^\top \Sigma^{-1}, \tag{C.16}
\]

where \(y_1\) and \(y_2\) are scalars given by

\[
y_1 = \varepsilon + V_p(p, \theta) \theta(p) \tag{C.17}
\]

\[
y_2 = \left( V_p(p, S) \theta(p) - v(p_I) \mathbb{E}_\varepsilon^E \left[ V_p \left( p + v(p) \times \left( -X \kappa - \mathbb{E}^I [-X_I \kappa | p] \right) + v(p) \varepsilon, S \right) \right] \right)
\]

Substituting in the unit-variance constraint , \(X \Sigma X^\top = 1\), we get,

\[
\zeta^{-2} (\alpha_S \mu \times y_1 + y_2 \times \lambda \kappa)^\top \Sigma^{-1} (\alpha_S \mu \times y_1 + y_2 \times \lambda \kappa) = 1. \tag{C.18}
\]

It follows that,

\[
\zeta = \sqrt{(\alpha_S \mu \times y_1 + y_2 \times \kappa)^\top \Sigma^{-1} (\alpha_S \mu \times y_1 + y_2 \times \kappa)}. \tag{C.19}
\]
Define $\pi = \frac{y_2}{y_1}$, then we can write

$$X(p, S) = \frac{(\mu \times \alpha_S + \pi \times \lambda \kappa)^\top \Sigma^{-1}}{\sqrt{(\mu \times \alpha_S + \pi \times \lambda \kappa)^\top \Sigma^{-1} (\mu \times \alpha_S + \pi \times \lambda \kappa)}}. \quad (C.20)$$

Now note that $\pi$ depends on the choice $X(p, \theta)$ through the dynamics of $p_{t+}$. So the above equation characterizes $X$ only implicitly. Because $X$ is of the same dimension as the number of assets, this fixed point problem in Equation (C.20) is a hard one to solve. I will now characterize the solution in terms of scalar incentives $\pi$.

So let me define

$$\chi^{\theta}(\pi) = \frac{(\mu \times \alpha^{\theta} + \pi \times \lambda \kappa)^\top \Sigma^{-1}}{\sqrt{(\mu \times \alpha^{\theta} + \pi \times \lambda \kappa)^\top \Sigma^{-1} (\mu \times \alpha^{\theta} + \pi \times \lambda \kappa)}}. \quad (C.21)$$

It must be the case that a given incentive $\pi$ imply a choice $\chi_S(\pi)$, which itself is consistent with incentive $\pi$. Formally for any $p$, $\pi$ must solve

$$\pi = V_p(p, S)\theta(p) - v(p_t)E^E \left[ V_p (p + v(p) \times (-\chi_S(\pi)\kappa - \mathbb{E}^I [-X_I\kappa|p]) + v(p)e, S) \right] \frac{\epsilon + V_p(p, S)\theta(p)}{\mathbb{E}^E_t \left[ -X_I\kappa | dJ_t = 1 \right]}, \quad (C.22)$$

which can be written as $\pi = F(p, \chi_S(\pi), S)$ as in item (i) from Proposition 6. This proves (i), the optimal policy for the skilled expert.

The opportunistic choice requires more work because sometimes the unit-variance constraint does not bind. Suppose it did, then for $\pi = 0$ we would have $\zeta = 0$ (From Equation (C.19) and $\alpha_O = 0$), in which case the optimal portfolio can no longer be written as $\chi_O(\pi)$, which assumes that the constraint binds. In particular note that $\chi_O(\pi)$ is undefined at $\pi = 0$. Going back to the original problem and substituting $\alpha_O = 0$ I obtain,

$$\sup_{\chi \in \Omega} \lambda V_p \phi(p_t)X\kappa + \lambda \mathbb{E}^E_t \left[ V \left( p + v(p) \times (-X\kappa - \mathbb{E}^I [-X_I\kappa|p]) + v(p)e, O \right) | dJ_t = 1 \right],$$

(C.23)

where the first term increasing and linear in the tail exposure choice $X\kappa$ and the second term is decreasing in the tail-exposure choice $X\kappa$. Suppose that the constraint does not bind, than it must be that the first order condition is satisfied with a multiplier.
equal to zero. Differentiating with respect to $X$ and rearranging I can write the FOC as
\[ \pi_O = 0, \]
where
\[ \pi_O = \mathbb{V}_p \mathbb{P}_t \left( p + \nu(p) \times \left( -\kappa_O - \mathbb{E}_t \left[ -X_t \kappa | p \right] \right) + \nu(p) \epsilon, O \right) | dJ_t = 1. \]  
(C.24)

The first term in the RHS is a constant (for a given reputation) and the second term is increasing in $\kappa_O$. Because $V$ is concave, the lower the tail event performance (higher $\kappa_O$), the higher the marginal value of reputation. So there is at most one solution $\kappa_O$ to the above equation. If this solution is consistent with $X \Sigma X^\top = 1$, then it is the optimal solution. Specifically it must be that $\kappa_O \in [-\kappa^+, \kappa^+]$ is order for it to be consistent with the unit-variance constraint. If that is the case than any $X$ such that $X \Sigma X^\top = 1$ and $X\kappa = \kappa_O$ is the solution of the opportunistic expert problem

If $\kappa_O > \kappa^+$ then the constraint binds, and $X = \frac{\kappa^+ \Sigma}{\sqrt{\kappa^+ \Sigma^\top \kappa}}$. The intermediary would like to increase his tail exposure more than the investment opportunity enables him to do it without increasing his portfolio volatility. If $\kappa_O < -\kappa^+$ then the constraint binds on the other side, and $X = -\frac{\kappa^+ \Sigma}{\sqrt{\kappa^+ \Sigma^\top \kappa}}$. The intermediary would like to reduce his tail exposure (to increase performance during tail events) more than the investment opportunity enables him to do it without increasing his portfolio volatility. This last case never happens in equilibrium. This proves item (ii) of Proposition 6.

Proposition 7.

Proof. Start from the opportunistic expert FOC when the constraint does not bind (Equation (C.24)) and impose equilibrium between investors beliefs and the opportunistic portfolio choice, $X_t(p, O) = X(p, O)$, and express the learning coefficients as a function of investors beliefs about the the skilled and the opportunistic portfolio choice. I obtain
\[ \alpha_s \mathbb{V}_p (p, O) = (\kappa_O - X(p, S) \kappa) \lambda \left( \mathbb{V}_p (p, O) + \frac{1}{\phi} \mathbb{E}_t \left[ \mathbb{V}_p \left( p - \frac{(\kappa_O - X(p, S) \kappa)^2}{2\phi} + \frac{\kappa_O - X(p, S) \kappa}{\phi} \epsilon, O \right) \right] \right), \]  
(C.25)

where $\kappa_O = X_t(p, O) \kappa$. Note that this equation is equivalent to Equation (12), but slightly rearranged. Note that because $\mathbb{V}_p (p, O) \geq 0$ and $\alpha_s > 0$ it must be the case that $\kappa_O - X \kappa \geq 0$ so the opportunistic type must always take more tail risk than the skilled in equilibrium.

In particular note that $\kappa_O > -\kappa^+$. Suppose that $\kappa_O = -\kappa^+$, then it must be that
\( X(p, S)\kappa = -\kappa^+ \) given the result I just proved. But than in this case the opportunistic first order condition does not hold and \( \pi_O \) from Equation (C.24) is positive, indicating that the opportunistic expert would like to take more tail risk, which is feasible. So \( \kappa_r = -\kappa^+ \) cannot be an equilibrium. This completes the proof. \( \square \)

**Corollary 7.1.**

**Proof.** From the proof of Proposition 7 we have that \( \kappa_O - X\kappa \geq 0 \), this implies that \( v(p) \geq 0 \). Rearranging Equation (12) I obtain

\[
\frac{\alpha_s X \mu}{\lambda} \times \frac{V_p(p, O)}{V_p(p, O) + \frac{1}{\phi} \mathbb{E}_\phi \left[ V_p \left( p - \frac{(\kappa_O - X(p, S)\kappa)^2}{2\phi} + \frac{\kappa_O - X(p, S)\kappa}{\phi} \epsilon, O \right) \right]} = \kappa_O - X(p, S)\kappa, \tag{C.26}
\]

which proves that \( \kappa_O - X(p, S)\kappa \leq \frac{\alpha_s X \mu}{\lambda} \) in any interior equilibrium. If the equilibrium is not interior then \( \kappa_O \) is smaller than the implied by Equation (C.26), so it must be the case that \( \kappa_O - X(p, S)\kappa \leq \frac{\alpha_s X \mu}{\lambda} \) holds. This completes the proof. \( \square \)

**Proposition 8.**

**Proof.** I now impose equilibrium between investors beliefs and the skilled expert behavior while taking as given the best response of the opportunistic type. Using Definition 1 and expressing the skilled expert choice and investors beliefs in terms of incentives \( \pi(p, S) \) I obtain Equation

\[
\pi(p, s) = \frac{V_p(p, S) \left( \alpha_s X_s(\pi(p, s))\mu - \lambda \delta(p, \pi(p, s)) \right) - \frac{\delta(p, \pi(p, s))}{\phi} \mathbb{E}_\phi \left[ V_p \left( p + \frac{1}{2\phi} \left( \delta(p, \pi(p, s)) \right)^2 + \frac{\delta(p, \pi(p, s))}{\phi} \epsilon, S \right) \right]}{\epsilon + \left( \alpha_s X_s(\pi(p, s))\mu - \lambda \delta(p, \pi(p, s)) \right) V_p(p, S)}, \tag{C.27}
\]

which says that an equilibrium incentive function \( \pi(p, S) \) induce beliefs and choices given by \( X_s(\pi(p, s)) \) and \( \delta(p, \pi(p, s)) \) that must induce this same incentive function. \( \square \)

**Corollary 8.1.** *(Equilibrium with noisy tail event returns)* Let \( \phi \to \infty \), then

(i) if \( \kappa^+ < \alpha_S \lambda^{-1} + \mu^T S^{-1} \kappa \), then \( v(p) = 0; \theta(p) > 0; X(p, O)\kappa = \kappa^+; \) and \( \pi(p, S) > 0 \). If it also holds that \( V_{pp}(p, S) < 0 \), then \( \frac{\partial \pi(p, S)}{\partial p} < 0 \) for any \( p \in [0, \infty) \);

(ii) if \( \kappa^+ \geq \alpha_S \lambda^{-1} + \mu^T S^{-1} \kappa \), then there is an equilibrium where \( v(p) = \theta(p) = 0, X(p, O)\kappa = \alpha_S \lambda^{-1} + \mu^T S^{-1} \kappa \) and \( \pi(p, S) = 0 \). In both (i) and (ii), \( \mathbb{E}[dp_t|S] \leq \frac{\alpha_S^2}{2} \).

**Proof.** Taking the limit \( \phi \to \infty \) of equation (13), I obtain
\[ \pi(p,s) = \frac{V_p(p,S)(\alpha_sX_s(\pi(p,s))\mu - \lambda\delta(p,\pi(p,s)))}{\epsilon + (\alpha_sX_s(\pi(p,s))\mu - \lambda\delta(p,\pi(p,s))) V_p(p,S)'} \]  

(C.28)

From equation (12) we know that either \( \lambda X(p,O)\kappa = \alpha_sX(p,S)\mu + \lambda X(p,S)\kappa \) and \( \delta(p) = 0 \) or \( X(p,O)\kappa = \kappa^+ \) and \( \delta(p) \geq 0 \). Let me start with case(ii) and conjecture that \( \pi(p,S) = 0 \) and \( \alpha_sX(p,S)\mu = \alpha_s \). Then if \( \alpha_s + \frac{\lambda\mu^\top\Sigma^{-1}\kappa}{\mu^\top\Sigma^{-1}\mu} \leq \lambda\kappa^+ \) there is an interior tail exposure choice for the opportunistic expert that makes performance completely uninformative. Rearranging this gives condition

\[ \kappa^+ > \alpha_s\lambda^{-1} + \mu_s^\top\Sigma^{-1}\kappa \]  

(C.29)

where I use the normalization \( \sqrt{\mu^\top\Sigma^{-1}\mu} = 1 \). The opportunistic portfolio choice that implement this equilibrium is simply \( X(p,O)\kappa = \alpha_s\lambda^{-1} + \mu_s^\top\Sigma^{-1}\kappa \).

In case, \( \kappa^+ < \alpha_s\lambda^{-1} + \mu_s^\top\Sigma^{-1}\kappa \), then the opportunistic type is constrained and chooses \( X(p,O)\kappa = \kappa^+ \). In this case the equilibrium incentive function \( \pi(p,s) \) satisfies \( \pi(p,s) = G(p,\pi(p,1)) \) where

\[ G(p,\pi) = \frac{V_p(p,S)(X_s(\pi)(\mu\alpha_s + \lambda\kappa) - \lambda\kappa^+)}{\epsilon + V_p(p,S)(X_s(\pi)(\mu\alpha_s + \lambda\kappa) - \lambda\kappa^+)}' \]  

(C.30)

Note that \( X_s(\pi)(\mu\alpha_s + \lambda\kappa) \) is increasing in \( \pi \) for any \( \pi < 1 \), as

\[ X_s(\pi)(\mu\alpha_s + \lambda\kappa) = \frac{(\mu\alpha_s + \lambda\kappa)^\top\Sigma^{-1}(\mu\alpha_s + \lambda\kappa)}{(\mu\alpha_s + \lambda\kappa)^\top\Sigma^{-1}(\mu\alpha_s + \lambda\kappa)^\top} \]  

(C.31)

which is the solution of \( \max X(\mu\alpha_s + \lambda\kappa) \) subject to \( X\Sigma^{-1}X^\top = 1 \).

Together with \( V_p > 0 \) and \( \epsilon > 0 \), this implies that \( G \) is increasing in \( \pi \). Further more, we know that at \( G(p,0) > 0 \) because \( \kappa^+ < \alpha_s\lambda^{-1} + \mu^\top\Sigma^{-1}\kappa \) holds, and \( G(p,1) < 1 \). Since both the \( G \) and the \( \pi \) are continuous functions, the intermediate value theorem implies that they cross at least once. This proves that it exists an incentive function consistent with equilibrium and as long as \( V_p > 0 \) it will imply \( \pi(p,S) > 0 \).

To prove that incentives are decreasing in reputation note that the \( \pi \) crosses \( G \) from below, so that \( G_{\pi} < 1 \). I now use the implicit function theorem to compute the local behavior of \( \pi \) with respect to \( p \).
\[ \pi_p(p,s) = -\frac{G_p}{1 - G_{\pi}} \]  

Since \( G_{\pi} < 1 \), the denominator is positive so it is enough to characterize the sign of the numerator, which can be written as,

\[-G_p = V_{pp}(p,S) \left( \chi_S(\pi(p,s))(\mu s + \lambda \kappa) - \lambda \kappa^+ \right) \varepsilon,\]

which is negative if \( V_{pp} < 0 \), that is if the skilled expert value function is concave.

In case (ii) reputation growth is zero, so it is trivially true that learning is slower than in the transparent case. In case (i), we have that

\[ E[d\pi|s] = \frac{1}{2} (\chi_S(\pi(p,s))(\mu s + \lambda \kappa) - \lambda \kappa^+)^2, \]

which also must be lower than \( \frac{1}{2} \alpha_S^2 \), the reputation growth in the transparent case. To see this note that \( \chi_S(\pi)(\mu s + \lambda \kappa) \) is increasing in \( \pi \) for any \( \pi < 1 \). Note further that at \( \pi = 1 \), we have \( \chi_S(1)(\mu s + \lambda \kappa) < \alpha_S + \lambda \kappa^+ \). That is, the portfolio that maximizes short-term performance neither maximizes expected returns or the tail exposure. It follows that \( \chi_S(\pi(p,s))(\mu s + \lambda \kappa) - \lambda \kappa^+ < \alpha_S \), what proves that reputation growth is slower than the transparent benchmark in case (i) as well.

**Proposition 9.**

**Proof.** We have from the proof of Proposition 7.1 that \( \frac{\delta(p,\pi)\lambda}{\alpha_S \chi_S(\pi) \mu} \in [0,1] \). Equation (16) implies that if \( \frac{1}{\lambda \varphi^2} < 1 \), then it must be the case that

\[ \mathbb{E}[dp|\theta = S] \leq \frac{\alpha_S^2}{2}, \]

which is the expected reputation growth in the equilibrium with transparent portfolios. This proves the result.

**D Numerical Solution**

I apply the finite-difference method to solve the integro partial differential equation (6). To solve for optimal policies I sequentially iterate until the value functions converges. The pair of value functions \( \{V(p,S), V(p,O)\} \), and incentive functions \( \{\pi(p,S), \pi(p,O)\} \) are
determined jointly. The state space consists of manager reputation in log-likelihood space \((p \in \mathbb{R}^+\)).

I first discretize the state space as follows. I construct a grid with limits \(\{0, \bar{p}\}\) and \(N\) grid points. Let \(z = (\bar{p} + 1)^{1/N}\), I populate the grid by setting \(p(j) = z^j - 1\). Using this grid I discretize Equation (6) using central differences as described in Candler (1998).

First I hold constant the incentive distortion at zero, \(\pi(p, \theta) = 0\), and iterate to find the solution to the transparent portfolio problem \(\{V(p, S), V(p, O)\}^{tr}\). The transparent liquidation threshold is a lower bound to the equilibrium liquidation threshold. Starting from the transparent liquidation policy and value function, I iterate on equation (6), and each time I solve for the pair of incentive functions at each step.

More specifically, the iteration procedure can be divided in steps.

1. Given incentives \([\pi(p, S)]^{i-1}\) and solve for \([X(P, \theta)]^i, [A(p)]^i, [f(p)]^i, [P]^i\), using Propositions 6, 7, 3, and 1.

2. Solve for \([V(p, S), V(p, O)]^i\) using the discretization of Equation (6) and the boundary condition \(V(0, \theta) = 0\).

3. Given \([V(p, S), V(p, O)]^i\) solve for \([\pi(p, S)]^i\) using Proposition 8 and Definition 2 (see discussion below).

4. If \(|(V(p, S))^i - (V(p, S))^{i-1}| < \epsilon\) and \(|(V(p, O))^i - (V(p, O))^{i-1}| < \epsilon\). If not satisfied, repeat.

The procedure converges extremely fast and takes no more than 5 iterations for a typical solution. The typical solution time is 30 seconds. Code is available upon request.

**D.1 Fixed point**

Step 3 consists of finding the fixed point of equation (13) that is consistent with the fastest rate of learning. For each reputation \(p_j\) grid value I construct a grid with limits \(\{\pi, 1\}\) and \(N_\pi\) grid points, where \(\pi\) is a low negative number. I populate the grid using equal spacing across grid points. This produces \(N \times N_\pi \pi_{j,l}\) values. I look for all values of \(\pi_{j,l}\), where the expression

\[\pi_{j,l} - G(p_j, \pi_{j,l})\]

switch sign. Where \(G\) is defined in Proposition 8. This identifies all incentive functions consistent with equilibrium. Specifically for each grid point \(p_j\), there us a set \(\Omega_j\) that
includes all incentives $\pi_{j,l}$ for which $\pi_{j,l} = G(p_j, \pi_{j,l})$. If for a given reputation $p_j$ the set $\Omega_j$ is singleton, this incentive $\pi_{j,l} \in \Omega_j$ defines the equilibrium for reputation $p_j$, that is $\pi_j = \pi$, where $\pi \in \Omega_j$. If there is more than one $\pi_{j,l} \in \Omega_j$, than I compute

$$\pi_j = \arg\max_{\pi \in \Omega_j} \left( \alpha_s \mathcal{X}(\pi) \mu - \lambda \delta(p_j, \pi) \right)^2 + \lambda \left( \frac{\delta(p_j, \pi)}{\phi} \right)^2$$  \hspace{1cm} (D.35)

according to Definition 2 and select the one with the largest learning rate.
Figure 1: Valuation, incentives, and portfolio choice

The top row shows valuations and incentives as a function of reputation. Panels (a) and (b) show valuations as function of reputation in log-likelihood space ($p$), and probability space ($\mathbb{P}(p)$). Panel (c) shows reach for yield incentives as a function of reputation in probability space ($\mathbb{P}(p)$). In the bottom row, Panel (d) shows the weights the MVE (circle) and the tail risk (asterisk) portfolio place on each technology, Panel (e) shows the relationship between expected returns and tail risk across technologies, and Panel (f) shows the equilibrium portfolio weight on the tail risk portfolio. Red lines denote the skilled expert and black lines the opportunistic expert. Continuous lines denote the opaque fund and dashed lines the transparent fund. The vertical bar denotes the threshold at which the opaque fund is liquidated.
Figure 2: Performance and learning

Panels (a), (b), and (c) show equilibrium performance during normal-times, tail-events, and fund expected returns. Panels (d) and (e) show the informativeness of performance during normal times and tail events. Panel (f) shows the expected growth of the skilled intermediary reputation. Red lines denote the skilled expert and black lines the opportunistic expert. Continuous lines denote the opaque fund and dashed lines the transparent fund. The vertical bar denotes the threshold at which the opaque fund is liquidated.
Figure 3: Capital immobility

Panel (a) shows the aggregate capital-a-risk as function of reputation. Panel (b) shows the expected aggregate inflow of capital for an skilled expert as function of the capital gap \( A(P = 1) - A(P_t) \). Panel (c) show the capital invested in three different technologies: with high tail risk (red), average tail risk (green), low tail risk (blue) as function of reputation. Panel (d) shows the rate of inflow in these technologies as a function of the technology capital gap \( X(P = 1, S)A(P = 1) - X(P_t, S)A(P_t) \) Continuous lines denote the opaque fund and dashed lines the transparent fund. The vertical bar denotes the threshold at which the opaque fund is liquidated.

Aggregate

Technology level
Figure 4: Empirical implications

Panels (a) to (c) show the expected aggregate inflow of capital for a skilled expert \( (E_t[dA|S]) \) as a function of the capital gap \( (A(P = 1) - A(P_t)) \) for alternative parameters. Panel (a) varies \( \kappa^+ \) – the maximum tail exposure feasible in the opportunity set; Panel (b) varies the tail event probability \( \lambda \); and Panel (c) the cross-sectional relationship between tail risk and expected returns. Panel (d) shows the portfolio weight of the skilled intermediary \( w_{\kappa}(P, S) \) on the tail risk portfolio for different levels of the risk-free interest rate.