What do we do?

1. Volatility managed portfolios: scale aggregate priced factor by $1/\sigma_t^2$

2. Apply to many asset pricing factors

3. Motivation: risky asset demand

$$w_t = \frac{1}{\gamma} \frac{E_t(R_{t+1})}{\text{Var}_t(R_{t+1})}$$

4. Volatility doesn’t forecast returns $\Rightarrow$ volatility timing beneficial

Moreira and Muir (2015)
What do we find?

Volatility managed portfolios

1. increase Sharpe ratios, generate large alpha on original factors

2. take less risk in recessions when $\sigma$ high

3. sells after market crashes (1929, 1987, 2008), selling typically viewed as mistake
Outline

1. Vol managed portfolios empirically

2. Who should volatility time?
   - Large utility benefits for mean variance investor
   - Long horizon investors

3. General equilibrium
   - Price of risk negatively related to vol, contrary to standard theories
Data

- Factors: Market, SMB, HML, Momentum, Profitability, ROE, Investment, Carry (FX)

- Daily and monthly data for each factor

- Sample: 1926-2015 (Mkt, SMB, HML, Momentum), Post 1960 for the rest

- Also include unconditional mean-variance efficient portfolio (MVE) from factors

- All numbers annualized
Managed volatility factors

1. Let $f_{t+1}$ be an excess return, construct

$$f_{t+1}^\sigma = \frac{c}{\sigma_t^2} \times f_{t+1}$$

- $\sigma_t$ previous month realized volatility
- choose $c$ so $f^\sigma$ has same unconditional variance as $f$

2. Regression:

$$f_{t+1}^\sigma = \alpha + \beta f_{t+1} + \epsilon_{t+1}$$

- Formally show: $\alpha$ large when vol doesn’t forecast returns
## Volatility managed factors: alphas

<table>
<thead>
<tr>
<th></th>
<th>(1) Mkt$\sigma$</th>
<th>(2) SMB$\sigma$</th>
<th>(3) HML$\sigma$</th>
<th>(4) Mom$\sigma$</th>
<th>(5) RMW$\sigma$</th>
<th>(6) CMA$\sigma$</th>
<th>(7) MVE$\sigma$</th>
<th>(8) FX$\sigma$</th>
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<th>(10) IA$\sigma$</th>
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<td>MktRF</td>
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<td>$\alpha$</td>
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<td>1,065</td>
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<td>16.58</td>
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</table>

*Moreira and Muir (2015)*
Volatility managed factors

How much do we increase Sharpe ratio / expand MVE frontier?

\[ \frac{\alpha}{\sigma_{e}} \]

- MKT (0.34), HML (0.20), MOM (0.88), Profitability (0.41), Carry (0.44), ROE (0.80), Investment (0.32)

- average increase of 75% in Sharpe ratios
MVE portfolios

\[ w_t = \frac{1}{\gamma} \frac{E_t(R_{t+1})}{\text{Var}_t(R_{t+1})} \]

Agent wants to vol time the MVE portfolio, but based on which factors?

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<tr>
<th></th>
<th>(1) Mkt</th>
<th>(2) FF3</th>
<th>(3) FF3,Mom</th>
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<td>(1.56)</td>
<td>(1.00)</td>
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<td>1.20</td>
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Moreira and Muir (2015)
Robustness of result

1. Look at 20 OECD indices, study many factors in US

2. Transactions costs / leverage
   - Survives transactions costs
   - No leverage constraint: results survive, also cuts down turnover

3. Other moments besides mean and variance?
   - Generally, 10th and 1st percentiles of vol managed returns are above those for unconditional returns, look at skewness kurtosis also

4. *Stronger* results when using expected vol rather than realized vol

5. Multi-factor regressions: include BAB, etc.
Cumulative performance for the market return

Moreira and Muir (2015)
“Why” does this work?

1. Define factor price of risk

\[ \gamma_t = \frac{E_t[R_{t+1}]}{\text{Var}_t(R_{t+1})} \]

2. It follows that

\[ \alpha \approx -\text{cov}(\gamma_t, \sigma^2_t) E[\gamma_t] \gamma - (\gamma - E[\gamma_t] + \gamma E[\sigma^2_t] \text{var}(\gamma_t)) \]

more negative cov. between variance and price of risk, larger alpha

3. Regression of future returns on variance recovers \( \text{cov}(\gamma_t, \sigma^2_t) \)
“Why” does this work?

Price of risk low when volatility is high

$$\text{cov}(\gamma_t, \sigma^2_t)$$

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<td>HML</td>
<td>-0.27</td>
<td>0.13</td>
<td>[-0.52, -0.01]</td>
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<td>Mom</td>
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<td>[-1.89, -1.12]</td>
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<td>MVE</td>
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<td>RMW</td>
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<td>0.08</td>
<td>[-0.36, -0.05]</td>
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<tr>
<td>CMA</td>
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<td>[-0.19, 0.07]</td>
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<td>Carry</td>
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<td>0.07</td>
<td>[-0.33, -0.06]</td>
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</table>

also hold at the quarterly / annual frequency

Moreira and Muir (2015)
Vol managed portfolios take less risk in recessions

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<td>0.83 (0.08)</td>
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<td>MktRF $\times 1_{\text{rec}}$</td>
<td>-0.51 (0.10)</td>
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<td>HML $\times 1_{\text{rec}}$</td>
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<td>-0.43 (0.11)</td>
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<td>Mom</td>
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<td>0.74 (0.06)</td>
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<td>Mom $\times 1_{\text{rec}}$</td>
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<td>-0.53 (0.08)</td>
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<td>RMW $\times 1_{\text{rec}}$</td>
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<td>-0.39 (0.08)</td>
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</table>

Observations | 1,065 | 1,065 | 1,060 | 621 | 621 | 362 | 575 | 575 |
R-squared     | 0.43  | 0.37  | 0.29  | 0.38 | 0.49 | 0.51 | 0.43 | 0.49 |

Moreira and Muir (2015)
The dynamics of the risk return tradeoff

Study response to vol shock

\[ w_t = \frac{1}{\gamma} \frac{E_t[R_{t+1}]}{Var_t[R_{t+1}]} \]

Vector Auto Regression with \( E_t[R_{t+1}] \) and \( Var_t[R_{t+1}] \)

- Expected returns formed using CAPE and BaaAaa spread
- Expected variance formed using 3 lags of variance in logs

How much should portfolio weight respond?

Moreira and Muir (2015)
Response to 1 std dev variance shock (in months)
Conventional wisdom: don’t panic

Advice After Stock Market Drop: Take Some Deep Breaths, and Don’t Do a Thing

“If you decide to put a bunch of money in cash...how will you know when to get back in?”

Vanguard (8/15): *What to do during market volatility? Perhaps nothing*

- Vol was at 30-40% in August

Oct/Nov 2008 op-eds Buffett (NYT), Cochrane (WSJ): buy!

- Vol was 60-80%
Our findings

Data: panic driven selling can be beneficial

- NYT: “If you decide to put a bunch of money in cash...how will you know when to get back in?”

*Our answer: get back in when volatility returns to normal* (on average 18 months)

- E.g., October 2008: reduce exposure by 90%

- Performance holds up in 1930, 1987, and 2008
Portfolio choice

1. Utility benefits for mean-variance investors

2. Portfolio choice for long-horizon investors
Utility benefits (for a mean-variance investor)

1. Expected return timing (Campbell and Thompson)

\[ \Delta U(\%) \approx 35\% \]

- Instability of predictability an issue (Goyal and Welch)

2. Volatility timing

\[ \Delta U(\%) \approx 75\% \]

- Out of sample vs in sample irrelevant
- Works for many factors, beyond the aggregate market
Investor Horizon

Many investors have *long* horizon. Should they time volatility?

- Know: long horizon can deviate from myopic / mean variance when returns not iid

- “Hedging demand” from transitory / mean reverting shocks ("discount rate" shocks)
Horizon effects: intuition

Suppose returns were very strongly mean reverting.
Horizon effects: intuition

Increase in “discount-rate” vol scarier for short-horizon investor
Horizon effects in practice

**Empirically**: mean reversion of transitory shocks very *slow*

1. Price dividend ratios highly persistent (Campbell and Shiller)
2. Sharpe ratios increase slowly with horizon (Poterba and Summers)

Returns also driven by permanent “cash flow” shocks (Campbell Shiller)
Optimal behavior of long horizon investor

1. CRRA investor, vary horizon
   - Paper: generalize to Epstein Zin

2. Calibrate process for returns, expected returns, volatility, etc

3. Consider cases where volatility is driven by: (1) discount rate vol, (2) cash flow vol, (c) constant mix of both
Portfolio choice: optimal portfolio

\[
\text{portfolio} = \text{Myopic} + \text{hedging demand}
\]

\[
w_t^* = \frac{1}{\gamma} \frac{\mu_t - r}{\sigma^2_{R,t}} + \frac{V_x}{\gamma} \frac{\kappa_x}{\gamma} DR_{\text{share},t}
\]

\[
DR_{\text{share},t} = \sigma_t(\text{discount rate shocks})/\sigma_{R,t}
\]

Moreira and Muir (2015)
Portfolio choice: optimal portfolio

**Numerical result:** optimal portfolio well approximated by

\[ w_t^* | \sigma_t^2 \approx a + b \frac{\mu_t}{\gamma} \times \frac{1}{\sigma_t^2} \]

- Weights, \( a \) and \( b \), describe optimal portfolio
- extent of volatility timing described by \( b \)
- Mean-variance or log: \( b=1 \) and \( a=0 \).
- Long horizon: \( b \) depends on driver of volatility (\( \frac{\partial DR_{share}}{\partial \sigma^2} \geq 0 \))
Optimal policy $b$ by horizon for CRRA w/ $\gamma = 10$

Moreira and Muir (2015)
General equilibrium models

$$E_t[R_{t+1}] \approx \gamma_t \sigma_t^2$$

1. **Models:** $\text{cov}(\gamma_t, \sigma_t^2) \geq 0$

   (habits, long run risk, rare disasters, intermediary models)

2. **Data:** $\text{cov}(\gamma_t, \sigma_t^2) < 0$ for all factors and MVE combinations of factors

   • Risk aversion / price of risk low in high vol states?
   
   • Paper: show how to estimate $\text{cov}(\gamma_t, \sigma_t^2)$, show high $\alpha \Rightarrow \text{cov} < 0$

*Moreira and Muir (2015)*
Risk aversion (price of risk) negatively correlated with vol

\[
\text{cov}(\gamma_t, \sigma_t^2)
\]

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also holds at the quarterly /annual frequency
Conclusion

Risky asset demand: \( w_t = E_t[R_{t+1}]/\sigma_t^2 \)

1. Vol managed portfolios across many factors
   - large \( \alpha \)'s
   - Sharpe ratios increase \( \approx 75\% \)
   - Take less risk in recessions and after market crashes

2. Portfolio choice: reduce exposure when volatility is high
   - Increase utility \( \approx 75\% \)
   - Long horizon still vol time (less for transitory vs permanent shocks)

3. General equilibrium puzzle: price of risk low when vol is high
Portfolio choice: stochastic environment

Returns: 
\[ dR_t = (r + x_t)dt + \sqrt{y_t}D_R dB_t + F_R dZ_t \]  \hspace{1cm} (1)

Expected returns: 
\[ dx_t = \kappa_x (\mu_x - x_t)dt + \sqrt{y_t}D_x dB_t + F_x dZ_t \]  \hspace{1cm} (2)

Variance: 
\[ dy_t = \kappa_y (\mu_y - y_t)dt + \sqrt{y_t}D_y dB_t \]  \hspace{1cm} (3)

- \( dB_t \) and \( dZ_t \) are 3 by 1 independent Brownian motions

- \( D_R, D_x, F_R, F_x \) consistent with no long run effect of \( dx_t \) shock

- Calibrate model to match the aggregate market factor

(1) one year return \( R^2 \), (2) persistence of expected return shocks, (3) volatility, (4) volatility of volatility, (5) persistence of volatility, (6) co-variance between volatility and future returns, (7) co-variance between volatility shocks and realized returns, and (8) unconditional Sharpe ratio.
Portfolio choice: preferences

Duffie-Epstein (Epstein Zin) utility

\[
J_t = E_t \left[ \int_t^\infty f(C_s, J_s) ds \right]
\]

\[
f(C, J) = \rho \frac{1 - \gamma}{1 - \frac{1}{\psi}} J \times \left[ \left( \frac{C}{((1 - \gamma)J)^{\frac{1}{1-\gamma}}} \right)^{1 - \frac{1}{\psi}} - 1 \right]
\]

- Value function has the usual form: \( J(W, x, y) = \frac{W^{1-\gamma}}{1-\gamma} e^{V(x,y)} \)
Portfolio choice: preferences

1. Use $\rho$ to capture variation in investment horizon
   - Perpetual youth model (Blanchard, Garleanu and Panageas)
   - $\rho$ consistent with half-life ranging from 5 to 30 years.

2. Risk-aversion: $\gamma \in \{5, 10\}$

3. IES: $\psi \in \{1/\gamma, 0.5, 1, 1.5\}$
Portfolio choice: optimal portfolio

Volatility mix view:

portfolio = Myopic + expect. return hedging dem. + vol. hedging dem.

\[ w_t^* = \frac{1}{\gamma} \frac{E_t[dR_t] - r}{\text{Var}_t(dR_t)} + \frac{V_x \kappa_x}{\gamma} DR_{share}(y_t) + \frac{V_y}{\gamma} \beta_{dy_t, dR_t} \]

\( DR_{share}(y_t) \): share of return vol driven by discount rate shocks

\[ DR_{share}(y_t) = \frac{D'_R D_x \times y_t F'_R F_x}{\kappa_x (D'_R D_R \times y_t F'_R F_R)} \]

- \ \frac{\partial DR_{share}(y)}{\partial y} > 0: volatility driven by discount-rate volatility

- \ \frac{\partial DR_{share}(y)}{\partial y} < 0: volatility driven by cash-flow volatility